EE446 / EE546

R. TYMERSKI

SMALL SIGNAL STATE SPACE AVERAGING

PROBLEM 1.

For the buck-boost converter of Fig. 1, determine the control-to-output, $\frac{\hat{v}}{\hat{d}}(s)$, transfer function, using the technique of state-space averaging. Hence determine the DC gain and the location of the poles and zeroes in terms of V_g, D and the circuit parameters, L, C and R, only. (In the case of a complex pole pair, you need only determine the resonant frequency and Q).

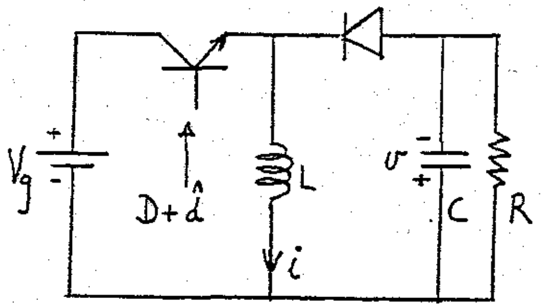

FIG. 1.

PROBLEM 2.

The output impedance of a system may be determined by driving the output with a perturbation current source, as shown in Fig. 2, and noting the voltage variations and then forming the quotient

$$Z_{out} = \frac{\hat{v}}{\hat{i}}$$

Using this information and the small-signal mathematical model of state-space averaging, determine the output impedance, Z_{out}, of the converter shown in Fig. 3.

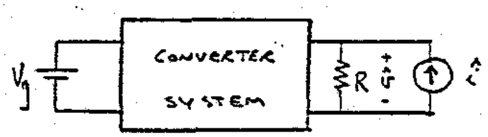

FIG. 2.

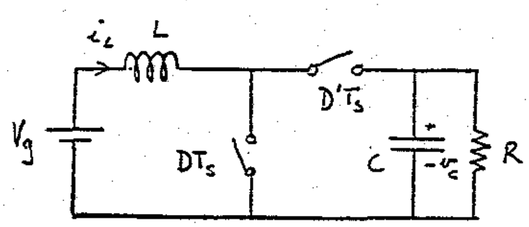

FIG. 3.

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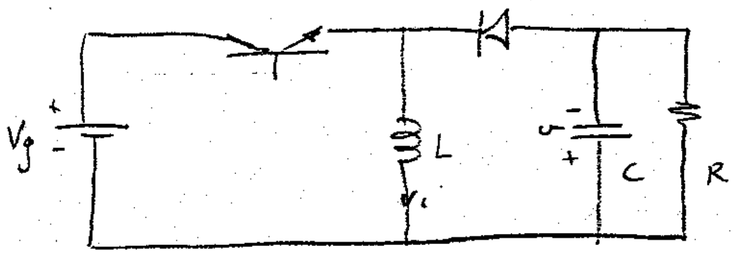
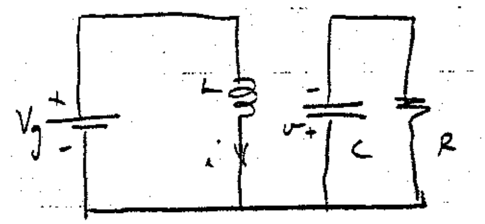
SMALL SIGNAL STATE SPACE AVERAGING

SOLUTION

PROBLEM 1



DURING DT_s



$$-V_g + L\dot{i} = 0 \Rightarrow \dot{i} = \frac{V_g}{L}$$

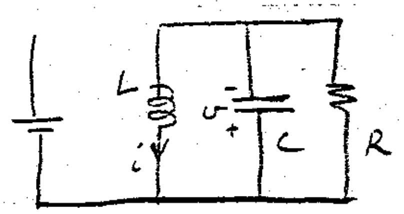
$$C\dot{v} = -\frac{v}{R} \Rightarrow \dot{v} = -\frac{v}{RC}$$

let $x = [i \quad v]^T$

$$\dot{x} = A_1 x + b_1 V_g$$

$$\frac{d}{dt}\begin{bmatrix} i \\ v \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_g$$

DURING $D'T_s$



$$v + L\dot{i} = 0 \Rightarrow \dot{i} = -\frac{v}{L}$$

$$C\dot{v} = i - \frac{v}{R} \Rightarrow \dot{v} = \frac{i}{C} - \frac{v}{RC}$$

$$\dot{x} = A_2 x + b_2 V_g$$

$$\frac{d}{dt}\begin{bmatrix} i \\ v \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_g$$

$$A = DA_1 + D'A_2 = \begin{bmatrix} 0 & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$b = Db_1 + D'b_2 = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}$$

$$X = -A^{-1} b V_g$$

$$\Rightarrow X = - \begin{bmatrix} 0 & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix}^{-1} \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} V_g$$

$$= - \frac{LC}{D'^2} \begin{bmatrix} -\frac{1}{RC} & \frac{D'}{L} \\ -\frac{D'}{C} & 0 \end{bmatrix} \begin{bmatrix} D/L \\ 0 \end{bmatrix} V_g .$$

$$= \frac{-LC}{D'^2} \begin{bmatrix} -\frac{1}{RCL} \\ -\frac{D'D}{LC} \end{bmatrix} V_g = \begin{bmatrix} \frac{D}{RD'^2} \\ \frac{D}{D'} \end{bmatrix} V_g$$

$$\frac{\hat{x}}{\hat{d}} = (sI - A)^{-1} b_d \qquad \text{where} \quad b_d = (A_1 - A_2) X + (b_1 - b_2) V_g$$

$$= \begin{bmatrix} s & +\frac{D'}{L} \\ -\frac{D'}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} \frac{D}{RD'^2} \\ \frac{D}{D'} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \right\} V_g$$

$$= \frac{1}{s\left(s + \frac{1}{RC}\right) + \frac{D'^2}{LC}} \begin{bmatrix} s + \frac{1}{RC} & -\frac{D'}{L} \\ +\frac{D'}{C} & s \end{bmatrix} \begin{bmatrix} \overbrace{\left(\frac{D}{D'} + 1\right)}^{\frac{1}{D'}} \frac{1}{L} \\ -\frac{D}{RCD'^2} \end{bmatrix} V_g$$

$$\frac{\hat{x}}{\hat{d}} = \frac{1}{s^2 + \frac{s}{RC} + \frac{D'^2}{LC}} \begin{bmatrix} \left(s + \frac{1}{RC}\right)\frac{1}{D'L} + \frac{D}{RCLD'} \\ \frac{1}{LC} - \frac{sD}{RCD'^2} \end{bmatrix} V_g$$

$$\hat{y} = c\hat{x} + (c_1 - c_2) X \hat{d}$$

$$y = v \Rightarrow c_1 = c_2 = c = [0 \quad 1]$$

$$\Rightarrow \frac{\hat{v}}{\hat{d}} = c \frac{\hat{x}}{\hat{d}} = \frac{\left(\frac{1}{LC} - \frac{sD}{RCD'^2}\right) V_g}{s^2 + \frac{s}{RC} + \frac{D'^2}{LC}}$$

$$= V_g \frac{\frac{1}{LC}\left(1 - \frac{sD}{RCD'^2} LC\right)}{\frac{D'^2}{LC}\left(\frac{s^2 LC}{D'^2} + \frac{LC}{D'^2}\frac{s}{RC} + 1\right)}$$

$$\boxed{\frac{\hat{v}}{\hat{d}} = \frac{V_g}{D'^2} \frac{1 - s\frac{LD}{RD'^2}}{\frac{LCs^2}{D'^2} + \frac{L}{D'^2 R}s + 1}}$$

DC gain $= \frac{V_g}{D'^2}$

RHP zero at $\omega_z = \frac{D'^2 R}{DL}$

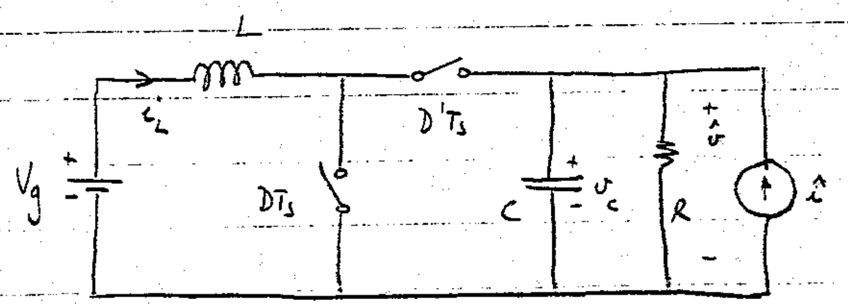
Complex pole pair at $\omega_o^2 = \frac{D'^2}{LC} \Rightarrow \omega_o = \frac{D'}{\sqrt{LC}}$

$$\frac{1}{\omega_o Q} = \frac{L}{D'^2 R} \Rightarrow Q = \frac{D'^2 R}{L}\frac{\sqrt{LC}}{D'} = D'R\sqrt{\frac{C}{L}}$$

SOLUTION

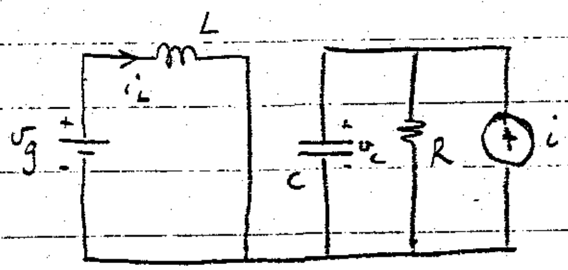
R. TYMERSKI

PROBLEM 1

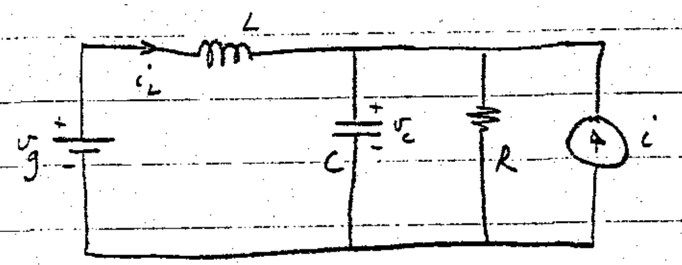


Let $x = \begin{bmatrix} i_L \\ v_c \end{bmatrix}$ $\quad u = \begin{bmatrix} v_g \\ i \end{bmatrix}$

DURING DT_s



DURING $D'T_s$



During DT_s:

$$-V_g + L\frac{di_L}{dt} = 0 \Rightarrow \frac{di_L}{dt} = \frac{V_g}{L}$$

$$i = C\frac{dv_c}{dt} + \frac{v_c}{R} \Rightarrow \frac{dv_c}{dt} = \frac{i}{C} - \frac{v_c}{RC}$$

During $D'T_s$:

$$-V_g + L\frac{di_L}{dt} + v_c = 0 \Rightarrow \frac{di_L}{dt} = \frac{V_g}{L} - \frac{v_c}{L}$$

$$i_L + i = C\frac{dv_c}{dt} + \frac{v_c}{R} \Rightarrow \frac{dv_c}{dt} = \frac{-v_c}{RC} + \frac{i_L}{C} - \frac{i}{C}$$

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} i_L \\ v_c \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}}_{A_1}\begin{bmatrix} i_L \\ v_c \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix}}_{B_1}\begin{bmatrix} v_g \\ i \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_c \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A_2}\begin{bmatrix} i_L \\ v_c \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} & 0 \\ 0 & +\frac{1}{C} \end{bmatrix}}_{B_2}\begin{bmatrix} v_g \\ i \end{bmatrix}$$

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} + \left[(A_1 - A_2)X + (B_1 - B_2)U\right]\hat{d}$$

$$\hat{y} = C\hat{x} + E\hat{u} + \left[(C_1 - C_2)X + (E_1 - E_2)U\right]\hat{d}$$

where $A = DA_1 + D'A_2$ $\qquad C = DC_1 + D'C_2$

$B = DB_1 + D'B_2$ $\qquad E = DE_1 + D'E_2$

we require $Z_{out} = \dfrac{\hat{v}}{\hat{i}}$

Set $\hat{d} = 0 \Rightarrow$

$$\hat{y} = \left[C(sI - A)^{-1}B + E\right]\hat{u}$$

Now $\hat{y} = \hat{v} = \hat{v}_c = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C_1 = C_2 = C} \hat{x} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{E_1 = E_2 = E} \hat{u}$

$$\Rightarrow \hat{v} = \left\{ \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & +\frac{D'}{L} \\ -\frac{D'}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \right\} \hat{u}$$

$$\hat{v} = \left\{ \frac{1}{s\left(s + \frac{1}{RC}\right) + \frac{D'^2}{LC}} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + \frac{1}{RC} & -\frac{D'}{L} \\ \frac{D'}{C} & s \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \right\} \hat{u}$$

$$= \left\{ \frac{1}{s^2 + s\frac{1}{RC} + \frac{D'^2}{LC}} \begin{bmatrix} \frac{D'}{C} & s \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \right\} \hat{u}$$

$$= \frac{1}{s^2 + s\frac{1}{RC} + \frac{D'^2}{LC}} \begin{bmatrix} \frac{D'}{LC} & \frac{s}{C} \end{bmatrix} \hat{u}$$

$$\hat{v} = \frac{\frac{D'}{LC}\hat{v}_g}{s^2 + s\frac{1}{RC} + \frac{D'^2}{LC}} + \frac{\frac{s}{C}\hat{i}}{s^2 + s\frac{1}{RC} + \frac{D'^2}{LC}}$$

$$\Rightarrow \left.\frac{\hat{v}}{\hat{i}}\right|_{\hat{v}_g = 0} = \frac{\frac{1}{C}}{\frac{D'^2}{LC}} \frac{s}{\frac{LC}{D'^2}s^2 + s\frac{1}{R\cancel{C}}\frac{L\cancel{C}}{D'^2} + 1}$$

$$\Rightarrow \boxed{Z_{out} = \frac{1}{D'^2} \frac{sL}{\frac{LC}{D'^2}s^2 + s\frac{L}{D'^2R} + 1}}$$

