C1 dc-to-dc voltage converter: Topology and Model

The schematic of the C1 dc-to-dc voltage converter is shown in Figure 1.

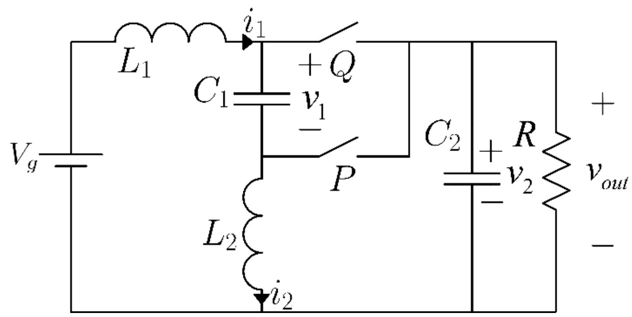

Fig. 1: C1 converter.

The switches Q and P operate cyclically with period T_s. During the first part of the period of length DT_s, where *D* is the steady state duty ratio, where $0 < D < 1$, switch Q in ON and switch P if OFF. This results in a circuit configuration which can be described in state space by the quadruple $\{A_1, B_1, C_1, E_1\}$. Note we will use the symbol *E* in the state space output equation here rather than *D* which has been reserved to represent duty ratio.

During the remainder of the period of length $(1-D)T_s$, switch Q in OFF and switch P if ON. The corresponding state space quadruple is $\{A_2, B_2, C_2, E_2\}$. Henceforth we will use for convenience the following: $D' = 1 - D$.

State Space Averaging (SSA) Model:

The large signal model is given by:

$$\dot{x} = Ax + Bu \qquad y = Cx + Eu$$

where

$$A = DA_1 + D'A_2 \qquad B = DB_1 + D'B_2$$

$$C = DC_1 + D'C_2 \qquad E = DE_1 + D'E_2$$

$$x = \begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} \qquad u = v_g$$

As is usual practice, the state variables are given by the inductor currents and capacitor voltage:

The state-space equations associated with DT_s are summarized:

$$\begin{aligned}
\frac{di_1}{dt} &= \frac{1}{L_1}(V_g - v_2) \\
\frac{di_2}{dt} &= \frac{1}{L_2}(v_2 - v_1) \\
\frac{dv_1}{dt} &= \frac{1}{C_1}(i_2) \\
\frac{dv_2}{dt} &= \frac{1}{C_2}(i_1 - i_2 - \frac{v_2}{R})
\end{aligned}$$

These equations yield the following matrices:

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{L_1} \\ 0 & 0 & -\frac{1}{L_2} & \frac{1}{L_2} \\ 0 & \frac{1}{C_1} & 0 & 0 \\ \frac{1}{C_2} & -\frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \qquad B_1 = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The state-space equations associated with $D'T_s$ are summarized:

$$\begin{aligned}
\frac{di_1}{dt} &= \frac{1}{L_1}(V_g - v_1 - v_2) \\
\frac{di_2}{dt} &= \frac{1}{L_2}(v_2) \\
\frac{dv_1}{dt} &= \frac{1}{C_1}(i_1) \\
\frac{dv_2}{dt} &= \frac{1}{C_2}(i_1 - i_2 - \frac{v_2}{R})
\end{aligned}$$

These equations yield the following matrices:

$$A_2 = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & -\frac{1}{L_1} \\ 0 & 0 & 0 & \frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ \frac{1}{C_2} & -\frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \qquad B_2 = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

SSA matrices:

Matrices A and B can then be evaluated:

$$A = \begin{bmatrix} 0 & 0 & -\frac{D'}{L_1} & -\frac{1}{L_1} \\ 0 & 0 & -\frac{D}{L_2} & \frac{1}{L_2} \\ \frac{D'}{C_1} & \frac{D}{C_1} & 0 & 0 \\ \frac{1}{C_2} & -\frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

X is the steady state vector and U is V_g, the DC value of the input voltage.

$$X = -A^{-1}BU = \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_g D^2}{R} \\ -\frac{V_g D D'}{R} \\ V_g \\ V_g D \end{bmatrix}$$

For the transient simulations we will use the small-signal SSA model:

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} + B_d\hat{d}$$

$$\hat{y} = C\hat{x} + E\hat{u} + E_d\hat{d}$$

where matrices A, B, C and E where previously defined and where

$$B_d = (A_1 - A_2)X + (B_1 - B_2)U$$

$$E_d = (C_1 - C_2)X + (E_1 - E_2)U$$