

TABLE 4-1 EQUIVALENT DISCRETE-TIME FILTERS FOR A CONTINUOUS-TIME FILTER $G(s) = a/(s + a)$

Mapping method	Mapping equation	Equivalent discrete-time filter for $G(s) = \frac{a}{s + a}$
Backward difference method	$s = \frac{1 - z^{-1}}{T}$	$G_D(z) = \frac{a}{\frac{1 - z^{-1}}{T} + a}$
Forward difference method	$s = \frac{1 - z^{-1}}{Tz^{-1}}$	This method is not recommended, because the discrete-time equivalent may become unstable.
Bilinear transformation method	$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$	$G_D(z) = \frac{a}{\frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} + a}$
Bilinear transformation method with frequency prewarping	$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$ $\left(\omega_A = \frac{2}{T} \tan \frac{\omega_D T}{2} \right)$	$G_D(z) = \frac{\tan \frac{aT}{2}}{\frac{1 - z^{-1}}{1 + z^{-1}} + \tan \frac{aT}{2}}$
Impulse-invariance method	$G_D(z) = T \mathcal{Z} [G(s)]$	$G_D(z) = \frac{Ta}{1 - e^{-aT}z^{-1}}$
Step-invariance method	$G_D(z) = \mathcal{Z} \left[\frac{1 - e^{-Ts}}{s} G(s) \right]$	$G_D(z) = \frac{(1 - e^{-aT})z^{-1}}{1 - e^{-aT}z^{-1}}$
Matched pole-zero mapping method	A pole or zero at $s = -a$ is mapped to $z = e^{-aT}$. An infinite pole or zero is mapped to $z = -1$.	$G_D(z) = \frac{1 - e^{-aT}}{2} \frac{1 + z^{-1}}{1 - e^{-aT}z^{-1}}$