

**ECE452/552**  
**HW #3**  
**SOLUTION**

Problems from Ogata, "Discrete-Time Control Systems" 2<sup>nd</sup> ed.

From pages 167-169

- 1) B-3-11 (plot analytical solution together with the Matlab approach solution)
  - 2) B-3-12
  - 3) B-3-15
  - 4) B-3-16
  - 5) B-3-18 (find  $c(kT)$  only)
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**SOLUTION:**

B-3-11.

$$G(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.5z^{-1}}{(1 - 0.3z^{-1})(1 + 0.7z^{-1})}$$

where

$$X(z) = \frac{1}{1 - z^{-1}}$$

The output  $Y(z)$  is given by

$$\begin{aligned} Y(z) &= \frac{1 - 0.5z^{-1}}{(1 - 0.3z^{-1})(1 + 0.7z^{-1})} \cdot \frac{1}{1 - z^{-1}} \\ &= \frac{3}{35} \frac{1}{1 - 0.3z^{-1}} + \frac{42}{85} \frac{1}{1 + 0.7z^{-1}} + \frac{50}{119} \frac{1}{1 - z^{-1}} \end{aligned}$$

Hence

$$y(k) = \frac{3}{35} (0.3)^k + \frac{42}{85} (-0.7)^k + \frac{50}{119}$$

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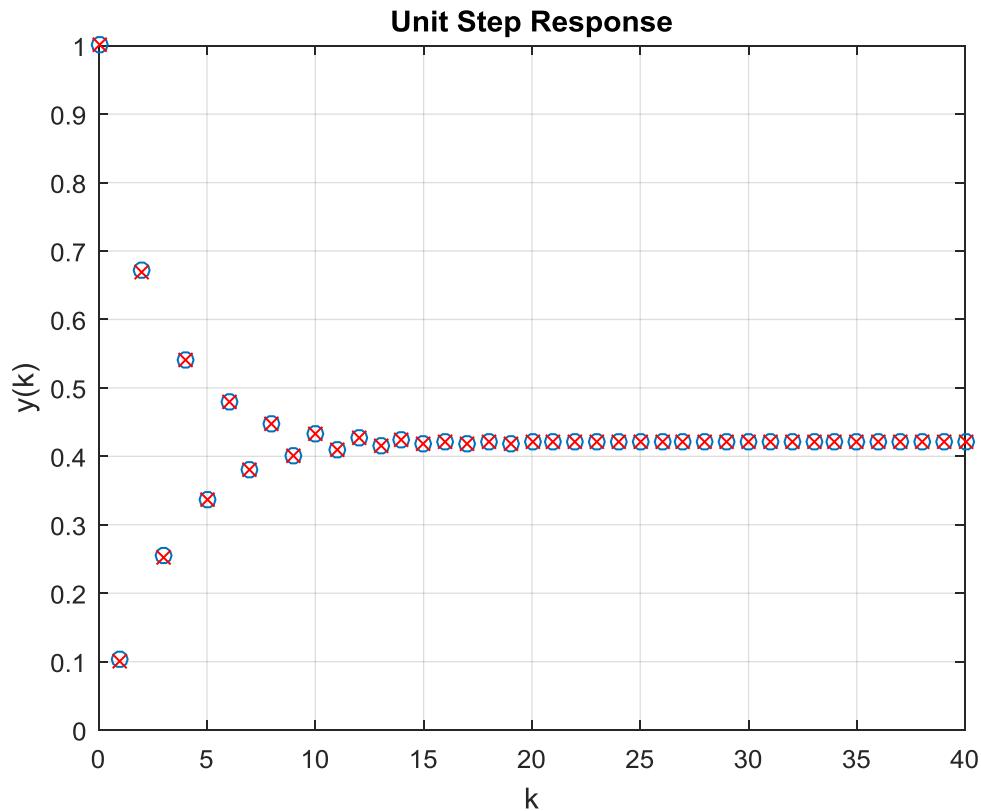
% Ogata B-3-11
format compact
close all
clear

% analytical solution
k=0:40;
y(k+1)= 3/35*(0.35.^k) + 42/85*((-0.7).^k) + 50/119;
plot(k,y,'o')
grid
title('Unit Step Response')
xlabel('k')
ylabel('y(k)')
hold on

