ECE452/552 Control Systems Design II

Notes from *True Digital Control: Statistical Modelling and Non-Minimal State Space Design* by C.James Taylor et al.

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1 Introduction

2 Discrete-Time Transfer Function

2.1 Discrete-Time TF Model

- $y(k)$: controlled output variable (see **Figure 2.1**)
- $u(k)$: control input variable (see **Figure 2.1)**
- k: sample number
- K: scalar gain element or multiplier

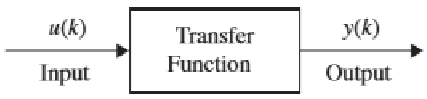

Figure 2.1 *Transfer Function* (TF) in block diagram form

2.1.1 The Backward Shift Operator

z^{-1} in **Equation 1** is called backward shift operator.

$$z^{-i}y(k) = y(k-i) \tag{1}$$

The inverse of **Equation 1** is $z^i y(k) = y(k+1)$ where z^i is the forward shift operator. $z^i y(k) = y(k+1)$ takes place of the differential operator, $s^i(t) = d^i y(t)/dt^i$, which is commonly used in continuous-time TF model. As depicted in **Figure 2.2**, the backward shift operator creates one sample delay between input and output.

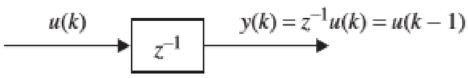

Figure 2.2 The backward shift operator

Example 2.1 Transfer Function Representation of a First Order System

Difference Equation:

$$y(k) + a_1 y(k-1) = b_1 u(k-1)$$

$$y(k) + a_1 z^{-1} y(k) = b_1 z^{-1} u(k)$$

TF representation: (The coefficients were arbitrarily chosen.)

$$y(k) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1}} u(k) = \frac{1.6 z^{-1}}{1 + 0.8 z^{-1}} u(k)$$

TF model: See **Figure 2.3**

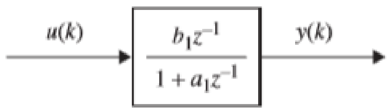

Figure 2.3 Block diagram form of the TF model (2.4)

Unit step response: See **Figure 2.4**

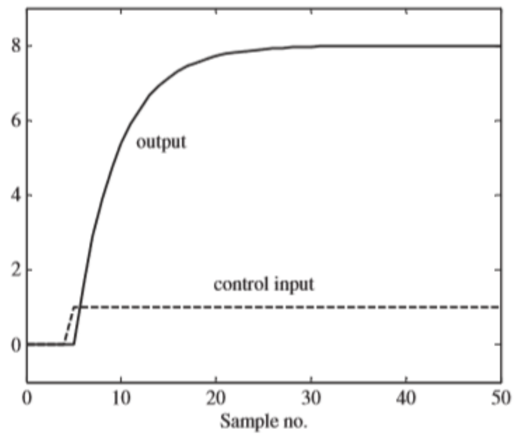

Figure 2.4 Unit step response of the first order TF model in Example 2.1

Note that there is a delay of one sample for the output to respond after the command input goes to one which is expressed in the difference equation; (k-1).

Example 2.2 Transfer Function Representation of a Third Order System

Differencel Equation:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) = b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3)$$

$$y(k) + a_1 z^{-1} y(k) + a_2 z^{-2} y(k) + a_3 z^{-3} y(k) = b_1 z^{-1} u(k) + b_2 z^{-2} u(k) + b_3 z^{-3} u(k)$$

TF representation: (The coefficients were from an exercise in CH.8.)

$$y(k) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} u(k) = \frac{27.4671 z^{-1} + 65.6418 z^{-2} - 91.1006 z^{-3}}{1 - 2.4425 z^{-1} + 2.2794 z^{-2} - 0.8274 z^{-3}} u(k)$$

TF model: See **Figure 2.4**

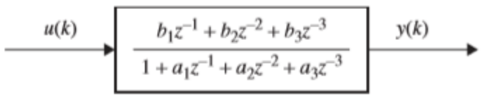

Figure 2.5 Block diagram form of the TF model (2.9)

Unit step response: See **Figure 2.6**

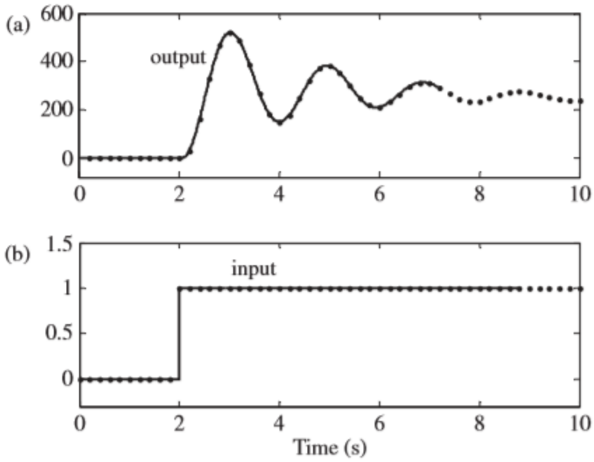

Figure 2.6 Unit step response of the wind turbine system in Example 2.2, comparing the discrete-time TF model (2.10) with the continuous-time simulation (solid trace)

2.1.2 General Discrete-time TF Model

The general difference equation for an nth order model without time delay:

$$y(k) + a_1y(k-1) + ... + a_ny(k-n) = b_0u(k) + b_1u(k-1) + ... + b_mu(k-m)$$

where $a_i(i = 1...n)$ and $b_i(i = 0...m)$ are constant coefficients. The $b_0u(k)$ term disappears since discrete-time control requires at least one sample delay; $\tau = 1$ or larger.

2.1.3 Steady-State Gain

Steady-state is where $y(k) - y(k-1) = 0$. This is equivalent to $y(k)(1 - z^{-1}) = 0$, where z^{-1} must be 1 in order to satisfy the equation. Thus, steady state gain, G of a discrete-time TF can be found by substituting z^{-1} with 1. For example, using the model introduced in Example 2.1,

$$G(z^{-1} = 1) = \frac{1.6z^{-1}}{1 - 0.8z^{-1}} = \frac{1.6}{1 - 0.8} = 8.0 \tag{2}$$

$$y(k \Rightarrow \infty) = Gu = 8.0u \tag{3}$$

Refer to **Figure 2.4**. The output reaches to 8 in the steady state.