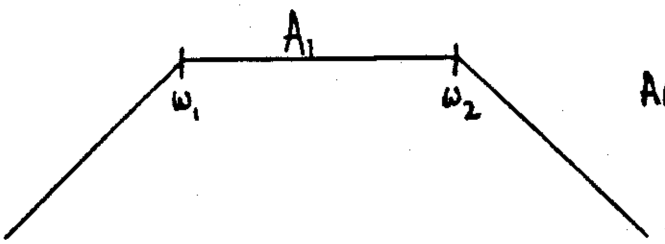
BODE PLOTS I

Problem

Review of Bode Diagrams

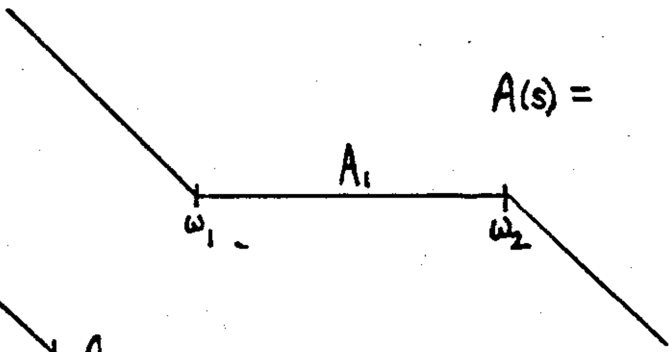
1. Express the gains in factored pole-zero form.

a)



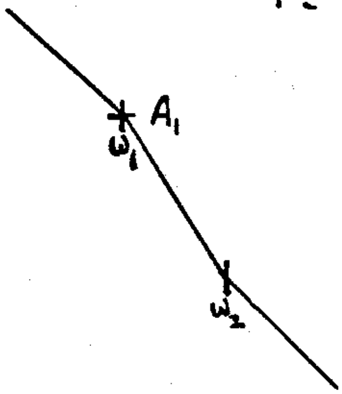
A(s) =

b)



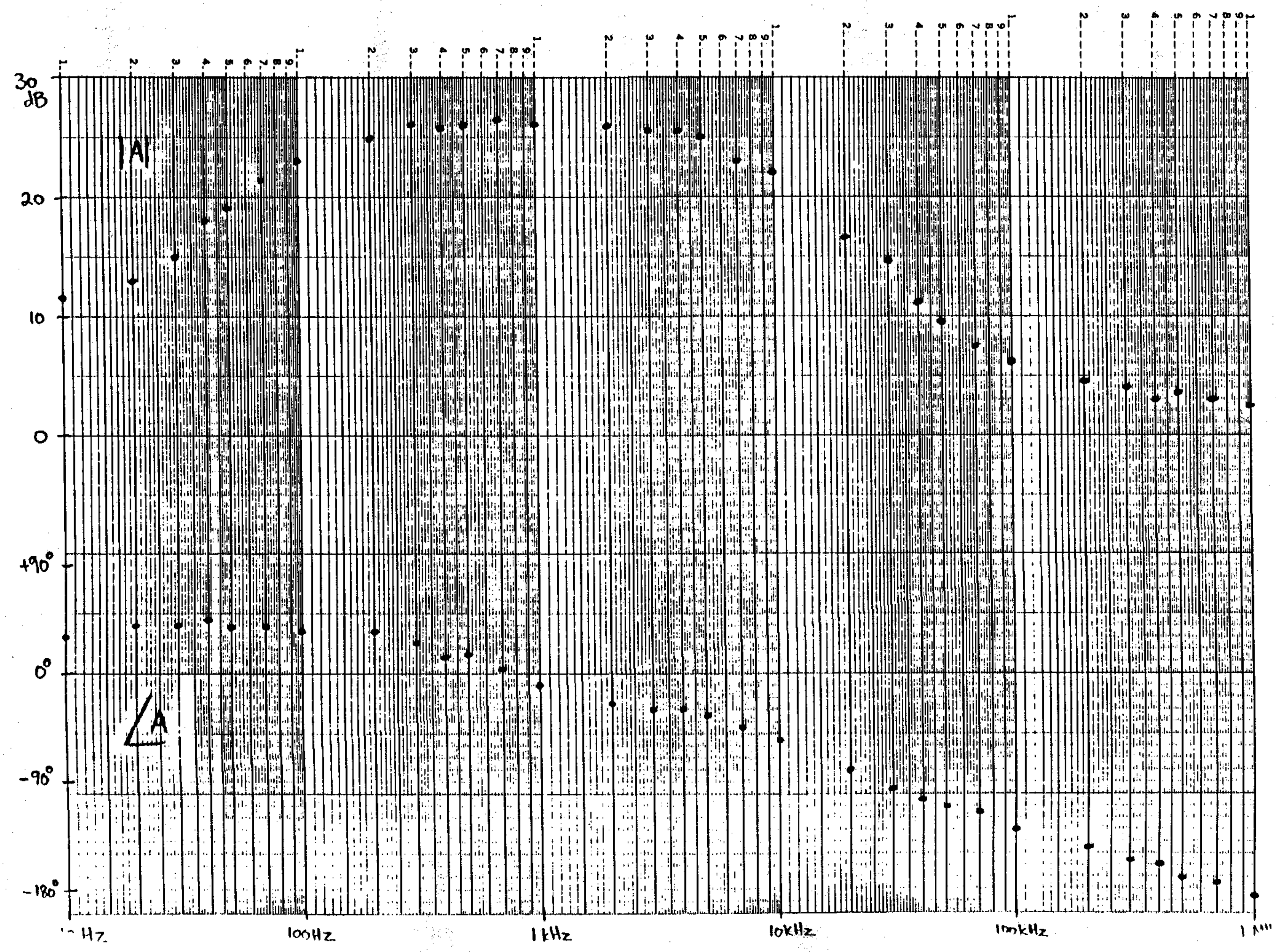
A(s) =

c)

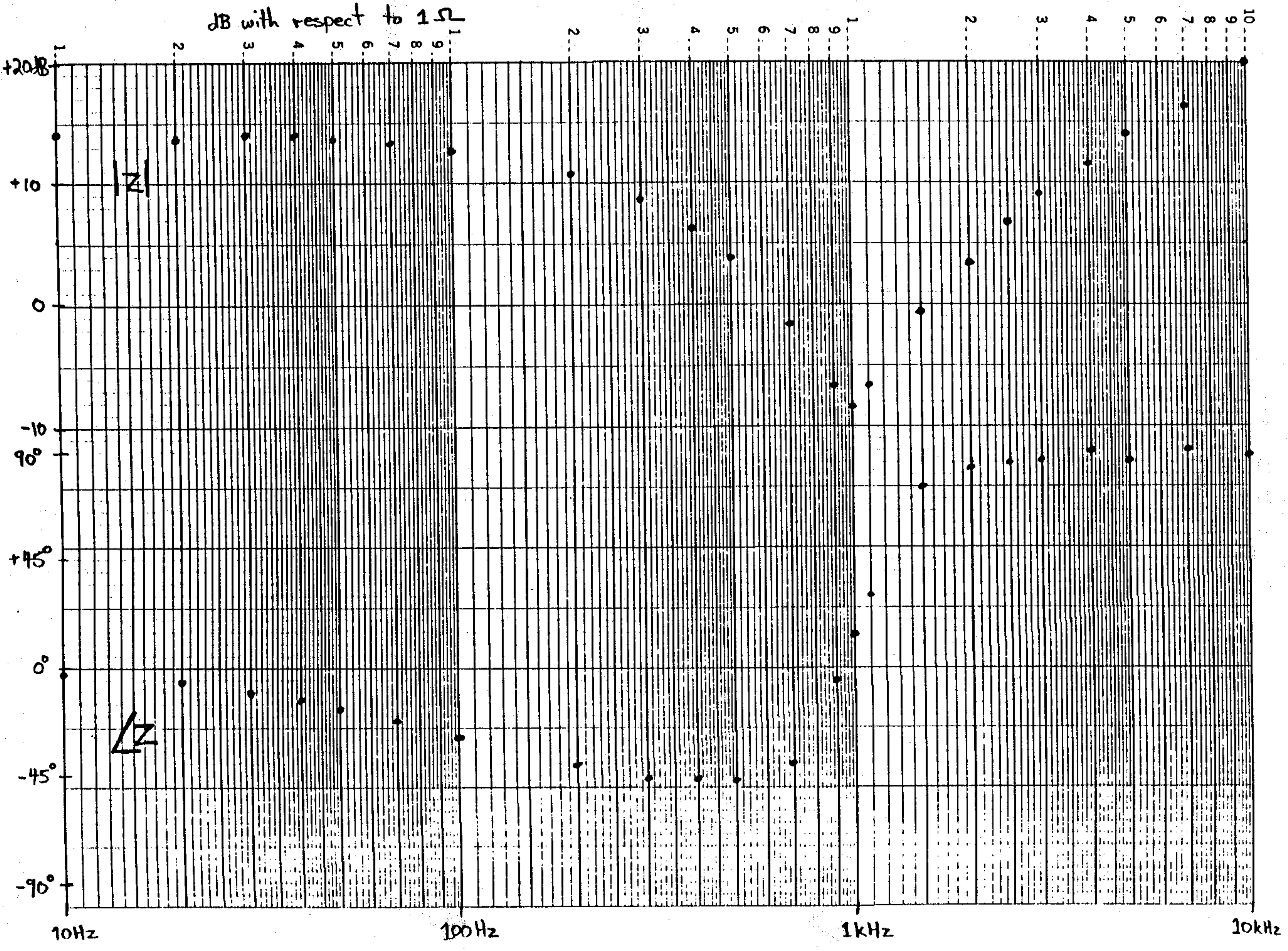


A(s) =

2. The accompanying graphs show experimental magnitude and phase data for a certain gain function A(S) and impedance Z(S). Draw appropriate straight-line asymptotes through the data points and hence deduce numerical values for the mid-frequency gain A_m and low-frequency impedance R_o, as well as for the poles, zeroes, and Q-factors in the corresponding analytic expressions for A(S) and Z(S).



30 dB
20
10
0
+90°
0°
-90°
-180°
|A|
∠A
100Hz
1kHz
10kHz
100kHz


dB with respect to 1 Ω
|Z|
∠Z
+20dB
+10
0
-10
90°
+45°
0°
-45°
-90°
10Hz
100Hz
1kHz
10kHz

SOLUTION

EE446/546

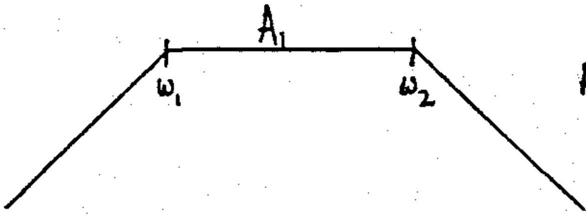
Power Electronics

Problem

Review of Bode Diagrams

1. Express the gains in factored pole-zero form.

a)



$$A(s) = \frac{A_1\omega_2}{s}\,\frac{1}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{\omega_1}{s}\right)}$$

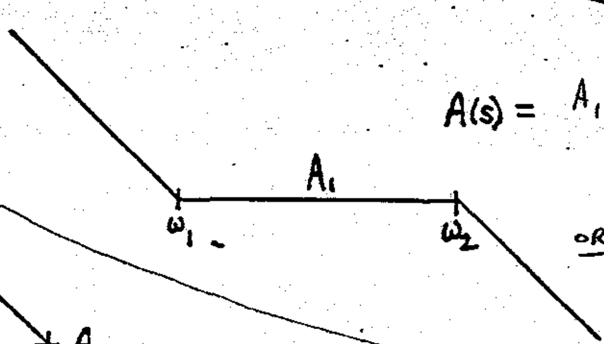
OR

$$A(s) = A_1\,\frac{1}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{\omega_1}{s}\right)}$$

OR

$$A(s) = A_1\,\frac{s/\omega_1}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_1}\right)}$$

b)



$$A(s) = A_1\,\frac{\omega_2}{s}\,\frac{1+\frac{\omega_1}{s}}{1+\frac{\omega_2}{s}}$$

OR

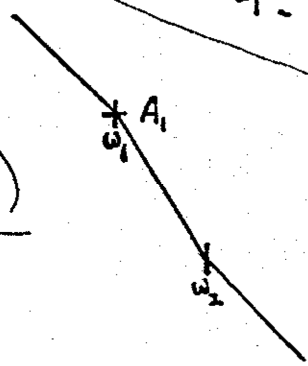
$$A(s) = A_1\,\frac{\omega_1\omega_2}{s^2}\left(\frac{1+\frac{s}{\omega_1}}{1+\frac{\omega_2}{s}}\right)$$

$$A(s) = A_1\,\frac{\left(1+\frac{\omega_1}{s}\right)}{1+s/\omega_2}$$

OR

$$A(s) = \frac{A_1\,\omega_1\left(1+\frac{s}{\omega_1}\right)}{s\left(1+\frac{s}{\omega_2}\right)}$$

c)



$$A(s) = A_1\,\frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{\omega_1}\right)^2}$$

$$A(s) = \frac{A_1\,\frac{\omega_1}{s}\left(1+\frac{s}{\omega_2}\right)}{1+\frac{s}{\omega_1}}$$

2. The accompanying graphs show experimental magnitude and phase data for a certain gain function A(S) and impedance Z(S). Draw appropriate straight-line asymptotes through the data points and hence deduce numerical values for the mid-frequency gain A_m and low-frequency impedance R_o, as well as for the poles, zeroes, and Q-factors in the corresponding analytic expressions for A(S) and Z(S).

