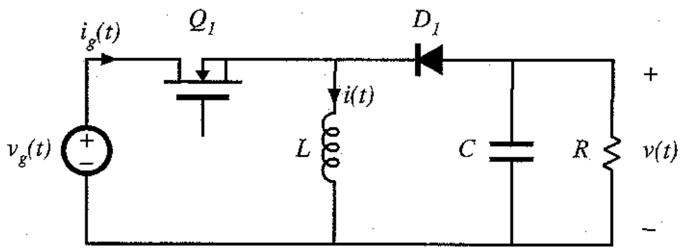
7.4.4. Example: State-space averaging of a nonideal buck-boost converter



Model nonidealities:

- MOSFET on-resistance R_{on}
- Diode forward voltage drop V_D

state vector

$$\mathbf{x}(t) = \begin{bmatrix} i(t) \\ v(t) \end{bmatrix}$$

input vector

$$\mathbf{u}(t) = \begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}$$

output vector

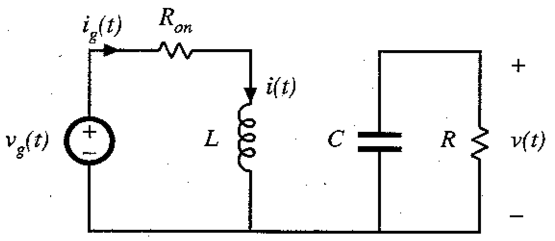
$$\mathbf{y}(t) = \begin{bmatrix} i_g(t) \end{bmatrix}$$

Subinterval 1

$$L\frac{di(t)}{dt} = v_g(t) - i(t)\,R_{on}$$

$$C\frac{dv(t)}{dt} = -\frac{v(t)}{R}$$

$$i_g(t) = i(t)$$

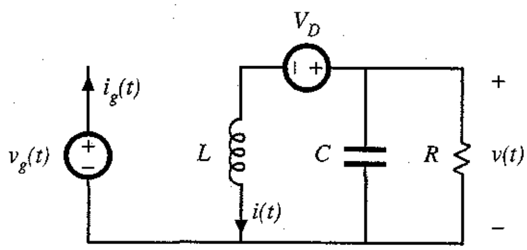


$$\underbrace{\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}}_{\mathbf{K}} \underbrace{\frac{d}{dt}\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\frac{d\mathbf{x}(t)}{dt}} = \underbrace{\begin{bmatrix} -R_{on} & 0 \\ 0 & -\frac{1}{R} \end{bmatrix}}_{\mathbf{A}_1} \underbrace{\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\mathbf{x}(t)} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{B}_1} \underbrace{\begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}}_{\mathbf{u}(t)}$$

$$\underbrace{\begin{bmatrix} i_g(t) \end{bmatrix}}_{\mathbf{y}(t)} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbf{C}_1} \underbrace{\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\mathbf{x}(t)} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{\mathbf{E}_1} \underbrace{\begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}}_{\mathbf{u}(t)}$$

Subinterval 2

$$L\frac{di(t)}{dt} = v(t) - V_D$$
$$C\frac{dv(t)}{dt} = -\frac{v(t)}{R} - i(t)$$
$$i_g(t) = 0$$



$$\underbrace{\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}}_{\mathbf{K}} \underbrace{\frac{d}{dt}\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\frac{d\mathbf{x}(t)}{dt}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -\frac{1}{R} \end{bmatrix}}_{\mathbf{A}_2} \underbrace{\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\mathbf{x}(t)} + \underbrace{\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{B}_2} \underbrace{\begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}}_{\mathbf{u}(t)}$$

$$\underbrace{\begin{bmatrix} i_g(t) \end{bmatrix}}_{\mathbf{y}(t)} = \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{\mathbf{C}_2} \underbrace{\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\mathbf{x}(t)} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{\mathbf{E}_2} \underbrace{\begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}}_{\mathbf{u}(t)}$$

Evaluate averaged matrices

$$\mathbf{A} = D\mathbf{A}_1 + D'\mathbf{A}_2 = D\begin{bmatrix} -R_{on} & 0 \\ 0 & -\frac{1}{R} \end{bmatrix} + D'\begin{bmatrix} 0 & 1 \\ -1 & -\frac{1}{R} \end{bmatrix} = \begin{bmatrix} -DR_{on} & D' \\ -D' & -\frac{1}{R} \end{bmatrix}$$

In a similar manner,

$$\mathbf{B} = D\mathbf{B}_1 + D'\mathbf{B}_2 = \begin{bmatrix} D & -D' \\ 0 & 0 \end{bmatrix}$$
$$\mathbf{C} = D\mathbf{C}_1 + D'\mathbf{C}_2 = \begin{bmatrix} D & 0 \end{bmatrix}$$
$$\mathbf{E} = D\mathbf{E}_1 + D'\mathbf{E}_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

DC state equations

$$\mathbf{0} = \mathbf{A}\,\mathbf{X} + \mathbf{B}\,\mathbf{U}$$
$$\mathbf{Y} = \mathbf{C}\,\mathbf{X} + \mathbf{E}\,\mathbf{U}$$

or,

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -DR_{on} & D' \\ -D' & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} D & -D' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

$$\begin{bmatrix} I_g \end{bmatrix} = \begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