2.2 Stability and the Unit Circle

Referring to **Figure 2.7**, for stability, all the poles p_i must have a magnitude less than unity, i.e.

$$|p_i| < 1 \qquad i = 1, 2, .., n$$

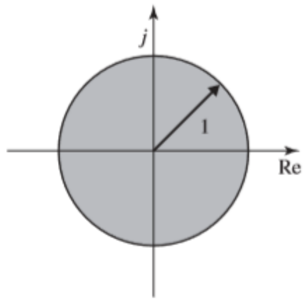

Figure 2.7 Unit circle on the complex z-plane with complex axis j and real axis Re. The shaded area shows the stable region with the magnitude of 1.0 highlighted by the arrow

Example 2.3 Poles.Zeros and Stability

Figure 2.8(a) shows the unit step response of second order TF mode:

$$y(k) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} u(k) = \frac{0.5z^{-1} - 0.4z^{-2}}{1 - 0.8z^{-1} + 0.15z^{-2}} u(k)$$

The denominator $1 - 0.8z^{-1} + 0.15z^{-2}$ multiplied by z^2 gives:

$$z^2 - 0.8z + 0.15 = (z - 0.3)(z - 0.5) = 0$$

The two poles are 0.3 and 0.5, both of which are less than unity. Hence, the system is stable. Now, changing the numerator to the following:

$$y(k) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} u(k) = \frac{-0.2z^{-1} + 0.3z^{-2}}{1 - 0.8z^{-1} + 0.15z^{-2}} u(k)$$

This system has one zero, which is outside the unit circle. The resulting plot is shown in **Figure 2.8(b)**. This type of system where the initial negative response can be seen is called *non-minimum phase*. Lastly, the response of the following system:

$$y(k) = \frac{-1.0z^{-1} + 2.0z^{-2}}{1 - 1.7z^{-1} + z^{-2}} u(k)$$

is shown in **Figure 2.8(c)**. This one is also *non-minimum phase*, and the poles are at one which resulted in *marginally stable* system.

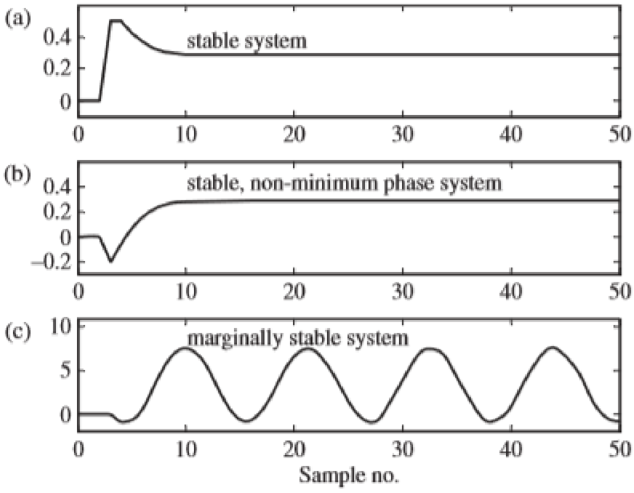

Figure 2.8 Unit step response of (a) stable (2.18), (b) stable non-minimum phase (2.20) and (c) marginally stable (2.21) TF models in Example 2.3

2.3 Block Diagram Analysis

Each of the equations in this section is represented by the diagram shown below it.

$$y(k) = G_1(z^{-1})G_2(z^{-1})u(k) = G_2(z^{-1})G_1(z^{-1})u(k)$$

$$y(k) = G_1(z^{-1})u(k) + G_2(z^{-1})u(k) = (G_1(z^{-1}) + G_2(z^{-1}))u(k)$$

$$y(k) = \frac{G_1(z^{-1})}{1 + G_1(z^{-1})G_2(z^{-1})}u(k)$$

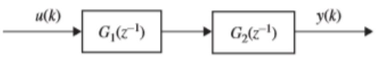

Figure 2.9 Two TF models connected in series

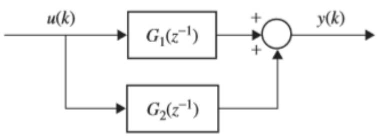

Figure 2.10 Two TF models connected in parallel

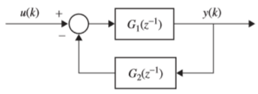

Figure 2.11 Two TF models connected in a negative feedback arrangement

2.4 Discrete-Time Control

Example 2.4 Proportional Control of a First Order TF Model

Control Algorithm:

$$u(k) = k_p(y_d(k) - y(k))$$

where $y_d(k)$ is the command input.

CLTF representation: (The coefficients the same as Example 2.1's.)

$$y(k) = \frac{k_p b_1 z^{-1}}{1 + a_1 z^{-1} + k_p b_1 z^{-1}} y_d(k)$$

Figure 2.12 and **Figure 2.13** are the TF model and response of the proportional control system, respectively. The steady state error can be seen in this system. This is referred to as a *Type 0* control system.

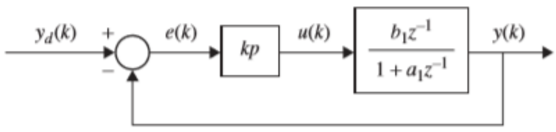

Figure 2.12 Proportional control of a first order TF model

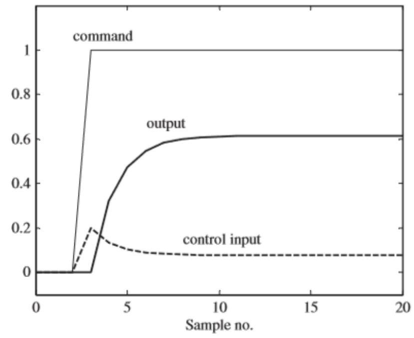

Figure 2.13 Closed-loop unit step response of the proportional control system in Example 2.4

Example 2.5 Integral Control of a First Order TF Model

Control Algorithm:

$$u(k) = \frac{k_I}{1 - z^{-1}}(y_d(k) - y(k))$$

CLTF representation:

$$y(k) = \frac{k_I b_1 z^{-1}}{1 + (k_I b_1 + a_1 - 1)z^{-1} - a_1 z^{-2}} y_d(k)$$

TF model: See **Figure 2.14**

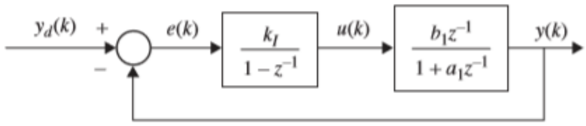

Figure 2.14 Integral control of a first order TF model

This system has unity steady state gain and called *Type 1 servomechanism.*

Example 2.6 Proportional Integral Control of a First Order TF Model

Control Algorithm:

$$u(k) = \frac{k_I}{1 - z^{-1}}(y_d(k) - y(k)) - f_0 y(k)$$

CLTF representation:

$$y(k) = \frac{k_I b_1 z^{-1}}{1 + (f_0 b_1 + a_1 - 1 + k_I b_1)z^{-1} - (a_1 - f_0 b_1)z^{-2}} y_d(k)$$

The transfer function model in **Figure 2.15** was simplified and shown in **Figure 2.16**. This one is also *Type 1 servomechanism.*

