Quiz 1

February 7, 2018

ECE 452/552: Control System Design II

- Closed book/closed notes except for a one page formula sheet (writing on front and back is OK)
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- Read the questions carefully, and for full credit, clearly show your approach in answering each question.

Student name: _____

Problem 1:

Obtain in the form of an analytical expression, the inverse z-transform of $X(z) = \frac{z(z+2)}{(z-1)^2}$

Problem 2:

For the discrete-time control system shown below, obtain the discrete-time output $Y(z)$ in terms of the input and the transfer functions of the blocks $H(s)$, $G_1(s)$ and $G_2(s)$.

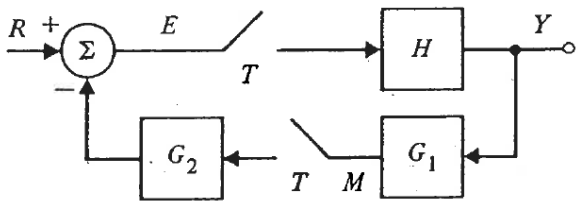

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n\text{-}k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$

Definition of the Z-transform

$$\mathcal{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

Important properties and theorems of the Z-transform

	$x(t)$ or $x(k)$	$\mathcal{Z}\{x(t)\}$ or $\mathcal{Z}\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t)+bx_2(t)$	$aX_1(z)+bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z)-zx(0)$
4.	$x(t+2T)$	$z^2X(z)-z^2x(0)-zx(T)$
5.	$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
6.	$x(t+kT)$	$z^kX(z)-z^kx(0)-z^{k-1}x(T)-\ldots-zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z)-z^kx(0)-z^{k-1}x(1)-\ldots-zx(k1-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz\frac{d}{dz}X(z)$
11.	$kx(k)$	$-z\frac{d}{dz}X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^{a})$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z\frac{d}{dz}X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z\to\infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z\to 1}\left[\left(1-z^{-1}\right)X(z)\right]$ if $\left(1-z^{-1}\right)X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k)=x(k)-x(k-1)$	$\left(1-z^{-1}\right)X(z)$
19.	$\Delta x(k)=x(k+1)-x(k)$	$(z-1)X(z)-zx(0)$
20.	$\sum_{k=0}^{n} x(k)$	$\frac{1}{1-z^{-1}}X(z)$
21.	$\frac{\partial}{\partial a}x(t,a)$	$\frac{\partial}{\partial a}X(z,a)$
22.	$k^mx(k)$	$\left(-z\frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^{n} x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$