ECE 451/551 Summary

- **Transfer Function (TF) to State Space (SS) form** TF → SS
  - Phase variable form
  - Dual phase variable form
- **SS → TF**

- **State Transformation (x = Pz) – invariance of TF**

- **Diagonalization of system matrix**
  - By state transformation $P = [x_1 \ x_2 \ \dots \ x_n]$
  - By partial fraction expansion ↑eigenvectors of A

- **Solution of state equation – time response of systems**
  - Matrix exponential $e^{At} = \mathcal{L}^{-1}\{[sI-A]^{-1}\}$

- **Stability**
  - Asymptotic $x(t) \to 0$ *as* $t \to 0$
    ⇔ eigenvalues have negative real parts
  - BIBO for $|u(t)| \le N < \infty \ \to \ |y(t)| \le M < \infty$
    ⇔ no poles in RHP or no complex pair on imaginary axis
    if no pole-zero cancellations occur → TF poles = eigenvalues
    ∴ BIBO stability ≡ asymptotic stability
  - Internal stability stronger than BIBO stability

- **Controllability**
  - Test by
    - diagonalization - no zero rows in B
    - $M_c = [B \ \ AB \ \ \dots \ \ A^{n-1}B]$ - full rank or det. $M_c \neq 0$

- **Observability**
  - Test by
    - diagonalization - no zero column in C
    - $M_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$ - full rank or det. $M_o \neq 0$

- **Controller Design**
  - Control law u=-Kx
  - Pole placement by
    - Brute force - equate actual coefficients of system characteristic equation with desired to find gains K
    - Use Ackerman's formula

$$K = [0 \quad \dots \quad 0 \quad 1] M_c^{-1} \Delta_d(A)$$

$$\Delta_d(A) = A^n + \alpha_1 A^{n-1} + \alpha_2 A^{n-2} + \dots + \alpha_n I$$

- **Integral Control**
  - Add integrator in forward path to assure zero steady-state error
  - Augment the system and reformulate

- **Observer Design**
  - Full Order Observer

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{Bu} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})$$

error: $\tilde{x} = x - \hat{x}$

$$\dot{\tilde{x}} = (A - LC)\tilde{x}$$

Determine gains L to place poles of estimate error equation

- **Duality**

Control	Estimation
A	A^T
B	C^T
C	B^T
M_c	M_o^T
K	L^T

- **Observer Pole Placement**
  - Use Ackerman's Formula

$$\mathbf{L} = \Delta_e(\mathbf{A})\mathbf{M}_o^{-1}\begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

- **Separation Principle**

- **Compensator Transfer Function**

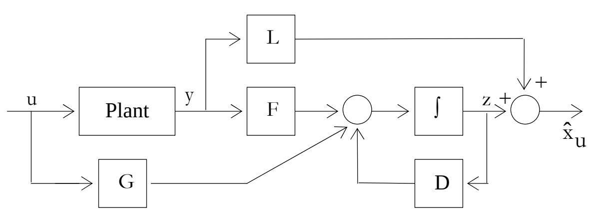
$$U(s) = \underbrace{-K(sI - A + BK + LC)^{-1}L}_{\mathbf{H(s)}}Y(s)$$

- **Reduced Order Observer**

$$\dot{\tilde{\mathbf{x}}}_u = (\mathbf{A}_{22} - \mathbf{L}\mathbf{A}_{12})\tilde{\mathbf{x}}_u$$

$$\dot{\mathbf{z}} = \mathbf{Dz} + \mathbf{Fy} + \mathbf{Gu}$$

$$\hat{\mathbf{x}}_u = \mathbf{z} + \mathbf{Ly}$$



$$\mathbf{D} = \mathbf{A}_{22} - \mathbf{L}\mathbf{A}_{12}$$

$$\mathbf{F} = \mathbf{DL} + \mathbf{A}_{21} - \mathbf{L}\mathbf{A}_{11}$$

$$\mathbf{G} = \mathbf{B}_2 - \mathbf{L}\mathbf{B}_1$$

  - Compensator TF

$$\mathbf{U(s)} = [\mathbf{C'}(\mathbf{sI} - \mathbf{A'})^{-1}\mathbf{B'} + \mathbf{D'}]\mathbf{Y(s)}$$

$$A' = D - GK_2$$

$$B' = F - GK_1 - GK_2L$$

$$C' = -K_2$$

$$D' = -(K_1 + K_2L)$$

- **Linear Quadratic Regulator (LQR)**
  - Minimize $J = \frac{1}{2}\int_0^\infty (x'Qx + u'Ru)dt$

    subject to $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$

    solution exists if

    1) (A,B) controllable
    2) R > 0
    3) Q=Cq'Cq where (C_q, A) is observable

solution: Algebraic Riccati Equation (ARE)

$$A'P + PA + Q - PBR^{-1}B'P = 0$$

$$u = -Kx \quad \textbf{where} \quad K = R^{-1}B'P$$

  - Properties of LQR

    good:
    1) > 60° phase margin
    2) infinite gain margin
    3) gain reduction tolerance of -6dB

    not good:
    4) -20dB/dec roll off rate at high frequency for closed loop gain

- **Optimal Observer – Kalman Filter**

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{w}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{v}$$

solution: $L = \Sigma C' R_o^{-1}$

where $\sum$ is the positive definite solution of

$$A\Sigma + \Sigma A' + Q_o - \Sigma C' R_o^{-1} C\Sigma = 0$$

$$Q_o \geq 0 \qquad R_o > 0$$

- **Linear Quadratic Gaussian (LQG) Properties**
  - No guaranteed stability margins
  - High frequency roll-off can be > 20dB/dec
  - LQG not robust

- **Robustness**
  - Robust stability
  - Robust performance

    Uncertainty modeling
    1) Structured
    2) Unstructured
       a. Additive uncertainty
       b. Multiplicative

  - Small-Gain Theorem (SGT)

- **LQG/LTR (Loop Transfer Recovery)**
  - Recover LQR properties by using Q_o and R_o as tuning parameters