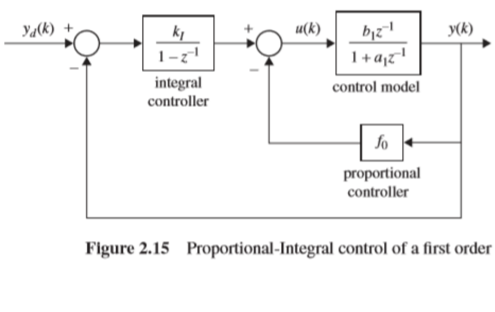

Figure 2.15 Proportional-Integral control of a first order TF model

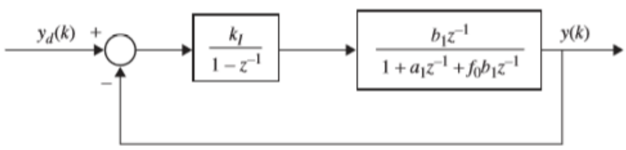

Figure 2.16 Reduced form of the control system in Figure 2.15

Example 2.7 Pole Assignment Design Based on PI Control Structure

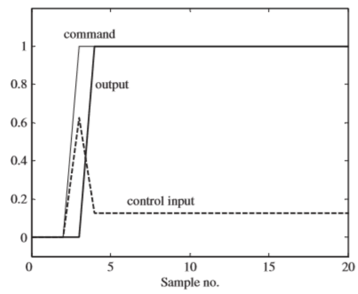

Figure 2.17 Closed-loop unit step response of the deadbeat PI control system using the characteristic equation (2.45) in Example 2.7

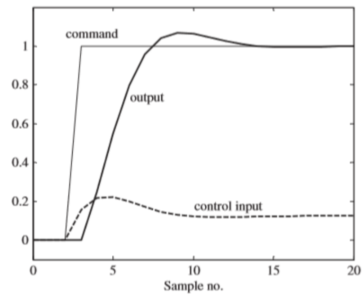

Figure 2.18 Closed-loop unit step response of the PI control system based on conjugate complex poles using the characteristic equation (2.43) in Example 2.7

2.5 Continuous to Discrete-Time TF Model Conversion

Is it important?

3 Minimal State Variable Feedback

Example 3.1 State Space Forms for a Third Order TF Model

$$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \end{bmatrix}$$

$$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(k-1)$$

$$\mathbf{y}(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

$$\mathbf{x}(k) = \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y(k-1) \\ y(k-2) \\ y(k-3) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(k-1)$$

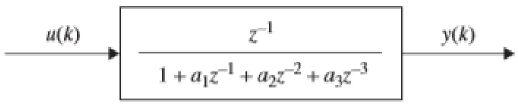

Figure 3.1 Block diagram form of the TF model (3.2)

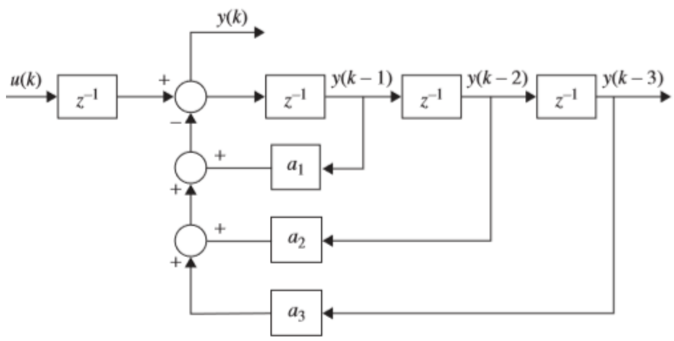

Figure 3.2 State space model described by equations (3.6)

$$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} -a_1x_1(k-1) + x_2(k-1) + u(k-1) \\ -a_2x_1(k-1) + x_3(k-1) \\ -a_3x_1(k-1) \end{bmatrix}$$

$$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(k-1)$$

$$\mathbf{y}(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

3.1 Controllable canonical Form

$$y(k) + a_1y(k-1) + \cdots + a_ny(k-n) = b_1u(k-1) + b_2u(k-2) + \cdots + b^nu(k-n)$$

$$y(k) = \frac{b_1z^{-1} + b_2z^{-2} + \cdots + b_nz^{-n}}{1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_nz^{-n}}u(k) = \frac{\mathbf{B}(z^{-1})}{\mathbf{A}(z^{-1})}u(k)$$

$$y(k) = \frac{b_1z^{-1} + b_2z^{-2} + \cdots + b_nz^{-n+1}z^{-1}}{1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_nz^{-n}}u(k)$$

$$w(k) = \frac{z^{-1}}{1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_nz^{-n}}u(k)$$

$$y(k) = (b_1z^{-1} + b_2z^{-2} + \cdots + b_nz^{-n+1})w(k)$$

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} = \begin{bmatrix} w(k) \\ w(k-1) \\ \vdots \\ w(k-n+2) \\ w(k-n+1) \end{bmatrix}$$

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ \vdots \\ x_{n-1}(k-1) \\ x_n(k-1) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u(k-1)$$

$$y(k) = \begin{bmatrix} b_1 & b_2 & \cdots & b_{n-1} & b_n \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u(k-1)$$

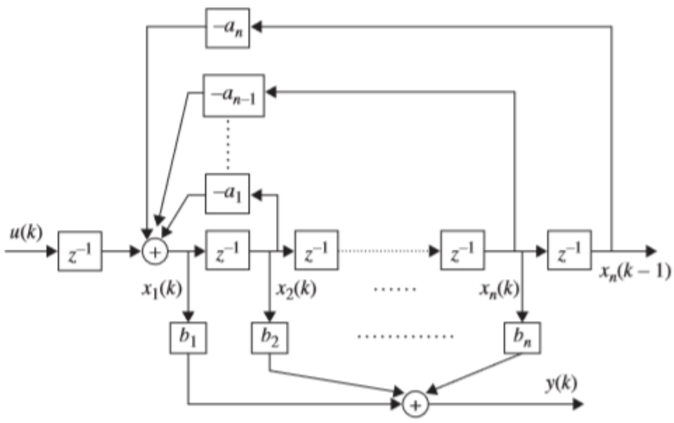

Figure 3.3 Controllable canonical form for the general discrete-time system (3.11)

$$w(k) = \frac{y(k)}{(b_1 + b_2 + \cdots + b_n z^{-n+1})}$$

$$w(k) = \frac{z^{-1}y(k)}{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}} = \frac{y(k-1)}{\mathbf{B}(z^{-1})}$$