R TYMERSKI

PROBLEM

The attached graph shows experimental data for a certain gain function A. Draw appropriate straight-line asymptotes through the data points and hence deduce numerical values for the low-frequency gain and for the poles and zeros in the corresponding analytic expression for A.

Draw horizontal asymptotes (1) & (2). This fixes maximum gain (A) at 26 dB and maximum +ve phase shift at ≈ 42°. The +ve phase shift confirms the existence of a LHP zero at a low frequency hinted at by the magnitude characteristic. With the gain and phase we can determine the low frequency gain (A_o) using $\phi_{max} = \tan^{-1}\left(\frac{k-1}{2\sqrt{k}}\right)$ where k is the ratio of the gains. We find $A_o \approx 12$ dB, . this is a little high considering the data points after drawing (3) (at +20dB/dec); draw in (4) at ≈10 to fit curve better. Draw in (5) at -20dB/dec. Draw in (6), (7) + (8), confirms a high frequency pole (≈ 7 kHz), phase slope -45/-90/-45° per dec. Draw in (9), with (10) and (11) indicates RHP zero at a high frequency ≈ 100 kHz.

The resulting analytic expression for A is

$$A(s) = A_o \frac{\left(1+\frac{s}{\omega_o}\right)\left(1-\frac{s}{\omega_3}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$

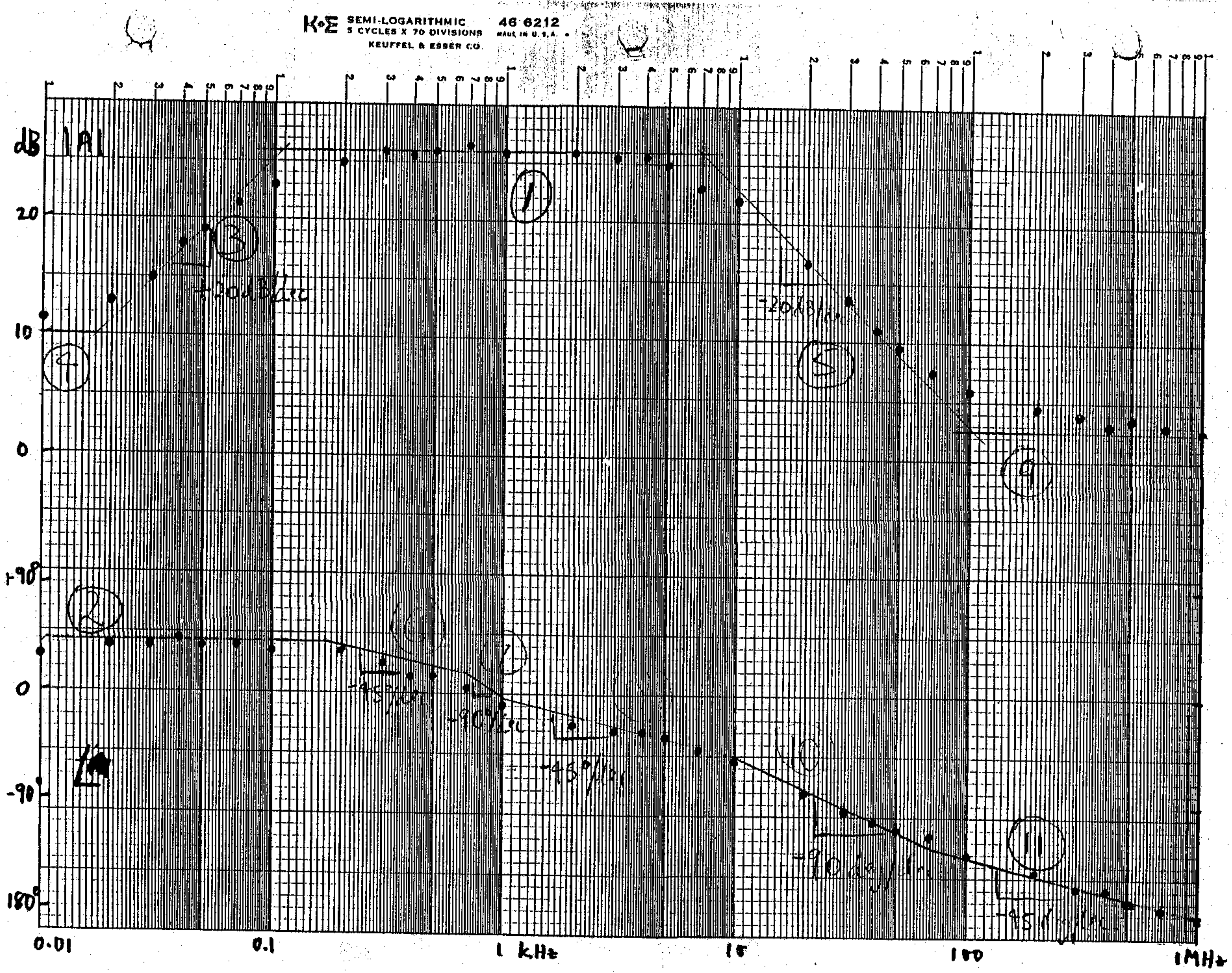
where A_o = 10 dB → 3.2

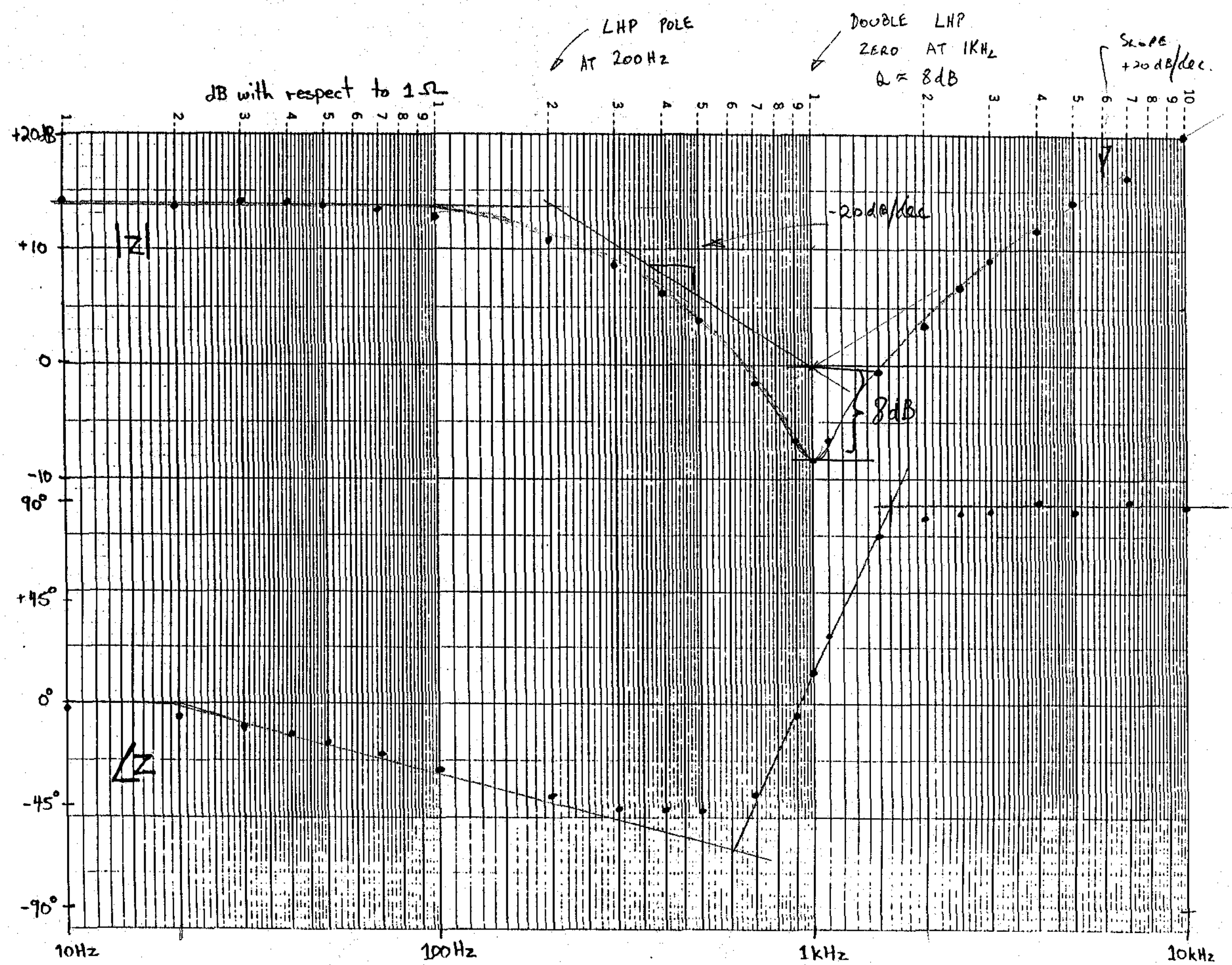
LHP zero $\omega_o \rightarrow f_o$ = 17 Hz

LHP pole $\omega_1 \rightarrow f_1$ = 110 Hz

LHP pole $\omega_2 \rightarrow f_2$ = 6.8 kHz

RHP zero $\omega_3 \rightarrow f_3$ = 100 kHz


K+E SEMI-LOGARITHMIC
5 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO.
46 6212
MADE IN U.S.A.
dB
|A|
20
10
0
+90°
0
-90°
180°
+20dB/dec
-20dB/dec
-45°/dec
-90°/dec
+45°/dec
-90°/dec
-45°/dec
0.01
0.1
1 kHz
10
100
1MHz


LHP POLE
AT 200Hz
DOUBLE LHP
ZERO AT 1KHz
Q ≈ 8dB
Slope
+20dB/dec.
dB with respect to 1Ω
+20dB
+10
0
-10
90°
+45°
0°
-45°
-90°
|Z|
∠Z
-20dB/dec
8dB
10Hz
100Hz
1kHz
10kHz

LHP POLE AT $2\pi(200)$ $\Rightarrow$ $\omega_p = 2\pi(200)$

DOUBLE LHP ZERO AT $2\pi(1000)$ $\Rightarrow$ $\omega_o = 2\pi(1000)$

$Q \approx 8\,dB \rightarrow 2.5$ $\Rightarrow$ $Q = 2.5$

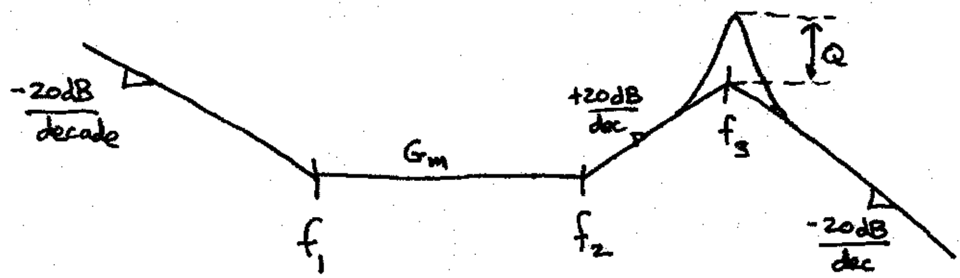
MIDBAND $Z_m = 14\,dB \rightarrow 5$ $\Rightarrow$ $Z_m = 5$

$$Z(s) = Z_m \frac{1 + \frac{s}{\omega_o Q} + \frac{s^2}{\omega_o^2}}{1 + s/\omega_p}$$

BODE PLOTS II

R.W. Erickson
Fundamentals of Power Electronics

A Solved Problem
similar to problems 8.1–8.4



a) Express the gain represented by the asymptotes above in factored pole-zero form. f_1, f_2, and f_3 are the corner frequencies in Hz. G_m is the magnitude of the midband asymptote, as illustrated.

b) Derive analytical expressions for each asymptote, using your result from part (a).

c) Compute the value of the asymptote at $f = f_3$

Solution to (a)

We are given the magnitude of the midband asymptote G_m. So reference $G(s)$ to G_m. Poles and zeroes at frequencies $f \le f_1$ should then be expressed in inverted form. Poles and zeroes at frequencies $f \ge f_2$ should be expressed in conventional non-inverted form.