% Computational solution with MATLAB
num = [1, -0.5, 0];
den = [1, 0.4, -0.21];
u = ones(1, 41);

y = filter(num, den, u);
plot(k,y,'rx')
hold off

```



B-3-12.

$$Y(s) = \frac{1}{(s+1)(s+2)} X^*(s) = \left(\frac{1}{s+1} - \frac{1}{s+2}\right) X^*(s)$$

By taking the starred Laplace transform of this last equation, we obtain

$$Y^*(s) = \left(\frac{1}{s+1}\right) * X^*(s) - \left(\frac{1}{s+2}\right) * X^*(s)$$

Hence

$$\begin{aligned} Y(z) &= \mathcal{Z}\left[\frac{1}{s+1}\right] X(z) - \mathcal{Z}\left[\frac{1}{s+2}\right] X(z) \\ &= \frac{1}{1 - e^{-T} z^{-1}} X(z) - \frac{1}{1 - e^{-2T} z^{-1}} X(z) \end{aligned}$$

Since

$$X(z) = \frac{1}{1 - z^{-1}}$$

we have

$$\begin{aligned} Y(z) &= \frac{1}{1 - e^{-T} z^{-1}} - \frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-2T} z^{-1}} + \frac{1}{1 - z^{-1}} \\ &= \frac{e^{-T}}{e^{-T} - 1} \frac{1}{1 - e^{-T} z^{-1}} + \frac{1}{1 - e^{-T}} \frac{1}{1 - z^{-1}} \\ &\quad - \frac{e^{-2T}}{e^{-2T} - 1} \frac{1}{1 - e^{-2T} z^{-1}} - \frac{1}{1 - e^{-2T}} \frac{1}{1 - z^{-1}} \end{aligned}$$

Hence

$$y(kT) = \frac{e^{-T}}{e^{-T} - 1} (e^{-T})^k + \frac{1}{1 - e^{-T}} - \frac{e^{-2T}}{e^{-2T} - 1} (e^{-2T})^k - \frac{1}{1 - e^{-2T}}$$

For  $T = 0.1$  we have

$$y(k) = -9.5083(0.9048)^k + 4.5167(0.8187)^k + 4.9917$$

B-3-15. From Figure 3-67 we obtain

$$C(s) = G(s)E^*(s)$$

$$E(s) = R(s) - H_2(s)M^*(s)$$

$$M(s) = H_1(s)G(s)E^*(s)$$

By taking the starred Laplace transforms of the preceding equations, we obtain

$$C^*(s) = G^*(s)E^*(s)$$

$$E^*(s) = R^*(s) - H_2^*(s)M^*(s)$$

$$M^*(s) = [GH_1(s)]^*E^*(s)$$

Hence

$$E^*(s) = R^*(s) - H_2^*(s) [GH_1(s)]^*E^*(s)$$

or

$$E^*(s) = \frac{R^*(s)}{1 + H_2^*(s) [GH_1(s)]^*}$$

and

$$C^*(s) = \frac{G^*(s)R^*(s)}{1 + H_2^*(s) [GH_1(s)]^*}$$

Thus,

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + H_2^*(s) [GH_1(s)]^*}$$

or

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + H_2(z)GH_1(z)}$$

B-3-16.

$$G(z) = \mathcal{Z} \left[ G_{h0}(s) \frac{\frac{K}{s+a}}{s+a} \right] = (1 - z^{-1}) \mathcal{Z} \left[ \frac{\frac{K}{s(s+a)}}{s(s+a)} \right]$$

$$= \frac{K}{a} \frac{(1 - e^{-aT})z^{-1}}{1 - e^{-aT}z^{-1}} = \frac{K}{a} \frac{(1 - e^{-a})z^{-1}}{1 - e^{-a}z^{-1}} \quad (T = 1)$$

Then

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z)} = \frac{K(1 - e^{-a})z^{-1}}{a + [K - (K + a)e^{-a}]z^{-1}}$$

B-3-18.

$$G(z) = \mathcal{Z} \left[ \frac{1 - e^{-Ts}}{s} \frac{\frac{K}{s}}{s} \right] = (1 - z^{-1}) \mathcal{Z} \left[ \frac{\frac{K}{s^2}}{s} \right]$$

$$= (1 - z^{-1}) \frac{\frac{Kt z^{-1}}{(1 - z^{-1})^2}}{(1 - z^{-1})^2}$$

Since  $T = 1$ , we have

$$G(z) = \frac{Kz^{-1}}{1 - z^{-1}}$$

Hence

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z)} = \frac{Kz^{-1}}{1 + (K - 1)z^{-1}}$$

Since

$$R(z) = \frac{1}{1 - z^{-1}}$$

we have

$$C(z) = \frac{Kz^{-1}}{1 + (K - 1)z^{-1}} \frac{1}{1 - z^{-1}}$$

$$= - \frac{1}{1 + (K - 1)z^{-1}} + \frac{1}{1 - z^{-1}}$$

The inverse  $z$  transform of  $C(z)$  gives

$$c(k) = - (1 - K)^k + 1$$

The output sequence  $c(k)$  will converge to 1 if  $0 < K < 2$ . Otherwise,  $c(k)$  approaches infinity as  $k$  approaches infinity.