Example 3.2 State Variable Feedback based on the Controllable Canonical Form

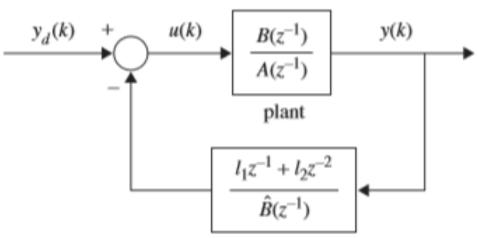

Figure 3.4 The closed-loop control system for Example 3.2 with model mismatch

Example 3.3 State Variable Feedback Pole Assignment based on the Controllable Canonical Form

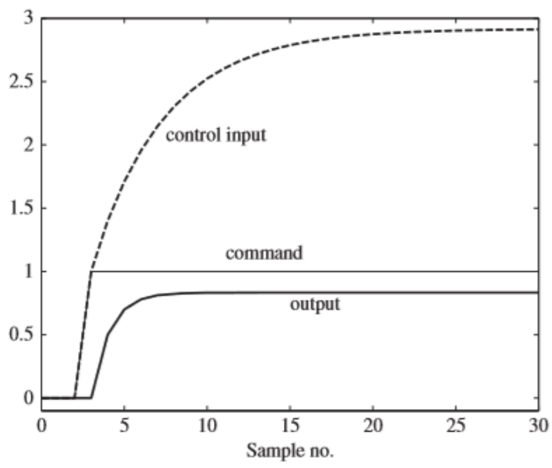

Figure 3.5 Closed-loop unit step response of the SVF control system in Example 3.3

3.1.1 State Variable Feedback for the General TF Model

3.2 Observable Canonical Form

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} = \begin{bmatrix} -a_1x_1(k-1) + x_2(k-1) + b_1u(k-1) \\ -a_2x_1(k-1) + x_3(k-1) + b_2u(k-1) \\ \vdots \\ -a_{n-1}x_1(k-1) + x_n(k-1) + b_{n-1}u(k-1) \\ -a_nx_1(k-1) + b_nu(k-1) \end{bmatrix}$$

$$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ a_n & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ \vdots \\ x_{n-1}(k-1) \\ x_n(k-1) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} u(k-1)$$

$$\mathbf{y}(k) = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix}$$

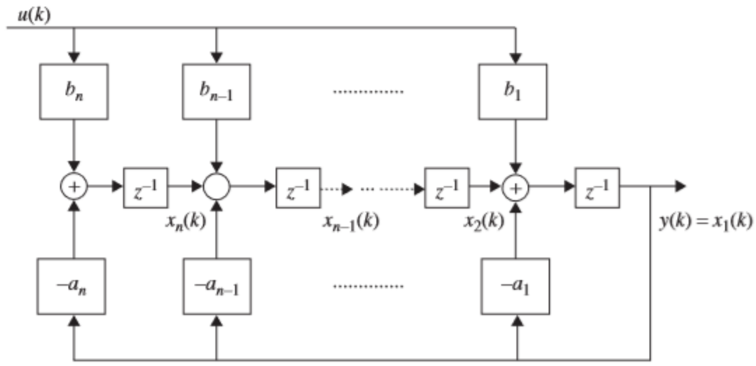

Figure 3.6 Observable canonical form for the general discrete-time system (3.11)

Example 3.4 State Variable Feedback based on the Observable Canonical Form

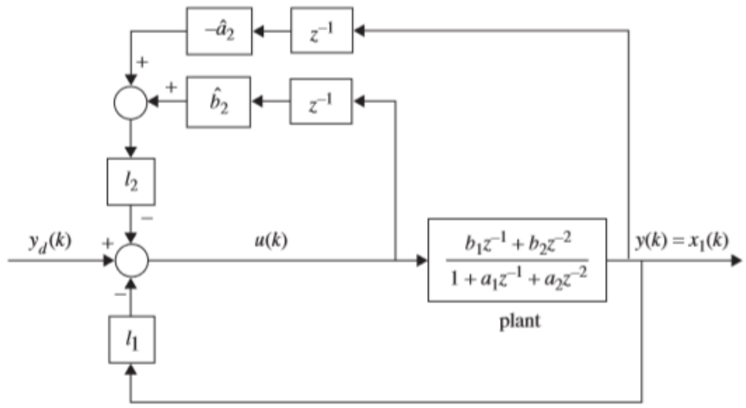

Figure 3.7 The closed-loop control system for Example 3.4 with model mismatch

3.3 General State Space Form

$$\mathbf{x}(k) = \mathbf{F}\mathbf{x}(k-1) + \mathbf{g}u(k-1)$$

$$y(k) = \mathbf{h}\mathbf{x}$$

3.3.1 Transfer Function Form of a State Space Model

$$\mathbf{x}(k) = \mathbf{F}z^{-1}\mathbf{x}(k) + \mathbf{g}z^{-1}u(k)$$

$$(\mathbf{I} - \mathbf{F}z^{-1})\mathbf{x}(k) = \mathbf{g}z^{-1}u(k)$$

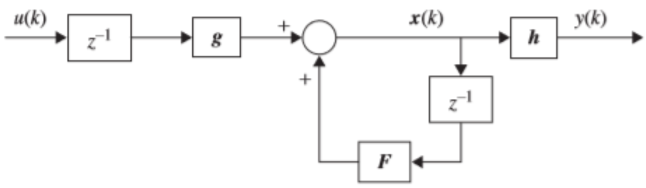

Figure 3.8 Block diagram form for the general state space system description {(3.46), (3.47)}

Example 3.5 Determining the TF from a State Space Model

3.3.2 The Characteristic Equation, Eigenvalues and Eigenvectors

Example 3.6 Eigenvalues and Eigenvectors of a State Space Model

3.3.3 The Diagonal Form of a State Space Model

3.4 Controllability and Observability

Controllability:

$$\mathbf{x(n) - F^n x(0)} = \begin{bmatrix} \mathbf{g} & \mathbf{Fg} & \cdots & \mathbf{F^{n-2}g} & \mathbf{F^{n-1}g} \end{bmatrix} \begin{bmatrix} u(n-1) \\ u(n-2) \\ \vdots \\ u(1) \\ u(0) \end{bmatrix}$$

$$\mathbf{S_1} = \begin{bmatrix} \mathbf{g} & \mathbf{Fg} & \cdots & \mathbf{F^{n-2}g} & \mathbf{F^{n-1}g} \end{bmatrix}$$