So $G(s)$ contains the following terms ;

G_m midband gain (Note that if the value of G_m is labeled in dB, then it must be converted for use in the expression for $G(s)$: $G_m = 10^{(\|G_m\|_{dB}/20)}$

$\left(1+\frac{\omega_1}{s}\right)$ inverted zero at $f = f_1$
$\omega_1 = 2\pi f_1$

$\left(1+\frac{s}{\omega_2}\right)$ zero at $f = f_2$
$\omega_2 = 2\pi f_2$

$$\frac{1}{1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2}$$

Complex poles at $f = f_3$
$\omega_3 = 2\pi f_3$
with Q-factor Q

(Note that if the value of Q is expressed in dB, then it must be converted for use in the expression for $G(s)$:

$$Q = 10^{(\|Q\|_{dB}/20)}$$

So

$$G(s) = G_m \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)}$$

An equivalent form that does not employ inverted zeroes:

$$G(s) = G_m \frac{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(\frac{s}{\omega_1}\right)\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)}$$

b) Analytical expressions for asymptotes

Given the solution to part (a)
(either version will work) :

$$G(s) = G_m \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)}$$

There are four asymptotes, one for each of the following frequency ranges:

1. $f \le f_1$
2. $f_1 \le f \le f_2$
3. $f_2 \le f \le f_3$
4. $f_3 \le f$

There are three frequency-dependent terms:

$\left(1+\frac{\omega_1}{s}\right)$

$\left(1+\frac{s}{\omega_2}\right)$

$\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)$

Each of these terms consists of the sum of terms. Over a given frequency range, the asymptote is derived by neglecting the smallest term or terms within each sum, retaining only the term having the largest magnitude. This process leads to an exact expression for the asymptote, which may be a good approximation for the actual function $\|G(j\omega)\|$.

1. For $f \le f_1$

Then $f \le f_1 < f_2 < f_3$

$\omega \le \omega_1 < \omega_2 < \omega_3$

Consider each term:

$$\underbrace{\left(1+\frac{\omega_1}{s}\right)}_{\text{inverted zero term}} \longrightarrow \underbrace{\frac{\omega_1}{s}}_{\text{asymptote for } f<f_1}$$

justification

$$\left\| 1+\frac{\omega_1}{s} \right\|_{s=j\omega} = \sqrt{1+\left(\frac{\omega_1}{\omega}\right)^2}$$

$$\approx \sqrt{\left(\frac{\omega_1}{\omega}\right)^2} = \frac{\omega_1}{\omega}$$

since for $\omega \ll \omega_1$, $1 \ll \left(\frac{\omega_1}{\omega}\right)^2$

so $\left(1+\frac{\omega_1}{s}\right) \approx \frac{\omega_1}{s}$ for $f \ll f_1$

zero term	asymptote for $f<f_1$
$\left(1+\frac{s}{\omega_2}\right)$	1

justification

$$\left\| 1+\frac{s}{\omega_2} \right\|_{s=j\omega} = \sqrt{1+\left(\frac{\omega}{\omega_2}\right)^2}$$

$\approx \sqrt{1} = 1$ since, for $\omega \ll \omega_1$, $1 \gg \left(\frac{\omega}{\omega_2}\right)^2$. So $\left(1+\frac{s}{\omega_2}\right) \approx 1$ for $f \ll f_1$

pole term	asymptote for $f<f_1$
$\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)$	1

justification

$$\left\| 1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2 \right\| = \sqrt{\left(1-\left(\frac{\omega}{\omega_3}\right)^2\right)^2+\left(\frac{\omega}{Q\omega_3}\right)^2}$$

$\approx \sqrt{1} = 1$ since, for $\omega \ll \omega_1$, $1 \gg \left(\frac{\omega}{\omega_3}\right)^2$ and $1 \gg \frac{\omega}{Q\omega_3}$.

So $\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right) \approx 1$ for $f \ll f_1$

Composite:
$$G(s) = G_m \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)} \rightarrow G_m \frac{\left(\frac{\omega_1}{s}\right)(1)}{(1)} = G_m \frac{\omega_1}{s}$$

let $s=j\omega$ and take magnitude: $\left(\frac{G_m \omega_1}{\omega}\right) = \left(\frac{G_m f_1}{f}\right)$ is the expression for the magnitude asymptote for $f<f_1$.

2. for $f_1 \le f \le f_2$

Then

$$G(s) = G_m \frac{\left(1+\cancel{\frac{\omega_1}{s}}\right)\left(1+\cancel{\frac{s}{\omega_2}}\right)}{\left(1+\cancel{\frac{s}{Q\omega_3}} + \cancel{\left(\frac{s}{\omega_3}\right)^2}\right)} \rightarrow G_m \frac{(1)(1)}{(1)} = G_m$$

3. for $f_2 \le f \le f_3$

Then

$$G(s) = G_m \frac{\left(1+\cancel{\frac{\omega_1}{s}}\right)\left(\cancel{1}+\frac{s}{\omega_2}\right)}{\left(1+\cancel{\frac{s}{Q\omega_3}} + \cancel{\left(\frac{s}{\omega_3}\right)^2}\right)} \rightarrow G_m \frac{(1)\left(\frac{s}{\omega_2}\right)}{(1)} = G_m \frac{s}{\omega_2}$$

which has magnitude $G_m \dfrac{\omega}{\omega_2} = G_m \dfrac{f}{f_2}$

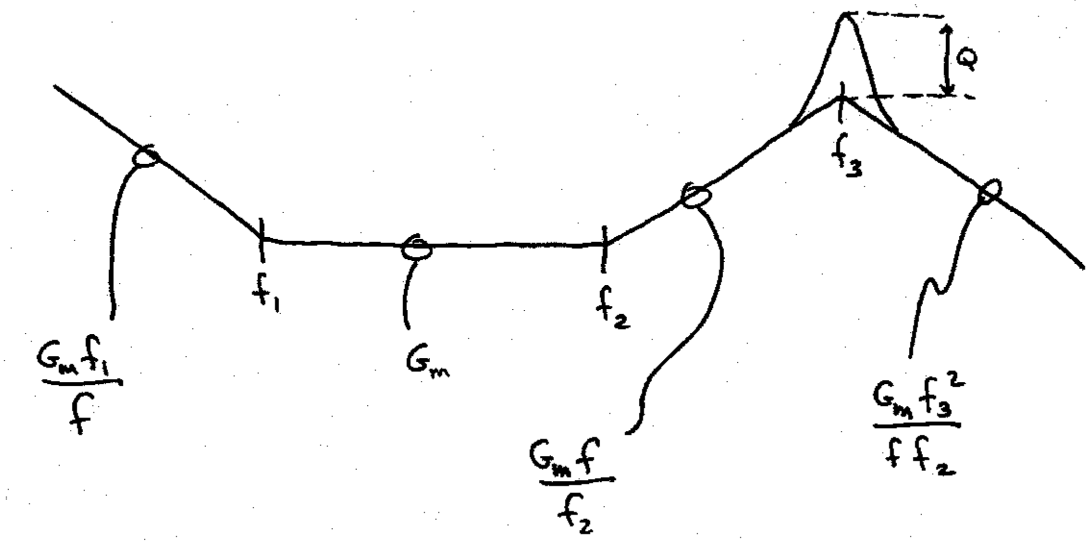
4. for $f_3 \le f$

Then

$$G(s) = G_m \frac{\left(1+\cancel{\frac{\omega_1}{s}}\right)\left(\cancel{1}+\frac{s}{\omega_2}\right)}{\left(\cancel{1}+\cancel{\frac{s}{Q\omega_3}} + \left(\frac{s}{\omega_3}\right)^2\right)} \rightarrow G_m \frac{(1)\left(\frac{s}{\omega_2}\right)}{\left(\frac{s}{\omega_3}\right)^2} = G_m \frac{\omega_3^2}{s\omega_2}$$

which has magnitude $G_m \dfrac{\omega_3^2}{\omega\,\omega_2} = G_m \dfrac{f_3^2}{f\,f_2}$

Summary of analytical expressions for asymptotes:



c) Compute the value of the asymptote at $f = f_3$, and estimate the value of the actual magnitude.

Plug into expression for either adjacent asymptote (either the $f_2 \le f \le f_3$ or the $f_3 \le f$ asymptote).

Result is

$$\frac{G_m f_3}{f_2}$$

At $f = f_3$, the magnitude of the actual curve is given approximately by

$$\frac{Q G_m f_3}{f_2}$$

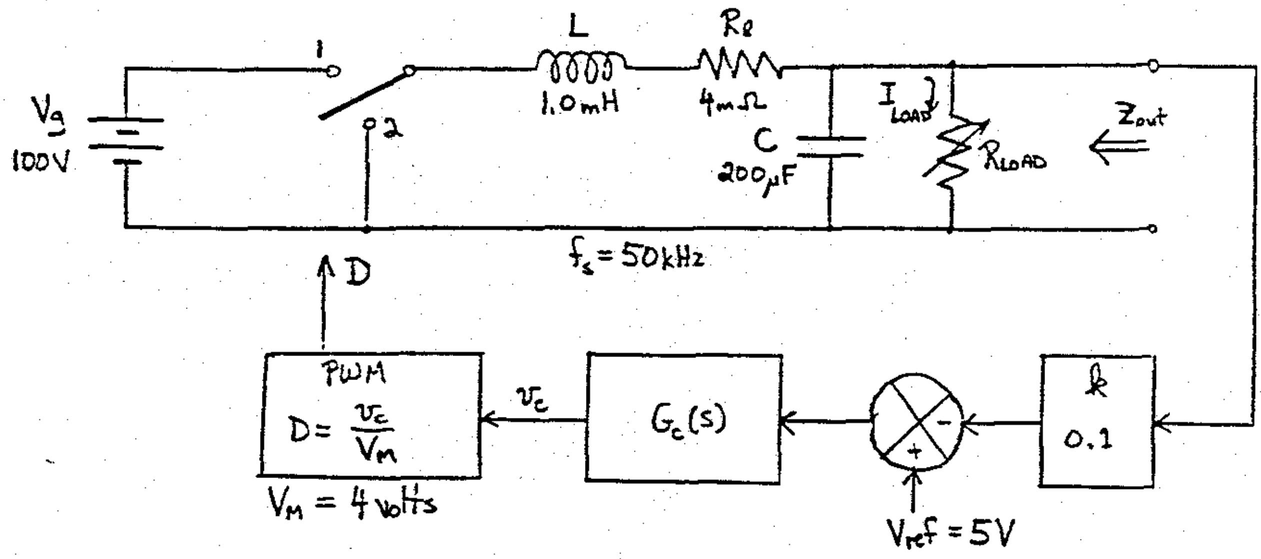
This neglects the (very small) deviation of the actual curve from the asymptotes caused by the zeroes at f_1 and f_2, but it includes the (significant) deviation caused by the Q-factor of the complex poles.

PROBLEM

FEEDBACK DESIGN

Power Electronics

Design of a buck regulator with specified closed-loop output impedance:



It is desired to design a compensation network $G_c(s)$ such that the closed-loop output impedance of the above regulator system is less than 0.2 Ohms over the entire frequency range 0-20kHz. Also, to ensure that the transient response is well-behaved, the Q of the closed-loop system must be less than 1 => 0dB. The load current I_{LOAD} can vary from 5 Amperes to 50 Amperes, and the above specifications must be met for every value of I_{LOAD} in this allowed range. For simplicity, you may assume that V_g does not vary.

Since the state-space averaging method is valid only for frequencies sufficiently less than the switching frequency, you should choose the loop gain crossover frequency f_c to be no greater than $f_s/4 = 12.5kHz$, to be sure that your model is valid.

1. Draw the open-loop output impedance $Z_{out}(s)$. Over what range of frequencies is the output impedance specification not met? Hence, deduce how large the minimum loop gain T(s) must be such that the closed-loop output impedance meets the specifications, and choose a suitable crossover frequency.

MORE ⟶ P.T.O

2. Design a compensation network $G_c(s)$ such that all specifications are met. You must give:

1) Your choice for the transfer function $G_c(s)$
2) The worst-case closed-loop Q.
3) Bode plots of the loop gain $T(s)$ and closed-loop $Z_{out}(s)$ for load currents of 5A and 50A. What effect does variation of R_{LOAD} have on the closed-loop behavior?

HINT: FOR THIS QUESTION, FROM THE SMALL-SIGNAL STATE SPACE AVERAGE MATHEMATICAL MODEL DRAW AN EQUIVALENT CIRCUIT MODEL. ALTERNATIVELY USE AS YOUR CIRCUIT MODEL THE CANONICAL MODEL.

