State of Control Theory knowledge quiz

Controls III is the fourth and final course that the ECE Department offers in control theory. The previous courses are in 1) classical control, 2) modern control and 3) digital control.

1) Your name: SOLUTION

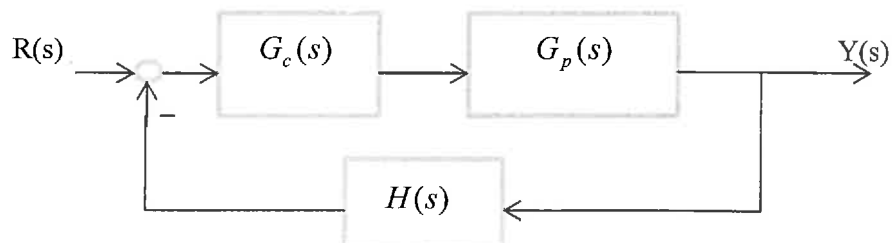
2) On a scale of 1 to 10 (10 being best) how would you rate your knowledge of classical control theory? ____

3) On a scale of 1 to 10 (10 being best) how would you rate your knowledge of modern control theory? ____

4) On a scale of 1 to 10 (10 being best) how would you rate your knowledge of digital control theory? _____

For the questions below, where possible, show your work/give reasons for your answers.

5) Consider the following standard feedback system, where G_p represents the transfer function of the plant.



a) What is the purpose of the gain block $G_c(s)$? FREQUENCY COMPENSATION OF THE LOOP

b) What is the purpose of the gain block $H(s)$? SETS THE CLOSED LOOP GAIN

c) With $G_p(s) = \dfrac{100}{s+5}$, $G_c(s) = \dfrac{A}{s}$ (where is A parameter yet to be determined) and $H(s) = \dfrac{1}{4}$, what is the gain $\dfrac{Y(s)}{R(s)}$?

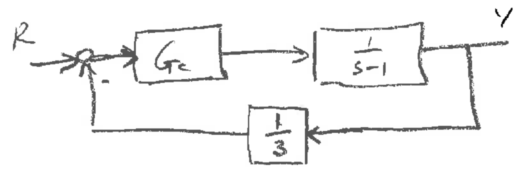
$$\frac{Y}{R} = \frac{G_c G_p}{1 + G_c G_p H} = \frac{\frac{100}{s+5}\frac{A}{s}}{1 + \frac{100}{s+5}\frac{A}{s}\frac{1}{4}} = \frac{400A}{4s^2 + 20s + 100A}$$

d) Find the range of parameter A for which the system is stable? $A > 0$

e) Find the range of parameter A for which the system has zero steady state error to a step input? $A \neq 0$ FOR ZERO SSE — $A > 0$

$A > 0$ FOR STABILITY

6) Consider the (unstable) plant with transfer function $G_p(s) = \dfrac{1}{s-1}$; design a closed loop (feedback) system which produces a low frequency (DC) gain of 3, to within a 10% tolerance. Be sure to sketch the complete block diagram of your system clearly labeling each block of your design.



$$\frac{Y}{R} = \frac{G_c \frac{1}{s-1}}{1 + G_c \frac{1}{s-1} \frac{1}{3}}$$

TRY PROPORTIONAL COMPENSATOR

$\Rightarrow G_c = K$

$$\Rightarrow \frac{Y}{R} = \frac{K \frac{1}{s-1}}{1 + K \frac{1}{s-1} \frac{1}{3}} = \frac{3K}{3s-3+K}$$

FOR STABILITI REQUIRE

$K - 3 > 0 \Rightarrow K > 3$

LOW FREQUENCY GAIN let $s = j\omega = j(0) = 0$

$$= \frac{3K}{K-3} = \frac{3}{1-\frac{3}{K}}$$

10% TOLERANCE $\Rightarrow$ $3 \pm 0.3 = [2.7, 3.3]$

$\Rightarrow$ 1) $\dfrac{3}{1-\frac{3}{K}} = 2.7 \Rightarrow 3 = 2.7 - \dfrac{3 \times 2.7}{K}$

$\Rightarrow K = -\dfrac{3 \times 2.7}{0.3} = -27$

2) $\dfrac{3}{1-\frac{3}{K}} = 3.3 \Rightarrow 3 = 3.3 - \dfrac{3 \times 3.3}{K}$