Observability:

$$\begin{bmatrix} \mathbf{h} \\ \mathbf{hF} \\ \vdots \\ \mathbf{hF}^{n-2} \\ \mathbf{hF}^{n-1} \end{bmatrix} x(0) = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(n-2) \\ y(n-1) \end{bmatrix}$$

$$\mathbf{S_0} = \begin{bmatrix} \mathbf{h} \\ \mathbf{hF} \\ \vdots \\ \mathbf{hF}^{n-2} \\ \mathbf{hF}^{n-1} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 0.8 & -0.15 & -0.4 \\ 1 & 0 & 0 \\ 0.49 & -0.12 & -0.32 \end{bmatrix}$$

$$\mathbf{g} = \begin{bmatrix} 0.5 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{h} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{S_1} = \begin{bmatrix} \mathbf{g} & \mathbf{Fg} & \mathbf{F^2g} \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & -0.075 \\ 0 & 0.5 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{S_0} = \begin{bmatrix} \mathbf{h} \\ \mathbf{hF} \\ \mathbf{hF^2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.8 & -0.15 & -0.4 \\ 0.49 & -0.12 & -0.32 \end{bmatrix}$$

4 Non-Minimal State Variable Feedback

4.1 The NMSS Form

4.1.1 The NMSS(Regulator) Representation

$$y(k) = \frac{\mathbf{B}(z^{-1})}{\mathbf{A}(z^{-1})} u(k)$$

$$\mathbf{x}(k) = \mathbf{Fx}(k-1) + \mathbf{g}u(k-1)$$

$$y(k) = \mathbf{hx}(k)$$

$$
\mathbf{F} = \begin{bmatrix}
-a_1 & -a_2 & \cdots & -a_{n-1} & -a_n & b_2 & b_3 & \cdots & b_{m-1} & b_m \\
1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \dots & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 & 0 \\
a_1 & a_2 & \dots & a_{n-1} & a_n & -b_2 & -b_3 & \dots & -b_{m-1} & -b_m
\end{bmatrix}
$$

$$
\mathbf{g} = \begin{bmatrix} b_1 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & -b^1 \end{bmatrix}^T
$$

$$
\mathbf{h} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}
$$

$$
\mathbf{x}(k) = \begin{bmatrix} y(k) & y(k-1) & \cdots & y(k-n+1) & u(k-1) & u(k-2) & \cdots & u(k-m+1) \end{bmatrix}^T
$$

4.1.2 The Characteristic Polynomial of the NMSS Model

Example 4.1 Non-Minimal State Space Representation of a Second Order TF Model

4.2 Controllability of the NMSS Model

$$
\mathbf{S_1} = \begin{bmatrix} \mathbf{g} & \mathbf{Fg} & \mathbf{F^2g} & \cdots & \mathbf{F^{n+m-2}g} \end{bmatrix}
$$

4.3 Proportional-Integral-Plus Control

$$
u(k) = -\mathbf{k}^T\mathbf{x}(k) + k_d y_d(k)
$$

$$
\mathbf{k}^T = \begin{bmatrix} f_0 & f_1 & \cdots & f_{n-1} & g_1 & \cdots & g_m - 1 & -k_I \end{bmatrix}
$$

$$
u(k) = -f_0 y(k) - f_1 y(k-1) - \cdots - f_{n-1}(k-n+1) - g_1 u(k-1) - \cdots - g_{m-1} u(k-m+1) - k_I z(k)
$$

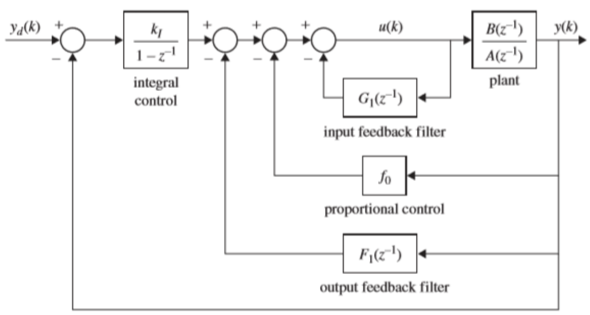

Figure 5.3 Block diagram of the univariate PIP control system explicitly showing the proportional and integral control action

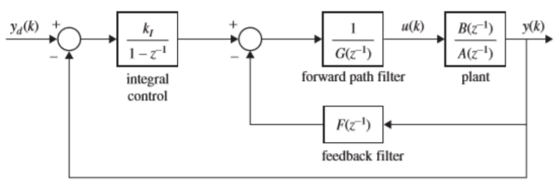

Figure 5.4 Block diagram of the univariate PIP control system in reduced form

$$\mathbf{F}(z^{-1}) = f_0 + f_1 z^{-1} + \cdots + f_{n-1} z^{-n+1}$$

$$\mathbf{G}(z^{-1}) = 1 + g_1 z^{-1} + \cdots + g_{m-1} z^{-m+1}$$

Example 4.2 Ranks Test for the NMSS Model

4.4 The Unity Gain NMSS Regulator

Example 4.3 Regulator Control Law for a NMSS Model with Four State Variables

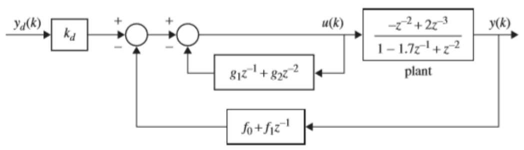

Figure 4.1 NMSS regulator control of Example 4.3 showing an explicit feedback of the input states

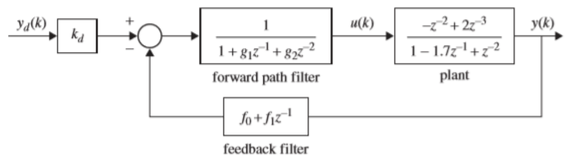

Figure 4.2 Simplified NMSS regulator control of Example 4.3 with a forward path filter

Example 4.4 Pole Assignment for the Fourth Order NMSS Regulator Example 4.5 Unity Gain NMSS Regulator for the Wind Turbine Simulation

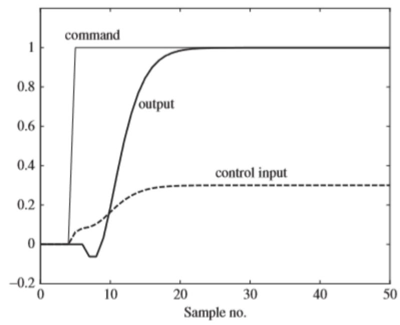

Figure 4.3 Closed-loop unit step response using the unity gain NMSS regulator of Example 4.4