DC solution:

$$\begin{bmatrix} I \\ V \end{bmatrix} = \left(\frac{1}{1 + \frac{D}{D'^2}\frac{R_{on}}{R}} \right) \begin{bmatrix} \frac{D}{D'^2 R} & \frac{1}{D' R} \\ -\frac{D}{D'} & 1 \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

$$\begin{bmatrix} I_g \end{bmatrix} = \left(\frac{1}{1 + \frac{D}{D'^2}\frac{R_{on}}{R}} \right) \begin{bmatrix} \frac{D^2}{D'^2 R} & \frac{D}{D' R} \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

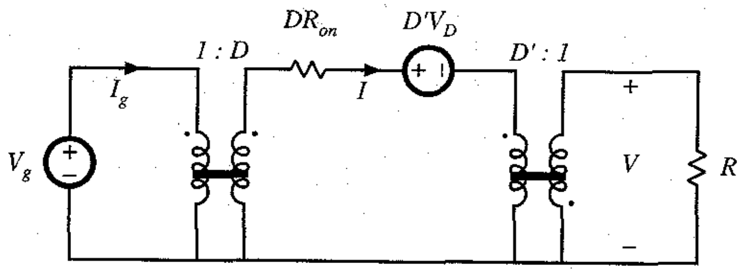
Steady-state equivalent circuit

DC state equations:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -DR_{on} & D' \\ -D' & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} D & -D' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

$$\begin{bmatrix} I_g \end{bmatrix} = \begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

Corresponding equivalent circuit:



Small-signal ac model

Evaluate matrices in small-signal model:

$$\left(\mathbf{A}_1-\mathbf{A}_2\right)\mathbf{X}+\left(\mathbf{B}_1-\mathbf{B}_2\right)\mathbf{U}=\begin{bmatrix}-V\\ I\end{bmatrix}+\begin{bmatrix}V_g-IR_{on}+V_D\\ 0\end{bmatrix}=\begin{bmatrix}V_g-V-IR_{on}+V_D\\ I\end{bmatrix}$$

$$\left(\mathbf{C}_1-\mathbf{C}_2\right)\mathbf{X}+\left(\mathbf{E}_1-\mathbf{E}_2\right)\mathbf{U}=\left[I\right]$$

Small-signal ac state equations:

$$\begin{bmatrix}L&0\\0&C\end{bmatrix}\frac{d}{dt}\begin{bmatrix}\hat{i}(t)\\ \hat{v}(t)\end{bmatrix}=\begin{bmatrix}-DR_{on}&D'\\ -D'&-\frac{1}{R}\end{bmatrix}\begin{bmatrix}\hat{i}(t)\\ \hat{v}(t)\end{bmatrix}+\begin{bmatrix}D&-D'\\0&0\end{bmatrix}\begin{bmatrix}\hat{v}_g(t)\\ \cancelto{0}{\hat{v}_D(t)}\end{bmatrix}+\begin{bmatrix}V_g-V-IR_{on}+V_D\\ I\end{bmatrix}\hat{d}(t)$$

$$\left[\hat{i}_g(t)\right]=\left[D\ \ 0\right]\begin{bmatrix}\hat{i}(t)\\ \hat{v}(t)\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}\begin{bmatrix}\hat{v}_g(t)\\ \cancelto{0}{\hat{v}_D(t)}\end{bmatrix}+\begin{bmatrix}0\\ I\end{bmatrix}\hat{d}(t)$$

Construction of ac equivalent circuit

Small-signal ac equations, in scalar form:

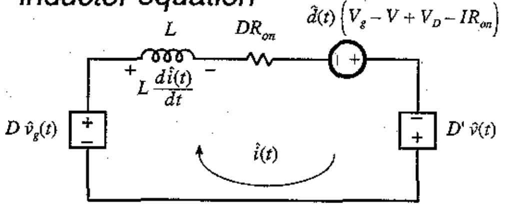
$$L\frac{d\hat{i}(t)}{dt}=D'\,\hat{v}(t)-DR_{on}\,\hat{i}(t)+D\,\hat{v}_g(t)+\left(V_g-V-IR_{on}+V_D\right)\hat{d}(t)$$

$$C\frac{d\hat{v}(t)}{dt}=-D'\,\hat{i}(t)-\frac{\hat{v}(t)}{R}+I\,\hat{d}(t)$$

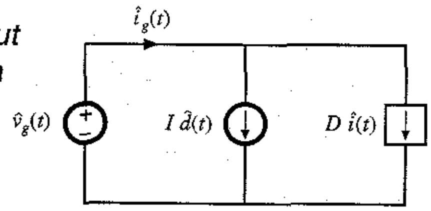
$$\hat{i}_g(t)=D\,\hat{i}(t)+I\,\hat{d}(t)$$

Corresponding equivalent circuits:

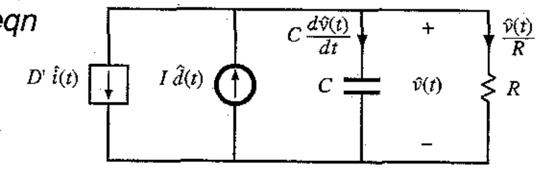
inductor equation



input eqn



capacitor eqn



Complete small-signal ac equivalent circuit

Combine individual circuits to obtain