Midterm #2

ECE 452/552: Control Systems Design II

- Closed book and closed notes, except as described below.
- One only ($8\frac{1}{2}'' \times 11''$) page of handwritten notes is permitted. (Written on both sides is OK).
- Calculators can be used.
- Scrap paper is not to be used. Show all work on the exam paper.

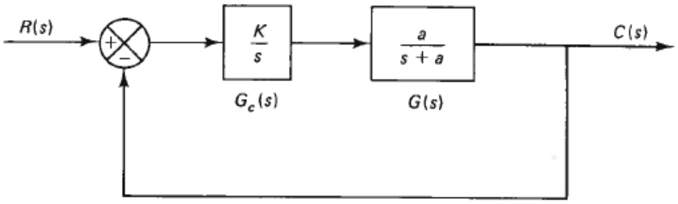
Student name: ______________________________________

Problem 1.
Use the Jury test to determine the stability of the following transfer function:

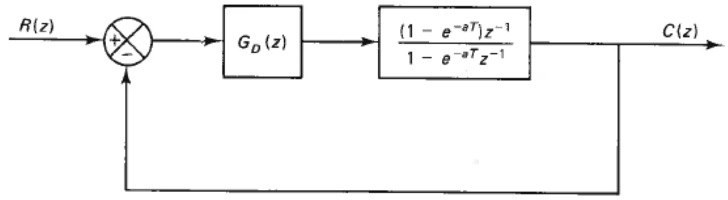
$$G(z) = \frac{1}{z^3 + 0.5z^2 - 1.34z + 0.24}$$

Problem 2.

Consider the analog control system shown in Figure (a), where the first order plant $a/(s+a)$ is controlled by the analog integral controller K/s. The plant preceded by a zero order hold has a pulse transfer function as shown in Figure (b). Obtain the equivalent digital controller $G_D(z)$ based on the matched pole-zero mapping method. **Hint:** Be sure to match the static velocity error coefficients for the analog and digital systems.



(a)



(b)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0 \quad$ for $t < 0$

$x(kT) = x(k) = 0 \quad$ for $k < 0$

Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$

TABLE 4–1 EQUIVALENT DISCRETE-TIME FILTERS FOR A CONTINUOUS-TIME FILTER $G(s) = a/(s + a)$

Mapping method	Mapping equation	Equivalent discrete-time filter for $G(s) = \dfrac{a}{s+a}$
Backward difference method	$s = \dfrac{1 - z^{-1}}{T}$	$G_D(z) = \dfrac{a}{\dfrac{1 - z^{-1}}{T} + a}$
Forward difference method	$s = \dfrac{1 - z^{-1}}{Tz^{-1}}$	This method is not recommended, because the discrete-time equivalent may become unstable.
Bilinear transformation method	$s = \dfrac{2}{T}\dfrac{1 - z^{-1}}{1 + z^{-1}}$	$G_D(z) = \dfrac{a}{\dfrac{2}{T}\dfrac{1 - z^{-1}}{1 + z^{-1}} + a}$
Bilinear transformation method with frequency prewarping	$s = \dfrac{2}{T}\dfrac{1 - z^{-1}}{1 + z^{-1}}$ $\left(\omega_A = \dfrac{2}{T}\tan\dfrac{\omega_D T}{2}\right)$	$G_D(z) = \dfrac{\tan\dfrac{aT}{2}}{\dfrac{1 - z^{-1}}{1 + z^{-1}} + \tan\dfrac{aT}{2}}$
Impulse-invariance method	$G_D(z) = T\,\mathscr{Z}[G(s)]$	$G_D(z) = \dfrac{Ta}{1 - e^{-aT}z^{-1}}$
Step-invariance method	$G_D(z) = \mathscr{Z}\left[\dfrac{1 - e^{-Ts}}{s} G(s)\right]$	$G_D(z) = \dfrac{(1 - e^{-aT})z^{-1}}{1 - e^{-aT}z^{-1}}$
Matched pole-zero mapping method	A pole or zero at $s = -a$ is mapped to $z = e^{-aT}$. An infinite pole or zero is mapped to $z = -1$.	$G_D(z) = \dfrac{1 - e^{-aT}}{2}\,\dfrac{1 + z^{-1}}{1 - e^{-aT}z^{-1}}$