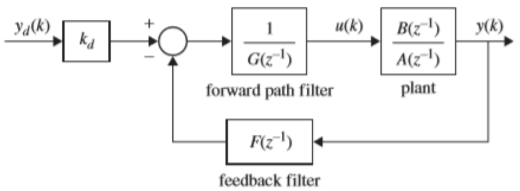

Figure 4.4 Block diagram representation of the unity gain NMSS regulator

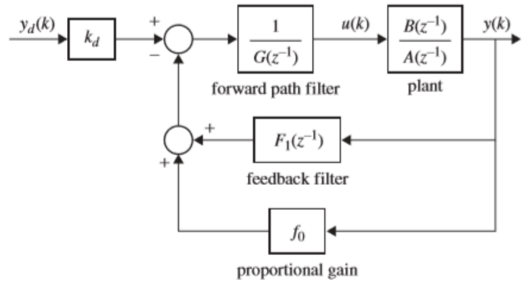

Figure 4.5 Unity gain NMSS regulator with separate proportional gain

Example 4.6 Mismatch and Disturbances for the Fourth Order NMSS Regulator

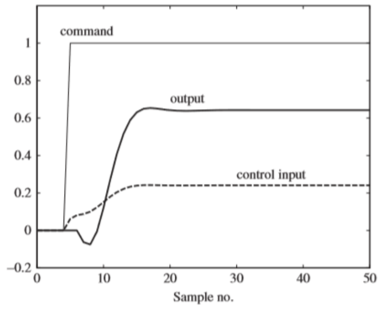

Figure 4.6 Closed-loop unit step response using the unity gain NMSS regulator of Example 4.4 when the model has a 10% error in one parameter

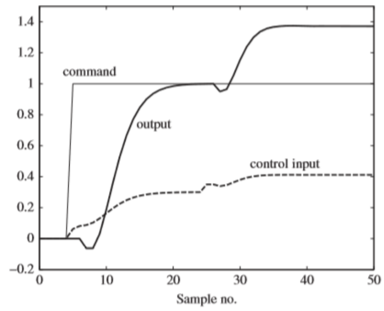

Figure 4.7 Closed-loop unit step response using the unity gain NMSS regulator of Example 4.4, with an input step disturbance of 0.05 at the 25th sample

4.5 Constrained NMSS Control and Transformation

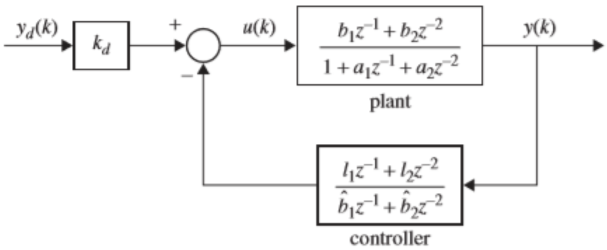

Figure 4.8 Unity gain (minimal) SVF regulator for Example 4.8 based on the controllable canonical form (cf. Figure 3.4)

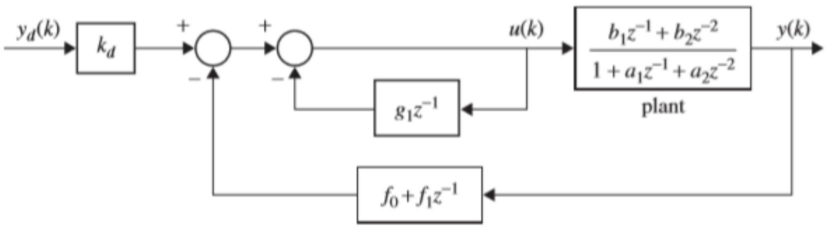

Figure 4.9 Unity gain NMSS regulator for Example 4.8

5 True Digital Control for Univariate Systems

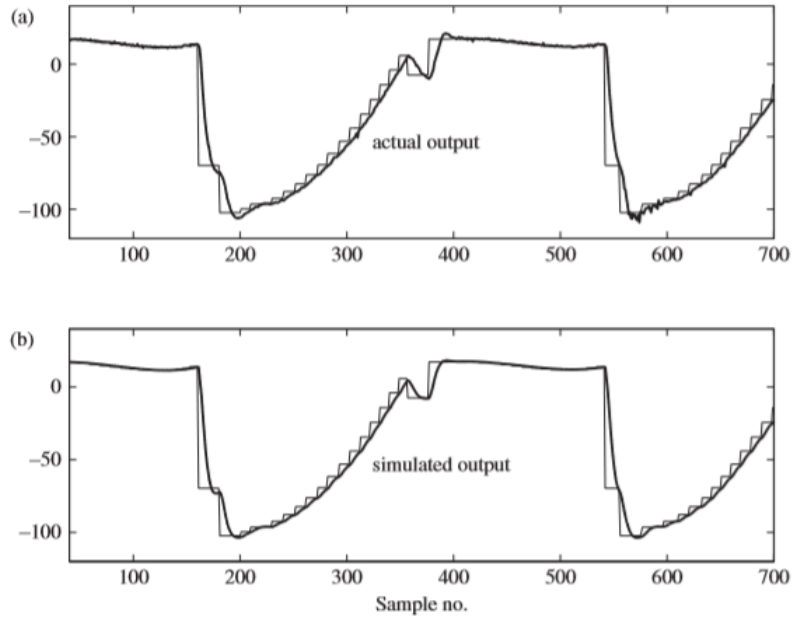

Figure 5.1 Closed-loop response using the PIP controller of Example 5.2, showing the output bucket joint angle in degrees (thick traces) and the time varying command input (thin traces). (a) Experimental data collected from the laboratory excavator and (b) the equivalent simulated response based on the TF model (5.1). The sampling rate is 0.11s

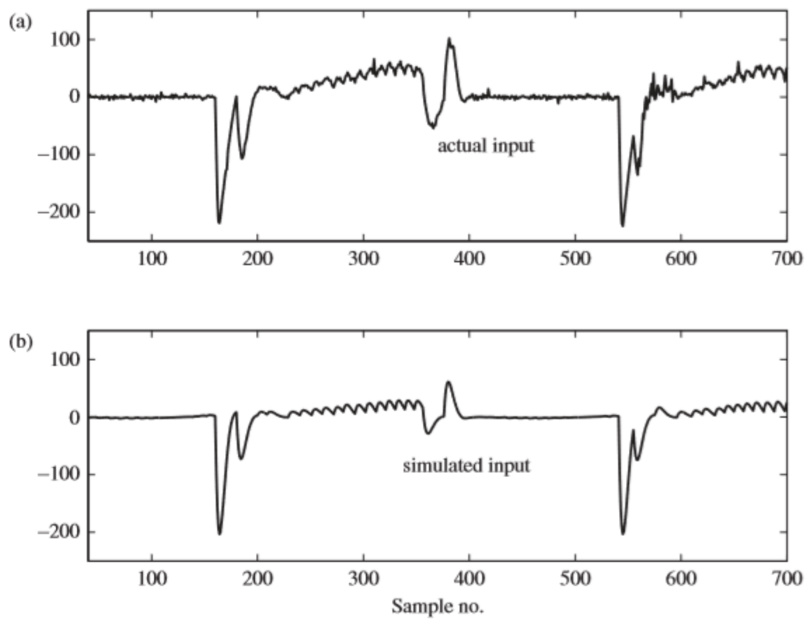

Figure 5.2 Control input signals associated with Figure 5.1, showing the scaled voltage. (a) Experimental data collected from the laboratory excavator and (b) the equivalent simulated response based on the TF model (5.1)