R. TYMERSKI

SOLUTION

Replace open loop converter with its canonical model

$e(s) = \frac{V_g}{D}$; $M = D$

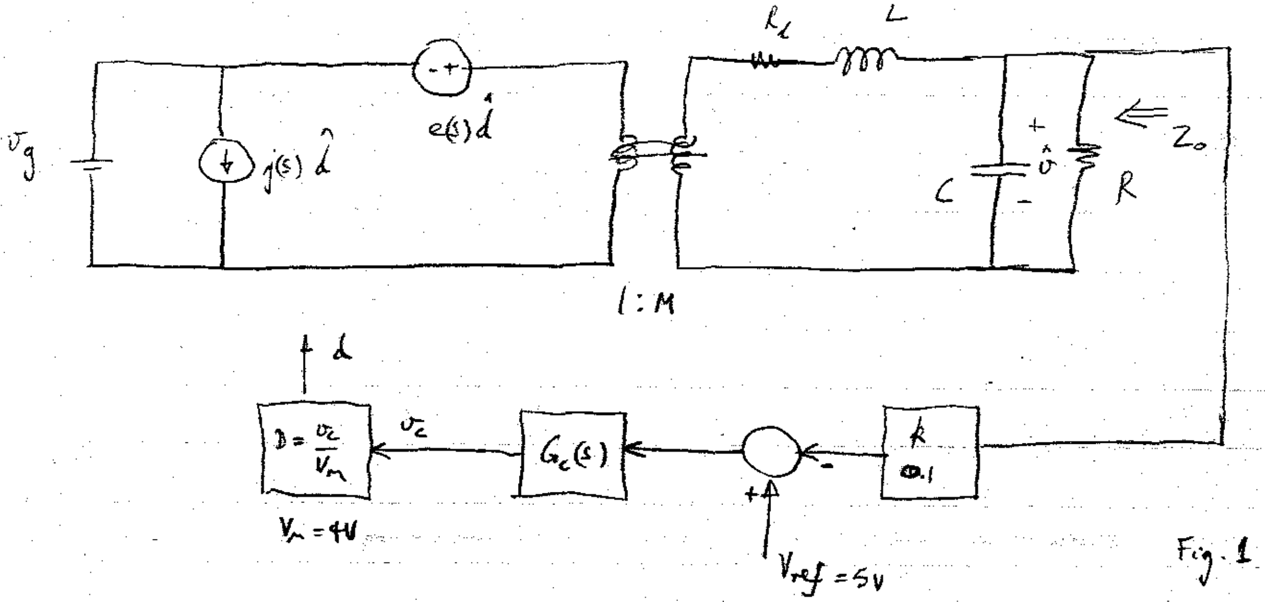
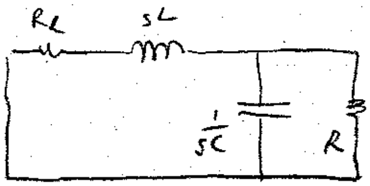


Fig. 1

1) Find Z_o (open loop)

Null all independent sources



$$Z_o = (R_L + sL) \parallel \frac{1}{sC} \parallel R$$

$$= (R_L + sL) \parallel \frac{R}{1+sRC}$$

$$= \frac{(R_L + sL)\frac{R}{1+sRC}}{(R_L + sL) + \frac{R}{1+sRC}}$$

$$= \frac{(R_\ell + sL)R}{R_L + sRCR_L + s^2 RCL + sL + R}$$

$$= \frac{RR_\ell}{R+R_\ell} \frac{\left(1 + s\frac{L}{R_\ell}\right)}{s^2 \frac{RCL}{R+R_\ell} + s\frac{(RCR_\ell + L)}{R+R_\ell} + 1}$$

$$Z_o = R\|R_\ell \frac{\left(1 + s\frac{L}{R_\ell}\right)}{s^2 LC\left(\frac{R\|R_L}{R_\ell}\right) + s\left[(R\|R_\ell)C + \frac{L}{R+R_\ell}\right] + 1}$$

$L = 1.0$ mH ; $C = 200\,\mu F$; $R_\ell = 4\,m\Omega$

$V_o = 50V$ $\quad I_{load} = 5$ to 50 $\Rightarrow$ $R = 1$ to $10\,\Omega$

$\Rightarrow$

$$Z_o = 4\times10^{-3} \frac{\left(1 + s/4\right)}{s^2(0.2\times10^{-6}) + s\underbrace{\left(0.8 + \frac{10^3}{R}\right)10^{-6}}_{\simeq \frac{10^{-3}}{R}} + 1}$$

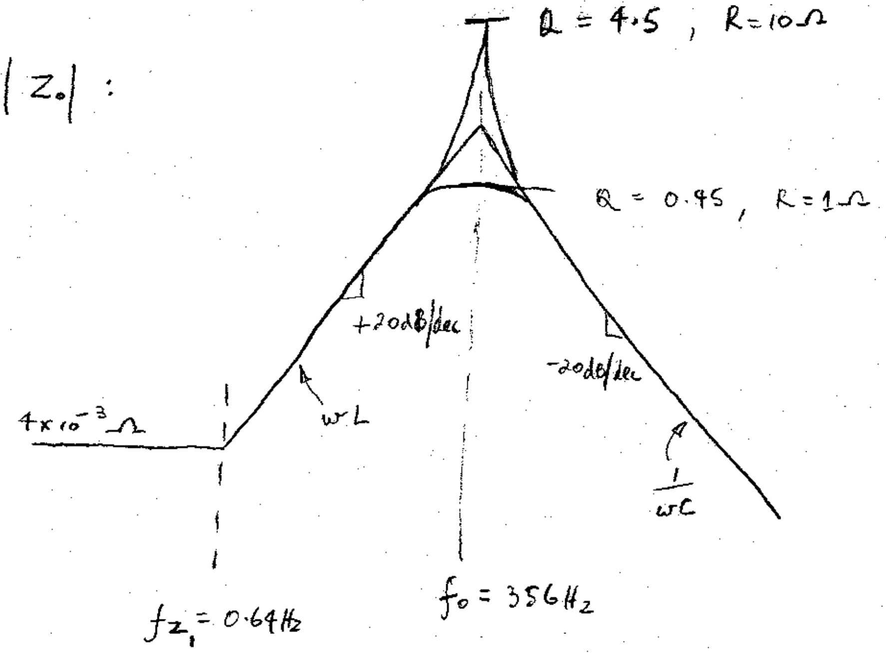
dc impedance $= 4\times10^{-3}\,\Omega$

a zero at $\omega_{z_1} = 4 \longrightarrow f_{z_1} = 0.64$ Hz

complex pole at $\omega_o = \sqrt{\frac{1}{0.2\times10^{-6}}} = 2236 \rightarrow f_o = 356$ Hz

$$\frac{1}{\omega_o Q} = \frac{10^{-3}}{R} \Rightarrow Q = \frac{R}{\omega_o 10^{-3}} = \frac{R}{2.236} \begin{cases} R=1 \Rightarrow Q = 0.45 \\ R=10 \Rightarrow Q = 4.5 \end{cases}$$

$|Z_o|$:



(3)

We require $|Z_o| < 0.2$ for $0 < f < 20\times10^3$

$$\Rightarrow \omega L < 0.2$$

$$\Rightarrow f < \frac{0.2}{2\pi \times 10^{-3}} = 32 \text{ Hz}$$

also

$$\frac{1}{\omega C} < 0.2$$

$$\Rightarrow f > \frac{1}{2\pi \times 0.2 \times 200\times10^{-6}} = 3979 \text{ Hz}$$

$\therefore$ output impedance spec. is not met for the frequency range

$$32 < f < 3979 \text{ Hz}$$

Magnitude of Z_o at intersection of asymptotes: (4)

at $\omega L = \frac{1}{\omega C} \Rightarrow \omega_o = \sqrt{\frac{1}{LC}}$

$\Rightarrow f_o = 356\ Hz$

$2\pi f_o L = 2\pi (356) \times 10^{-3} = 2.24\ \Omega$

The Q can be as high as 4.5

$\Rightarrow |Z_o|_{max} = 4.5 \times 2.24 = 10\ \Omega.$

We need to reduce $|Z_o|_{max} < 0.2$

Now in the presence of feedback (with loop gain $T(s)$) the output impedance Z_{of} becomes

$$Z_{of} = \frac{Z_o}{1 + T(s)}$$

For T large we require

$$T \approx \frac{Z_o}{Z_{of}} = \frac{10}{0.2} = 50$$

From Fig. 1:

$$T(s) = \frac{\hat{v}}{\hat{d}} \frac{\hat{d}}{\hat{v}_c} G_c(s)\, k$$

Now $\frac{\hat{d}}{\hat{v}_c} = M_d = \frac{1}{4}$; $k = 0.1$

$$\frac{\hat{v}}{\hat{d}} = V_g \frac{R}{R_\ell + R} \frac{1}{s^2 LC\left(\frac{R}{R_\ell+R}\right) + s\left(R_\ell C + \frac{L}{R}\right)\left(\frac{R}{R_\ell+R}\right) + 1}$$

$$\approx V_g \frac{1}{s^2 LC + s\frac{L}{R} + 1}$$

since $R_\ell << R$

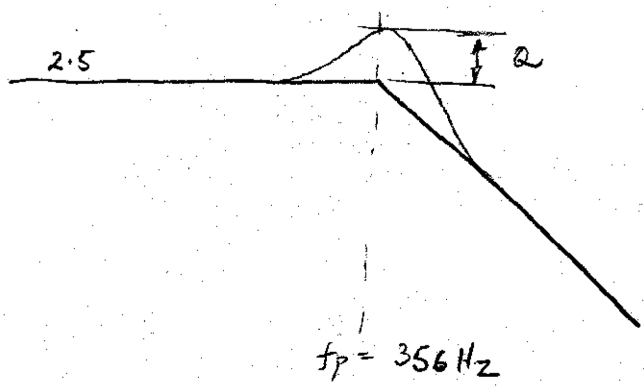
$R_\ell = 0.004$

$R = 1$ ~~to~~ 10

$$\Rightarrow T(s) = G_c(s) \frac{2.5}{s^2\left(0.2\times10^{-6}\right) + s\frac{10^{-3}}{R} + 1}$$

$R = 1$ to $10\,\Omega$

$\left|\frac{T(s)}{G_c(s)}\right|$



Let us choose $G_c(s)$ to be of the form

$$G_c(s) = G_m \frac{1 + s/\omega_z}{1 + s/\omega_p}$$

this is lead - lag compensation.

$\therefore$ $T(s)$ becomes

$$T(s) = \overbrace{k\, M_d\, G_m V_g}^{T_m} \frac{1 + \frac{s}{\omega_z}}{1 + s/\omega_p} \quad \frac{1}{1 + \frac{s}{\omega_o Q} + \frac{s^2}{\omega_o^2}}$$

where $\omega_o < \omega_z < \omega_p$

and $T_m = k\, M_d\, G_m V_g > 50$ $\qquad k = 0.1,\ M_d = \frac{1}{4}$

We can sketch $|T(s)|$, $|1+T(s)|$, $|Z_o|$ and $|Z_{of}|$ as follows for the two cases $R_{LOAD} = 1$ and $10\ \Omega$

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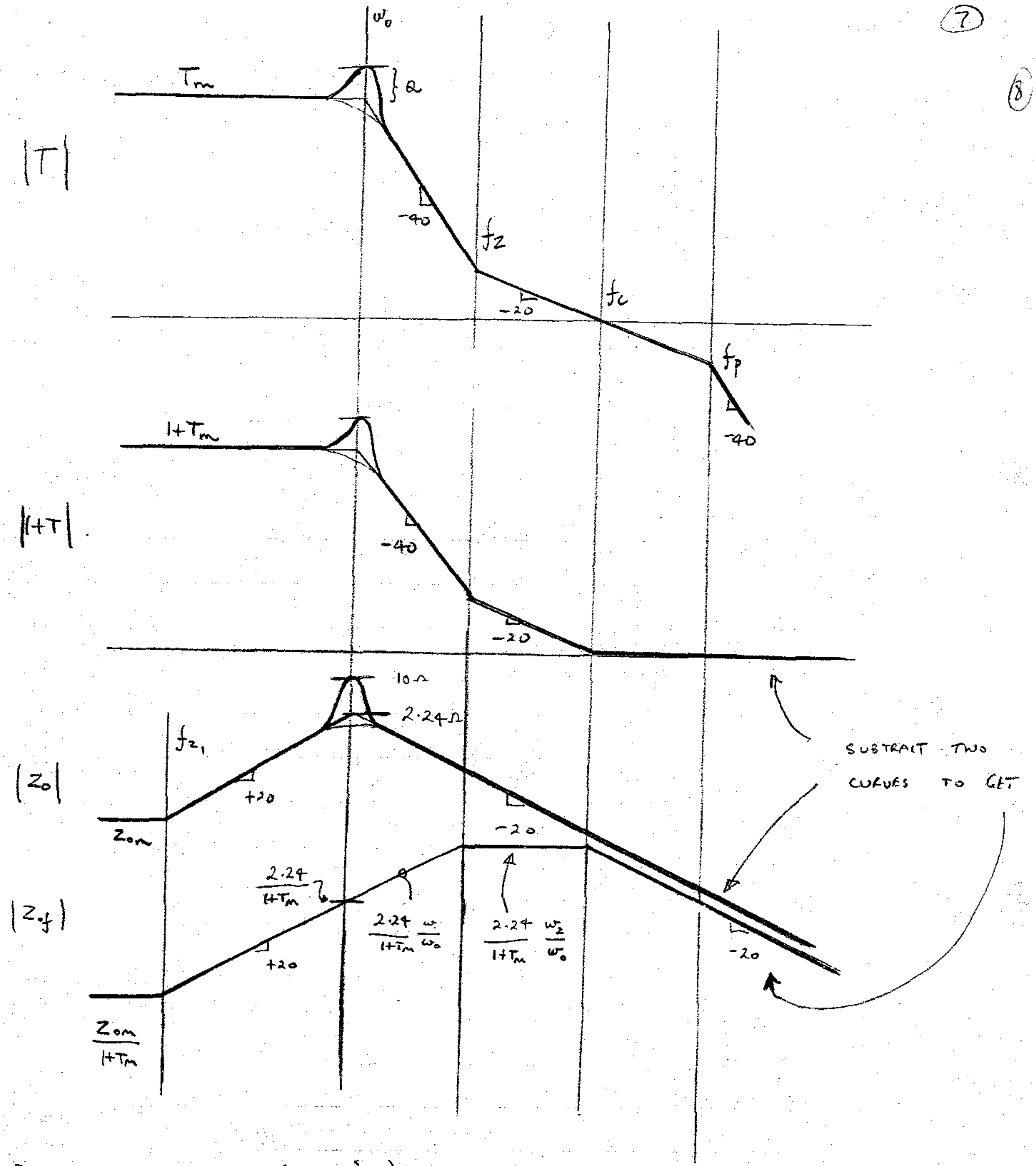


$$\Rightarrow \quad Z_{of} = \frac{Z_{om}}{1+T_m}\,\frac{\left(1+\frac{s}{\omega_{z_1}}\right)}{\left(1+\frac{s}{\omega_z}\right)\left(1+\frac{s}{\omega_c}\right)}$$

