ALTERNATIVE SOLUTION TO STEFANI PROB. 8.14D

GIVEN $T(s) = \dfrac{\begin{bmatrix} s \\ -3s^2-4 \\ 8 \end{bmatrix}}{s^3+3s^2+2s}$ FIND A STATE SPACE REPRESENTATION WHICH HAS A DIAGONAL A MATRIX.

SOLUTION:

WE WILL THE EIGVALUE/EIGENVECTOR METHOD.

FIRST REPRESENT IN PHASE VARIABLE FORM AND THEN USE A STATE TRANSFORMATION TO DIAGONALIZE A

PHASE VARIABLE FORM (BY INSPECTION):

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 0 & -3 \\ 8 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

NOW FIND THE EIGVALUES λ and EIGENVECTORS OF A.

THE CHARACTERISTIC EQUATION IS THE DENOMINATOR OF $T(s)$ SET TO 0 $\Rightarrow$ $s^3+3s^2+2s=0 \Rightarrow s(s^2+3s+2)=0$.

$\Rightarrow s(s+1)(s+2) = 0 \Rightarrow \underline{\lambda = 0, -1, -2}$ $\leftarrow$ DISTINCT EIGENVALUES

EIGENVECTORS: $\lambda v = Av$

1) $\underline{\lambda = 0}$:

$$0\begin{bmatrix} v_1 \\ v_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} v_2 \\ v_3 \\ -2v_2-3v_3 \end{bmatrix} \Rightarrow \left.\begin{matrix} v_2 = 0 \\ v_3 = 0 \\ \Rightarrow v_1 = 1 \end{matrix}\right\} v^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

2) $\underline{\lambda = -1}$:

$$-1\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \Rightarrow \left.\begin{matrix} -v_1 = v_2 \\ -v_2 = v_3 \end{matrix}\right\} \quad v^{(2)} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

3) $\underline{\lambda = -2}$:

$$-2\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \Rightarrow \left.\begin{matrix} -2v_1 = v_2 \\ -2v_2 = v_3 \end{matrix}\right\} \quad v^{(3)} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$$\Rightarrow \quad P = \begin{bmatrix} v^{(1)} & v^{(2)} & v^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

↑ TRANSFORMATION MATRIX.

NOW FIND P^{-1}

$$|P| = 1 \begin{vmatrix} -1 & -2 \\ 1 & 4 \end{vmatrix} = -4+2 = -2 \qquad \Big| \qquad P^{-1} = \frac{1}{|P|} \operatorname{adj}(P)$$

↖ DETERMINANT OF P

INVERSE OF P IS FOUND TERM BY TERM

$$(P^{-1})_{11} = (-1)^{(1+1)} \begin{vmatrix} -1 & -2 \\ 1 & 4 \end{vmatrix} = -4+2 = -2$$

$$(P^{-1})_{12} = (-1)^{(1+2)} \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = (-1)(4-1) = -3$$

$$(P^{-1})_{13} = (-1)^{(1+3)} \begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = -2+1 = -1$$

$$(P^{-1})_{21} = (-1)^{(2+1)} \begin{vmatrix} 0 & -2 \\ 0 & 4 \end{vmatrix} = 0$$

$$(P^{-1})_{22} = (-1)^{(2+2)} \begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} = 4$$

$$(P^{-1})_{23} = (-1)^{(2+3)} \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} = (-1)(-2) = 2$$

$$(P^{-1})_{31} = (-1)^{(3+1)} \begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} = 0$$

$$(P^{-1})_{32} = (-1)^{(3+2)} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = (-1)1 = -1$$

$$(P^{-1})_{33} = (-1)^{(3+3)} \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = -1$$

$$P^{-1} = \frac{1}{-2} \begin{bmatrix} -2 & -3 & -1 \\ 0 & 4 & 2 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1.5 & 0.5 \\ 0 & -2 & -1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

$$\bar{A} = P^{-1} A P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \quad \bar{B} = P^{-1} B = \begin{bmatrix} 1 & 1.5 & 0.5 \\ 0 & -2 & -1 \\ 0 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1 \\ 0.5 \end{bmatrix}$$

$$\bar{C} = CP = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 0 & -3 \\ 8 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 \\ -4 & -7 & -16 \\ 8 & 8 & 8 \end{bmatrix}, \quad \bar{D} = D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$