5.1 The NMSS Servomechanism Representation

$$y(k) = \frac{\mathbf{B}(z^{-1})}{\mathbf{A}(z^{-1})}u(k) = \frac{b_1 z^{-1} + \cdots + b_m z^{-1}}{1 + a_1 z^{-1} + \cdots + a_n z^{-1}}u(k)$$

$$\mathbf{x}(k) = \mathbf{F}\mathbf{x}(k-1) + \mathbf{g}u(k-1) + \mathbf{d}y_d(k)$$

$$y(k) = \mathbf{h}\mathbf{x}(k)$$

$$\mathbf{x}(k) = \begin{bmatrix} y(k) & y(k-1) & \cdots & y(k-n+1) & u(k-1) & u(k-2) & \cdots & u(k-m+1) & z(k) \end{bmatrix}^T$$

$$\mathbf{F} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n & b_2 & b_3 & \cdots & b_{m-1} & b_m & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 & 0 & 0 \\ a_1 & a_2 & \dots & a_{n-1} & a_n & -b_2 & -b_3 & \dots & -b_{m-1} & -b_m & 1 \end{bmatrix}$$

$$\mathbf{g} = \begin{bmatrix} b_1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & -b_1 \end{bmatrix}^T$$

$$\mathbf{d} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$\mathbf{h} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\mathbf{S_1} = \begin{bmatrix} \mathbf{g} & \mathbf{Fg} & \mathbf{F^2g} & \cdots & \mathbf{F^{n+m-1}g} \end{bmatrix}$$

5.2 Proportional-Integral-Plus Control

$$\mathbf{k}^T = \begin{bmatrix} f_0 & f_1 & \cdots & f_{n-1} & g_1 & \cdots & g_m - 1 & -k_I \end{bmatrix}$$

$$u(k) = -f_0 y(k) - f_1 y(k-1) - \cdots - f_{n-1}(k-n+1) - g_1 u(k-1) - \cdots - g_{m-1} u(k-m+1) - k_I z(k)$$

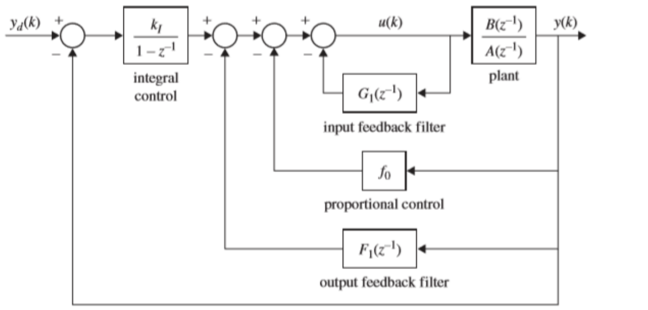

Figure 5.3 Block diagram of the univariate PIP control system explicitly showing the proportional and integral control action

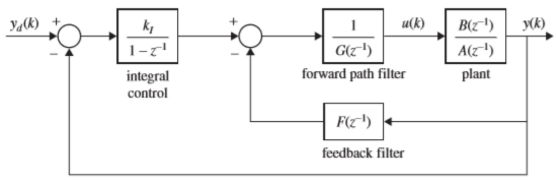

Figure 5.4 Block diagram of the univariate PIP control system in reduced form

$$\mathbf{F}(z^{-1}) = f_0 + f_1 z^{-1} + \cdots + f_{n-1} z^{-n+1}$$

$$\mathbf{G}(z^{-1}) = 1 + g_1 z^{-1} + \cdots + g_{m-1} z^{-m+1}$$

5.2.1 The Closed-Loop Transfer Function

Example 5.5 Proportional-Integral-Plus Control System Design for NMSS Model with Five State Variables

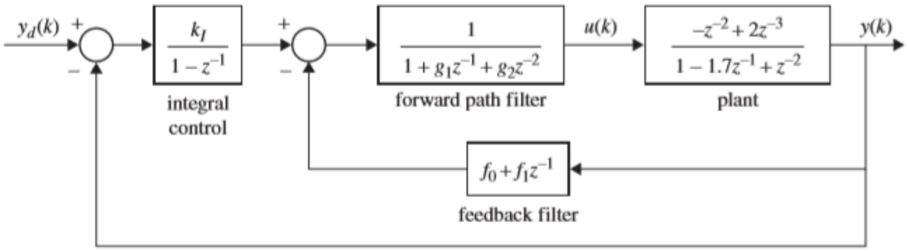

Figure 5.5 PIP control of the non-minimum phase oscillator in Example 5.5

5.3 Pole Assignment for PIP Control

5.3.1 State Space Derivation

Example 5.6 Pole Assignment Design for the NMSS Model with Five State Variables

Example 5.7 Implementation Results for FACE system with Disturbances

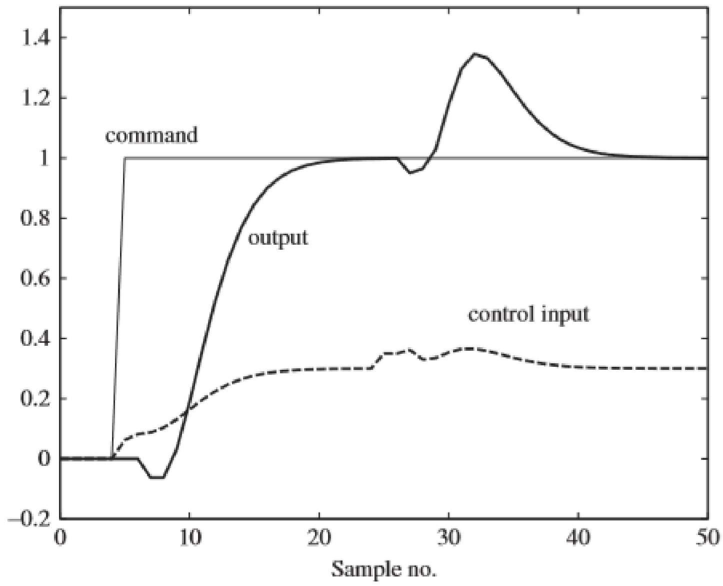

Figure 5.6 Closed-loop unit step response using the PIP controller of Example 5.6, with an input step disturbance of 0.05 at the 25th sample (cf. Figure 4.7 using the unity gain NMSS regulator)