Choice of compensation parameters: $G_c(s) = G_m \dfrac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}$

we require $T_m = k M_d G_m V_g > 50$

$$k = 0.1 \;;\; M_d = \tfrac{1}{4} \;;\; V_g = 100$$

$$\Rightarrow \quad G_m > \frac{50 \times 4 \times 10}{100} = 20 \quad \to 26\ dB$$

Let us choose a unity gain crossover frequency of $f_c = 10\ kHz$. Let us place $f_z = 800 < \frac{f_c}{10}$

The phase margin can be controlled by placing ω_p appropriately:

$$\varphi_m = \text{phase margin} \approx 180^\circ - \left[-\tan^{-1}\left(\frac{10{,}000}{800}\right) + \tan^{-1}\left(\frac{10{,}000}{f_p}\right) + 180^\circ\right]$$

$\approx$ phase contribution due to double pole at ω_o

$$\varphi_m = 85 - \tan^{-1}\left(\frac{10{,}000}{f_p}\right)$$

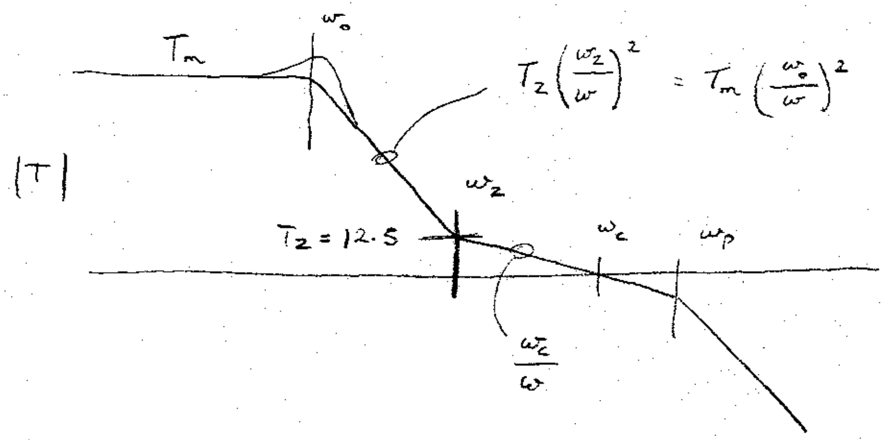
$$\Rightarrow \tan^{-1}\frac{10{,}000}{f_p} = -\varphi_m + 85 \quad \Rightarrow f_p = \frac{10{,}000}{\tan\left[85 - \varphi_m\right]}$$

for a phase margin of $\varphi_m = 60^\circ$

$$\Rightarrow f_p = 21{,}445$$

For $\varphi_m = 60 \Rightarrow$ closed loop $Q \approx -1.75\ dB$
(read from chart from class notes)
this figure is independent of the load.

$\therefore$ Final design for $G_c(s) = G_m \dfrac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}$



$$T_z \equiv \text{Gain at } \omega_z = \frac{\omega_c}{\omega_z} = \frac{10{,}000}{800} = 12.5$$

$$T_m \equiv \text{Gain at } \omega_o = T_z\left(\frac{\omega_z}{\omega_o}\right)^2 = 12.5\left(\frac{800}{356}\right)^2 = 63 > 50$$

$$\left(\text{Check: } T_m\left(\frac{\omega_o}{\omega_z}\right)^2 = 63\left(\frac{356}{800}\right)^2 = 12.5\right)$$

$$T_m = k M_d V_g G_m$$

$$\Rightarrow G_m = \frac{T_m}{k M_d V_g} = \frac{63}{0.1 \times \frac{1}{4} \times 100} = 25.2 \longrightarrow 28\,dB$$

$\therefore$
$$G_m = 25$$
$$\omega_z = 2\pi(800)$$
$$\omega_p = 2\pi(21{,}445)$$

CHECK

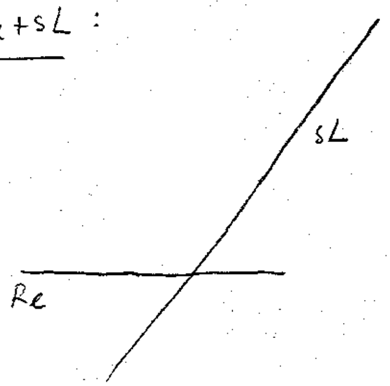
$$\text{Peak } z_{of} = \frac{2.24}{4T_m}\frac{\omega_z}{\omega_o} = \frac{2.24}{64}\frac{2\pi(800)}{2\pi(356)} = 0.079\,\Omega < 0.2\,\Omega$$

APPENDIX :

SEMI - GRAPHICAL METHOD FOR CONSTRUCTING BODE PLOT OF $|Z_o|$

$$Z_o = (R_L + sL) \parallel \frac{1}{sC} \parallel R$$

$R_L + sL$:



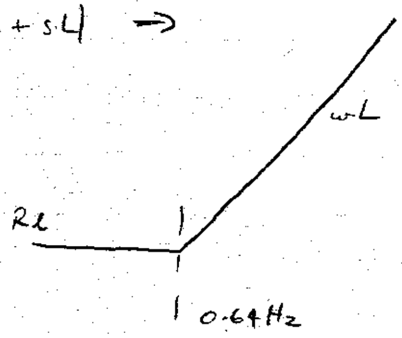
Take the bigger of the two

at crossover $R_L = sL$

$$\Rightarrow \omega_z = \frac{R_L}{L} = \frac{4 \times 10^{-3}}{1 \times 10^{-3}} = 4$$

$$\Rightarrow f_z = \frac{4}{2\pi} = 0.64\ Hz$$

$\therefore \quad |R + s \cdot L| \rightarrow$



$(R_\ell + sL) \| \frac{1}{sC} \| R$: parallel combination; take the smallest quantity

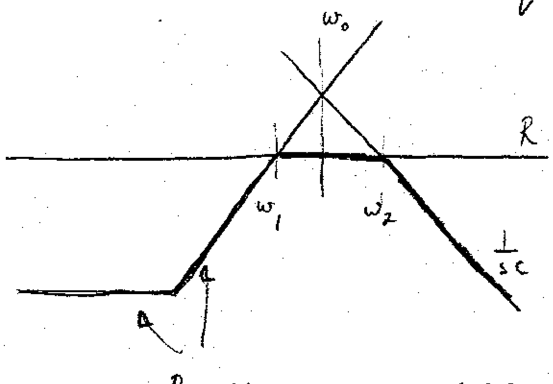


FIG. A

OR

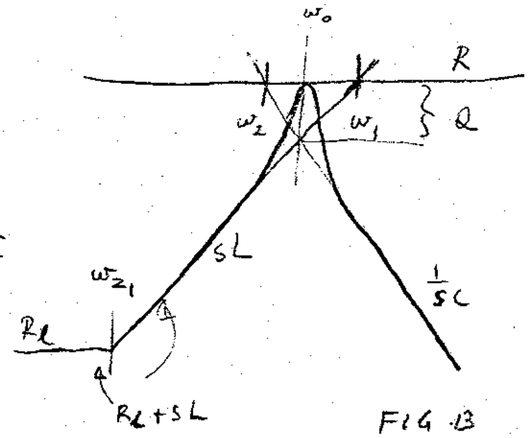


FIG. B

at crossover #1: $sL = R$

$$\Rightarrow \omega_1 = \frac{R}{L} = \begin{cases} 10^3, & R=1 \\ 10^4, & R=10 \end{cases}$$

at crossover #2: $\frac{1}{sC} = R$

$$\omega_2 = \frac{1}{RC} = \begin{cases} 5\times10^3, & R=1 \\ 5\times10^2, & R=10 \end{cases}$$

From the values for ω_1 and ω_2 we see that the poles will change as R changes. Clearly for R=1, Fig A applies as $\omega_1 < \omega_2$. However, for R=10, Fig B applies as $\omega_2 < \omega_1$.

At the crossing point of the asymptotes

$$sL = \frac{1}{sC} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

the impedance $= \omega_o L \Rightarrow \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}} \equiv$ characteristic impedance

$$Q = \frac{R}{\sqrt{\frac{L}{C}}} \Rightarrow \begin{cases} \frac{1}{2.24} = 0.45 & \text{for } R = 1 \\ \frac{10}{2.25} = 4.5 & \text{for } R = 10 \end{cases}$$

$$\left(\sqrt{\frac{L}{C}} = \sqrt{\frac{10^{-3}}{200 \times 10^{-6}}} = 2.24\,\Omega \right)$$

From the Bode plot of Fig. 8 we can write the factored pole zero expression for Z_o as

$$Z_o(s) = R_L \frac{\left(1 + \frac{s}{\omega_{z_1}}\right)}{1 + \frac{s}{\omega_o Q} + \frac{s^2}{\omega_o^2}}$$

$$\omega_{z_1} = \frac{R_L}{L} = 2\pi\,(0.64)$$

$$\omega_o = \sqrt{\frac{1}{LC}} = 2\pi\,(356)$$

$$Q = \frac{R}{\sqrt{\frac{L}{C}}} = \begin{cases} 0.45, & R = 1 \\ 4.5, & R = 10 \end{cases}$$

$R_L = 0.004$

EXAMPLE EE446/EE546 FINAL QUESTIONS

(20 marks) PROBLEM 1.

For the converter of Fig. 1, (assuming continuous conduction operation),

(15 marks) a) Determine an expression for Z_{in}, the linearized, average impedance seen by the source V_g.

(5 marks) b) Sketch the magnitude straight-line asymptote of Z_{in}, noting the salient features such as the values of the break frequencies, level of constant impedance values and peaking levels.

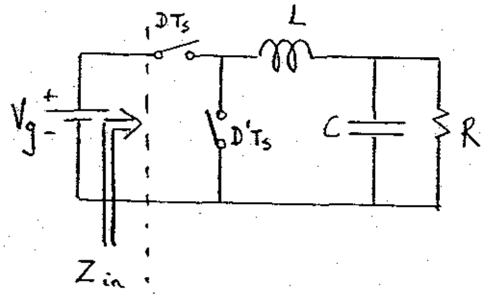


FIG. 1

(10 MARKS) PROBLEM 3.

(5) a) The transfer function, A(s), of a converter was determined to be

$$A(s) = A_m \frac{1 - s/\omega_z}{(1 + s/\omega_p)(1 + s/\omega_c)}$$

$$A_m = 1.53$$

$$\omega_z = 2\pi(2.29)kHz$$

$$\omega_p = 2\pi(16.4)Hz$$

$$\omega_c = 2\pi(7.69)kHz$$

A compensation network with transfer function, $G_c(s)$, where

$$G_c(s) = G_m(1 + \omega_1/s)$$

$$G_m = 30$$

$$\omega_1 = 2\pi(9.5)Hz$$

completes a feedback loop as shown in Fig. 2. Using a graphical technique or otherwise, determine the factored pole-zero expression for the feedback factor, 1 + T, where T is the loop gain. On a sketch of |1+T| label the numerical values of any poles and zeroes and constant gain regions.

(5) b) The open loop output impedance, Z_o, of the converter is given as

$$Z_o = R_{om} \frac{1}{1 + s/\omega_p}$$

$$R_{om} = 4.88\Omega$$

$$\omega_p = 2\pi(16.4)Hz$$

Using a graphical technique or otherwise, determine the factored pole-zero expression for the closed loop output impedance, Z_{of}, for the loop gain given in part (a). On a sketch of $|Z_{of}|$ label the numerical values of any poles and zeroes and constant gain regions.