$\Rightarrow K = \dfrac{3 \times 3.3}{+0.3} = 33$

$\Rightarrow$ $\underline{K > 33}$ to guarantee tolerance

TRY AN INTEGRAL COMPENSATOR

$\Rightarrow G_c = \dfrac{K}{s}$

$$\Rightarrow \frac{Y}{R} = \frac{\frac{K}{s} \frac{1}{s-1}}{1 + \frac{K}{s} \frac{1}{s-1} \frac{1}{3}} = \frac{3K}{3s^2 - 3s + K}$$

$\Rightarrow$ $\underline{\text{CAN'T BE STABILIZED}}$

7) The following feedback system, where G_p represents the plant and has state equations:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

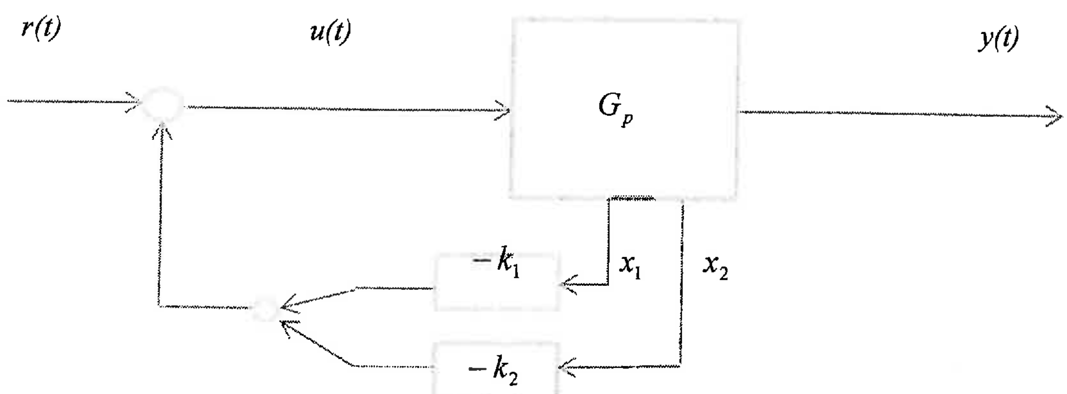
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{Y}{U} = C(sI-A)^{-1}B = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix}}{s^2}\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

i) Find the transfer function $\frac{Y(s)}{U(s)}$.

$$= \frac{\begin{bmatrix} 1 & 0 \end{bmatrix}}{s^2}\begin{bmatrix} 1 \\ s \end{bmatrix} = \underline{\underline{\frac{1}{s^2}}}$$

ii) Find the values of k_1 and k_2 so that $\frac{Y(s)}{R(s)} = \frac{A}{(s+1)(s+2)}$, where A is some constant.



char. eqn =

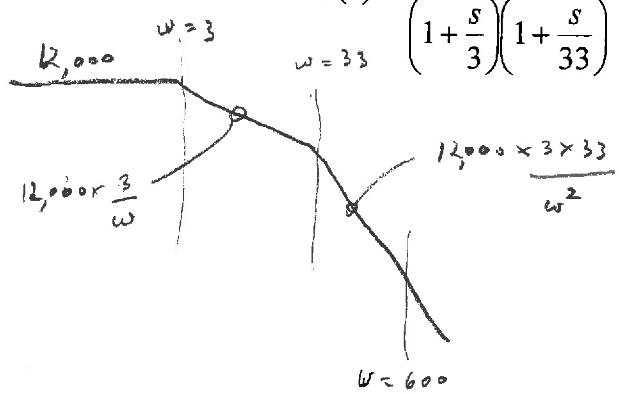
$$|sI - A + BK| = \left|\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix}\right| = \left|\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}\right| = \begin{vmatrix} s & -1 \\ k_1 & s+k_2 \end{vmatrix} = s^2 + k_2 s + k_1$$

$$\frac{Y}{R} = \frac{A}{s^2+3s+2} \Rightarrow \underline{k_1 = 2} \text{ and } \underline{k_2 = 3}$$

8) Find the approximate (absolute) magnitude at a frequency of $\omega = 600 rds/s$ of the following

transfer function: $G(s) = \frac{12{,}000}{\left(1+\frac{s}{3}\right)\left(1+\frac{s}{33}\right)}$

$|G(j\omega)| =$



$$@\ \omega = 600 \quad |G(j\omega)| \approx \frac{12{,}000 \times 3 \times 33}{600 \times 600} = \frac{36{,}000 \times 33}{36{,}000 \times 10} = \underline{\underline{3.3}}$$