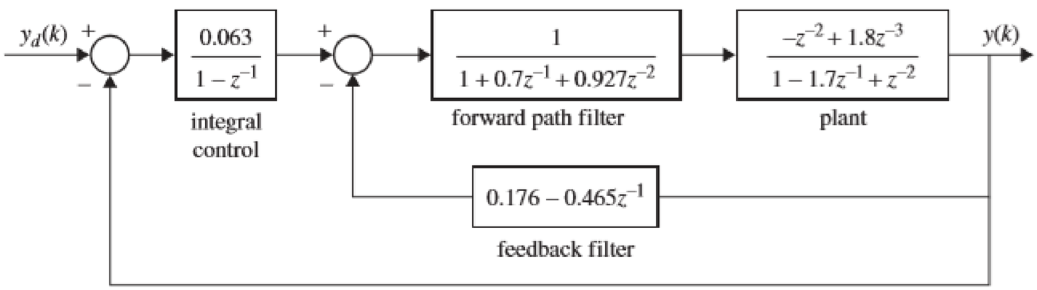

Figure 5.7 PIP control of the non-minimum phase oscillator in Example 5.6 with model mismatch

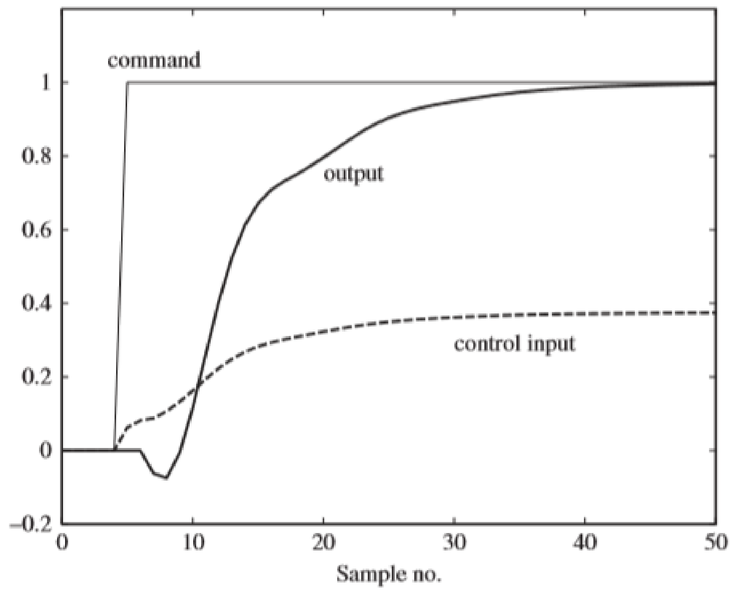

Figure 5.8 Closed-loop unit step response using the PIP controller of Example 5.6, when the model has a 10% error in one of the parameters (cf. Figure 4.6 using the unity gain NMSS regulator)

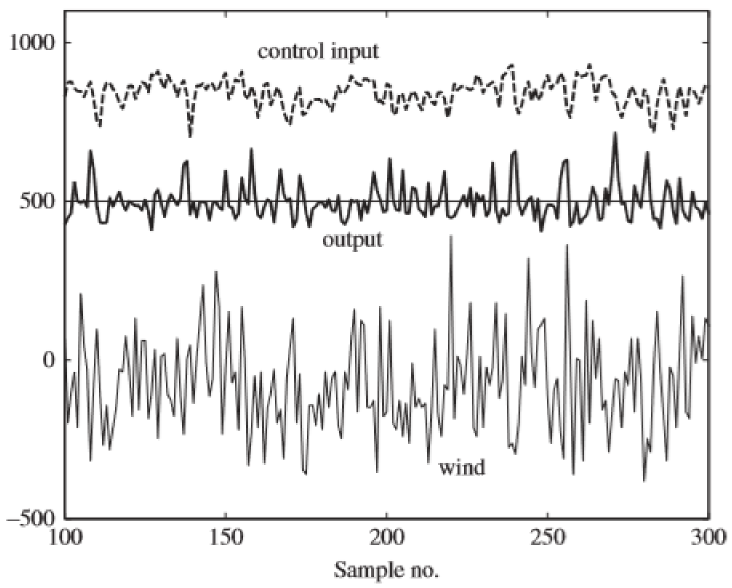

Figure 5.9 Closed-loop response using the PIP controller (5.74), showing the output CO_2 concentration (ppm), the command input at 500 ppm, the control input (a scaled voltage regulating a mass flow valve) and a scaled voltage representing changes in wind velocity (for which zero represents the mean wind speed). The sampling rate is 10 s

5.4 Optimal Design for PIP Control

$$\mathbf{J} = \sum_{k=0}^{\infty} \mathbf{x}(k)^T \mathbf{Q} \mathbf{x}(k) + r(uk(k)^2)$$

5.4.1 Linear Quadratic Weighting Matrices

$$\mathbf{Q} = diag(q_1 \quad q_2 \quad \dots \quad q_n \quad q_{n+1} \quad \dots \quad q_{n+m-1} \quad q_{n+m})$$

$q_1 \quad q_2 \quad ... \quad q_n$ are called the user-defined output weighting parameters and usually $q_y = W_y/n$ is used. Likewise, $q_{n+1} \quad ... \quad q_{n+m-1}$ are the input weighting parameters and substituted with $q_u = W_u/m$. The last term q_{n+m} is $q_e = 1$.

5.4.2 The LQ Closed-loop System and Solution of the Riccati Equation

$$\mathbf{k}^T = (r + \mathbf{g}^T\mathbf{P}\mathbf{g})^{-1}\mathbf{g}^T\mathbf{P}\mathbf{F} \tag{4}$$

$$\mathbf{P} - \mathbf{F}^T\mathbf{P}\mathbf{F} + \mathbf{F}^T\mathbf{P}\mathbf{g}(r + \mathbf{g}^T\mathbf{P}\mathbf{g})^{-1}\mathbf{g}^T\mathbf{P}\mathbf{F} - \mathbf{Q} = \mathbf{0} \tag{5}$$

Example 5.8 PIP-LQ Design for the NMSS Model with Five State Variables

$\mathbf{P} = \mathbf{0}; \mathbf{k}(N) = \mathbf{0}$

$$\mathbf{k}^T(i) = (r + \mathbf{g}^T\mathbf{P}(i+1)\mathbf{g})^{-1}\mathbf{g}^T\mathbf{P}(i+1)\mathbf{F} \tag{6}$$

$$\mathbf{P}(i) = \mathbf{Q} + \mathbf{F}^T\mathbf{P}(i+1)\mathbf{F} - \mathbf{F}^T\mathbf{P}(i+1)\mathbf{g}\mathbf{k}^T(i) \tag{7}$$

$\mathbf{Q}$ in **Equation 7** was defined as:

$$\mathbf{Q} = diag(q_y \quad q_y \quad q_y \quad q_y \quad q_u \quad q_u \quad q_u \quad q_e) \tag{8}$$