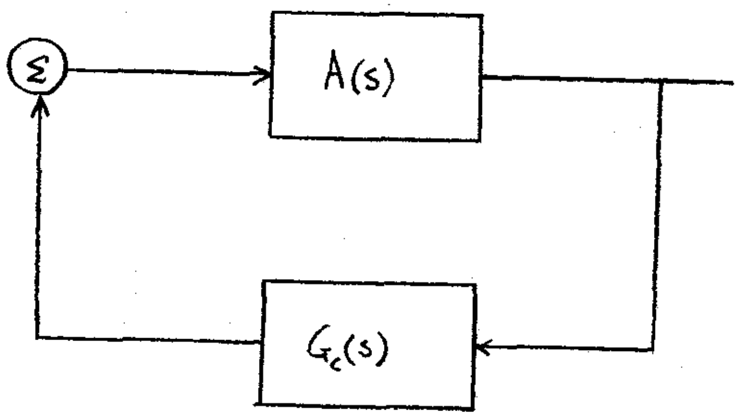


FIG. 2

(20 marks) PROBLEM 4.

The control-to-output transfer function of a certain converter is given by the following

$$\frac{\hat{v}}{\hat{d}} = \frac{V_g}{D'^2} \frac{1 - s\dfrac{L}{D'^2 R}}{1 + s\dfrac{L}{D'^2 R} + s^2 \dfrac{LC}{D'^2}}$$

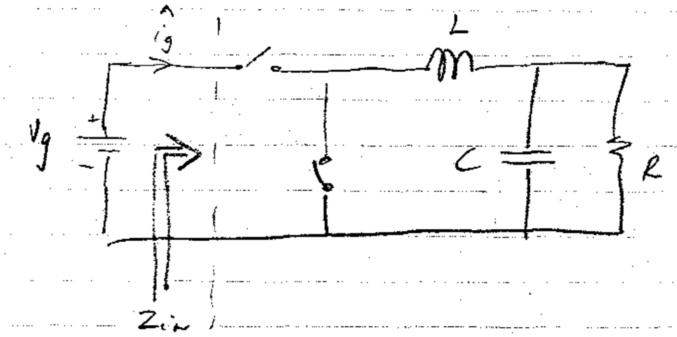
For the values, $D = 0.5, R = 10, V_g = 30, L = 160\mu H$ and $C = 160\mu F$.

a) Draw a Bode plot (magnitude and phase) using asymptotic straight line segment approximations. Label all salient features, such as, frequency breakpoints, constant gain/phase regions, etc.

b) Using your Bode plot, or otherwise, determine the unity gain frequency and the phase shift at this frequency.

SOLUTION

PROBLEM 1 :



$$Z_{in} = \frac{\hat{v}_g}{\hat{i}_g}$$

Now $\dot{\hat{x}} = A\hat{x} + B\hat{v}_g + B_d\hat{d}$

$\hat{y} = C\hat{x} + E\hat{v}_g + E_d\hat{d}$

Now $\hat{d} = 0$ and let $\hat{y} = \hat{i}_g$

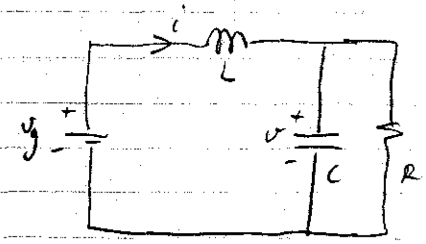
$$\Rightarrow \frac{\hat{i}_g}{\hat{v}_g} = \frac{1}{Z_{in}} = \frac{C\hat{x}}{\hat{v}_g} + E$$

where $\frac{\hat{x}}{\hat{v}_g} = (sI - A)^{-1}B$

Now lets find the appropriate A, B, $C + E$

$A = DA_1 + D'A_2$
$B = DB_1 + D'B_2$
$C = DC_1 + D'C_2$
$E = DE_1 + D'E_2$

DURING DT_s



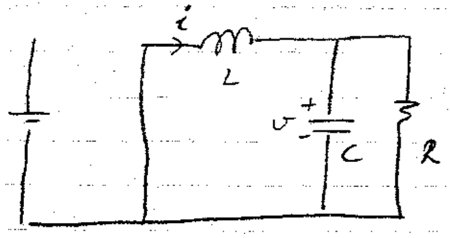
$$-V_g + L\frac{di}{dt} + v = 0$$

$$\Rightarrow \frac{di}{dt} = \frac{-v}{L} + \frac{V_g}{L}$$

$$i = C\frac{dv}{dt} + \frac{v}{R}$$

$$\Rightarrow \frac{dv}{dt} = \frac{i}{C} - \frac{v}{RC}$$

DURING $D'T_s$



$$\frac{di}{dt} = \frac{-v}{L}$$

$$\frac{dv}{dt} = \frac{i}{C} - \frac{v}{RC}$$

---

let $x = [i \quad v]^T$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A_1} x + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{B_1} V_g$$

$$i_g = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_1} x + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{E_1} V_g$$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A_2} x + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{B_2} V_g$$

$$i_g = \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{C_2} x + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{E_2} V_g$$

③

$$\Rightarrow \quad A = A_1 = A_2$$

$$B = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} D & 0 \end{bmatrix}$$

$$E = E_1 = E_2 = 0$$

Now $\quad \dfrac{\hat{i}_g}{\hat{v}_g} = \dfrac{1}{Z_{in}} = C\,(sI - A)^{-1} B + E$

$$\Rightarrow \quad \frac{1}{Z_{in}} = \begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} s & \frac{1}{L} \\ -\frac{1}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} s + \frac{1}{RC} & -\frac{1}{L} \\ \frac{1}{C} & s \end{bmatrix} \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}}{s\left(s + \frac{1}{RC}\right) + \frac{1}{LC}}$$

$$= \frac{\begin{bmatrix} D\left(s + \frac{1}{RC}\right) & -\frac{D}{L} \end{bmatrix} \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$= \frac{\frac{D^2}{L}\left(s + \frac{1}{RC}\right)}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$= \frac{\frac{D^2}{\cancel{L}} \frac{1}{R\cancel{C}} \left(1 + RCs\right)}{\frac{1}{L\cancel{C}} \left(LCs^2 + L\cancel{C} \frac{s}{R\cancel{C}} + 1\right)}$$

$$\frac{1}{Z_{in}} = \frac{D^2}{R} \frac{1 + RCs}{LCs^2 + \frac{sL}{R} + 1}$$

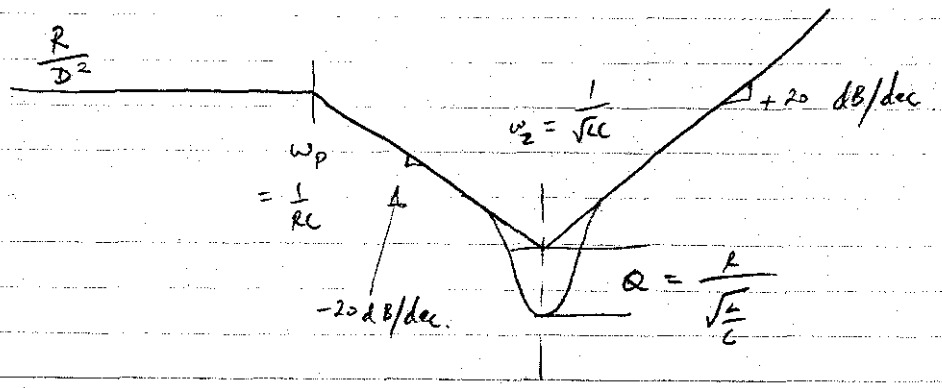
$$\Rightarrow \boxed{Z_{in} = \frac{R}{D^2} \frac{LCs^2 + s\frac{L}{R} + 1}{1 + RCs}}$$

$\Rightarrow$ Pole at $\omega_p = \frac{1}{RC}$

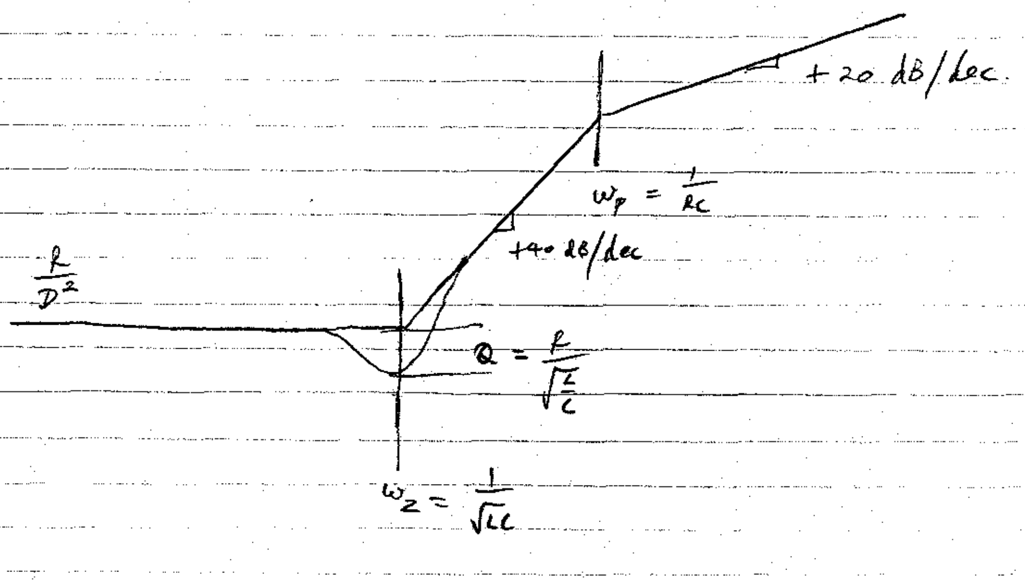
zeros at $\omega_z = \frac{1}{\sqrt{LC}}$ $\qquad Q = \frac{R}{\sqrt{\frac{L}{C}}}$

b) TWO CASES

1. $\omega_p < \omega_z$



2. $\omega_p > \omega_z$

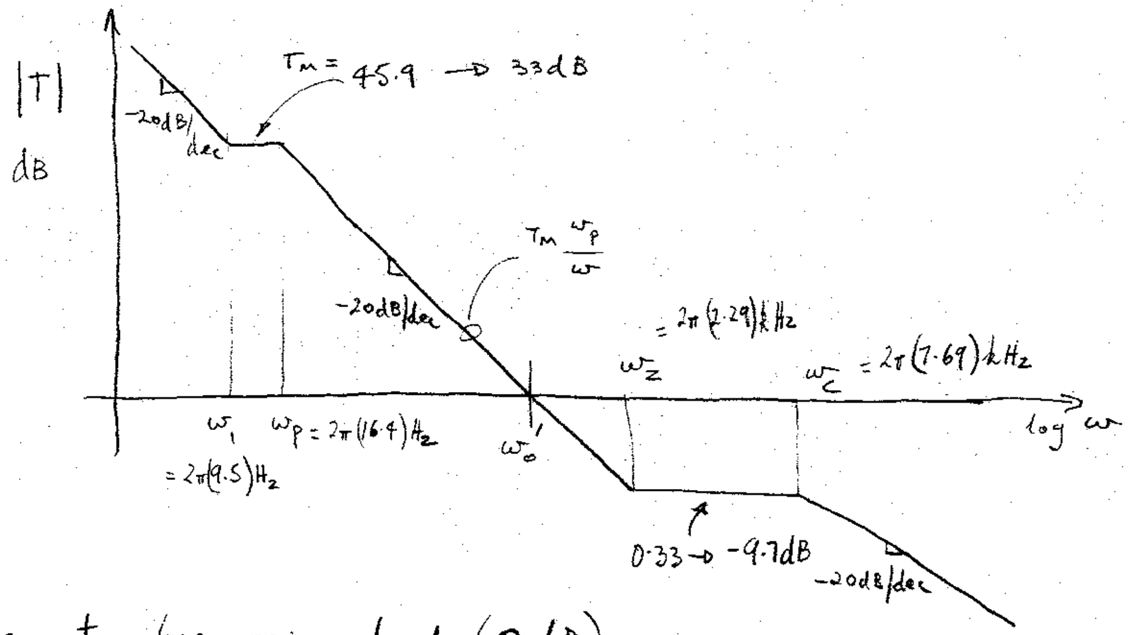


SOLUTION

PROBLEM 3

$$T(s) = A(s)\,G_c(s) = \frac{A_m G_m \left(1 + \omega_1/s\right)\left(1 - s/\omega_z\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + s/\omega_c\right)}$$

$$T_m = A_m G_m = 30 \times 1.53 = 45.9 \longrightarrow 33\,dB$$



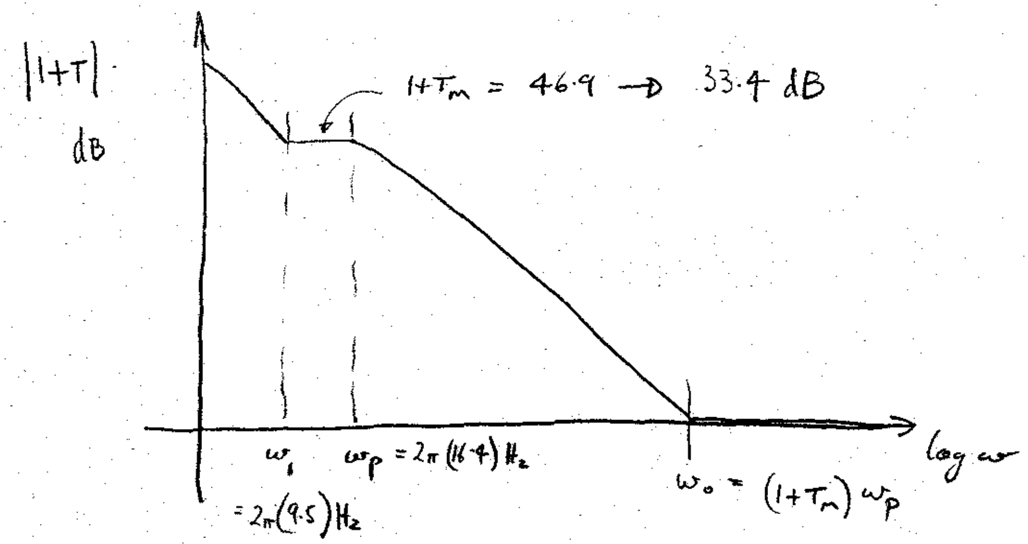
ω_o is freq. at loop gain of 1 (0 dB)

$$1 = \frac{T_m \omega_p}{\omega_o'}$$

$$\Rightarrow \omega_o' = T_m \omega_p = 45.9 \times 2\pi\,(16.4) = 2\pi\,(753)\ Hz$$

Gain at freq. ω_z: $\quad T_z = \frac{T_m \omega_p}{\omega_z} = 45.9 \times \frac{16.4}{2290}$

$$= 0.33 \rightarrow -9.7\,dB$$



$$\omega_o = (1+T_m)\,\omega_p$$

$$= (46.9)\,2\pi\,(16.4) = 2\pi\,(769)\ Hz$$

From the plot of $|1+T|$ we can write

$$1+T(s) = (1+T_m)\,\frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_o}\right)}{\left(1+\frac{s}{\omega_p}\right)}$$

$$1+T_m = 46.9 \longrightarrow 33.4$$

$$\omega_1 = 2\pi\,(9.5)\ Hz$$
$$\omega_p = 2\pi\,(16.4)\ Hz$$
$$\omega_o = (1+T_M)\,\omega_p = 2\pi\,(769)\ Hz$$

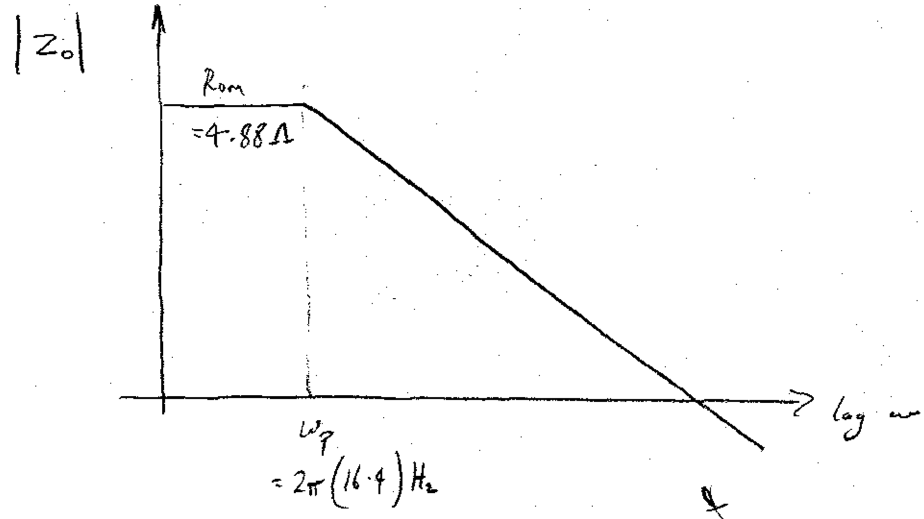
b)

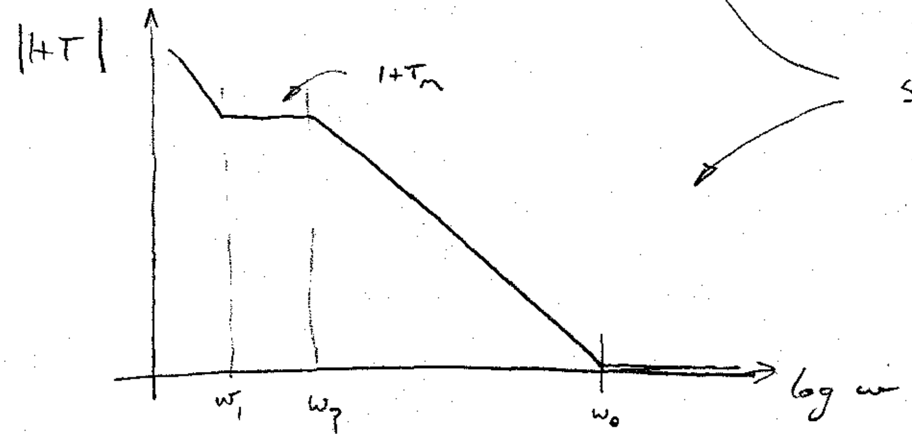
$$Z_o = R_{om}\,\frac{1}{1+\frac{s}{\omega_p}}$$

$$R_{om} = 4.88\,\Omega$$
$$\omega_p = 2\pi\,(16.4)\ Hz$$

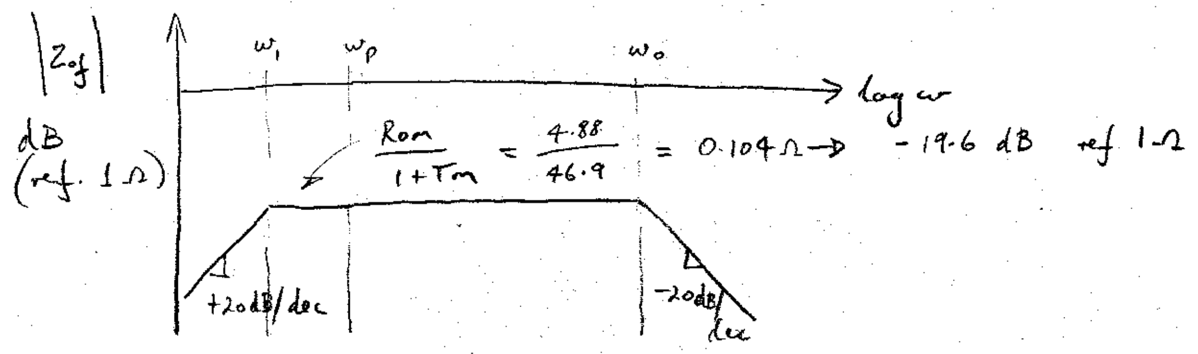
See plot, over the page.

$$Z_{of} = \frac{Z_o}{1+T} \quad \leftarrow \text{from feedback theory}$$





SUBTRACT THESE TWO CURVES TO GET Z_{of}



From the plot of $|Z_{of}|$ we can write

$$Z_{of} = \frac{R_{om}}{1+T_m} \frac{1}{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_o}\right)}$$

$\omega_1 = 2\pi\,(9.5)\,Hz$

$\omega_o = (1+T_M)\,\omega_p$

$= 2\pi\,(769)\,Hz$

PROBLEM 4 — SOLUTION

a) This problem is taken from the lecture notes from where we see

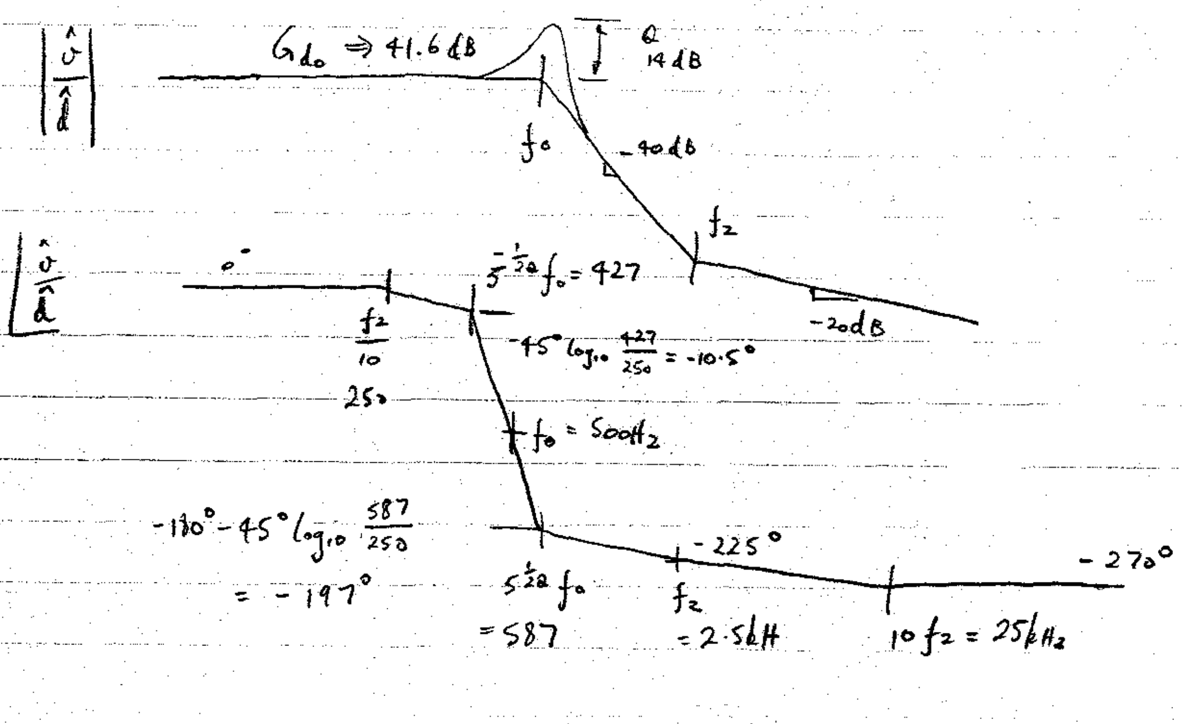
$$\frac{\hat{v}}{\hat{d}} = \frac{V_g}{D'^2} \; \frac{1 - s\frac{L}{D'^2 R}}{1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}} \equiv G_{do} \; \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

where $G_{do} = 120 \Rightarrow 41.6\ dB$

$f_z = 2.5\ kHz$ (RHP) $(15{,}708\ rads/sec)$

$f_o = 500\ Hz$ $(3{,}142\ rads/sec)$

$Q = 5 \Rightarrow 14\ dB$



b) The magnitude along the -40dB slope away from the resonant frequency is given by

$$G_{do}\left(\frac{\omega_o}{\omega}\right)^2$$

$\therefore$ at f_z, $\left|\frac{\hat{v}}{\hat{d}}\right|_{f_z = 2.5kH} \cong 120\left(\frac{500}{2500}\right)^2 = 4.8$

The magnitude along the -20dB slope is given by

$$G_{f_z}\ \frac{\omega_z}{\omega}$$

$\therefore$ the unity gain frequency $= f_u = G_{f_z} f_z$

$$= 4.8 \times 2500$$
$$= 12\ kHz \quad (75400\ rad/sec$$

Using MATLAB (see program later) we find $f_u = 12{,}202\ Hz$

To determine the phase we realize that since 12 kHz is very much higher than the double pole the phase contribution is very close to -180 degrees $\Rightarrow$

phase at 12kHz $= -180 - \tan^{-1}\frac{12000}{2500}$ ← the contribution from the RHP zero

$= -180 - 78.23$

$= -258.23$ ← MATLAB gives -258.01

Alternatively using the straight line asymptote approach for the phase we realize the phase at f_z = $-180 \quad -45 \quad = -225$

phase due to double pole

phase due to zero

Now at $-45°/\text{dec}$ slope the phase change from $f_z = 2500\ Hz$ to $f_u = 12000\ Hz$ is given by

$$-45° \log_{10} \frac{12000}{2500} = -30.66°$$

$\therefore$ total phase $= -225 - 30.66$

$= -255.66$ ← thus a slight error from $-258°$ is introduced using the straight line approximations

```
% This m-file plots the magnitude and phase response of the transfer
% function given in Question 4.
% The unity gain frequency and the phase at this frequency are also
% determined.

vg=30;r=10;d=0.5;l=160e-6;c=160e-6;
dp=1-d;

m=vg/(dp*dp);
a0=1;a1=-l/(dp*dp*r);a2=0;
b0=1;b1=l/(dp*dp*r);b2=(l*c)/(dp*dp);

num=m*[a2 a1 a0];
den=[b2 b1 b0];

w=logspace(2,6,1000);
[mag,phase]=bode(num,den,w);

[Gm,Pm,Wcg,Wcp]=margin(mag,phase,w)

magdb=20*log10(mag);
semilogx(w,magdb)
pause
semilogx(w,phase)
pause
f_unity=Wcp/(2*pi)      % the unity gain frequency
phase_u=-180+Pm         % the phase at the unity gain frequency
```