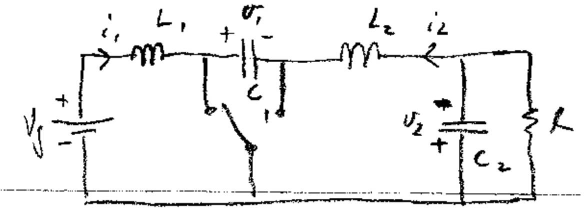
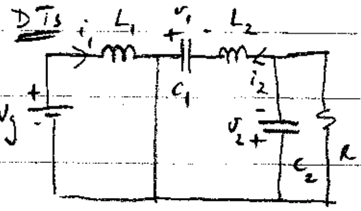
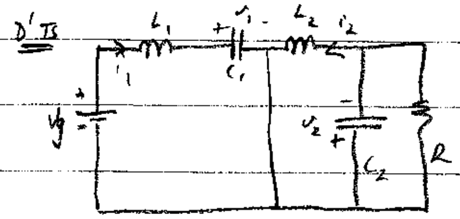
CUK







$$-V_g + L_1 \frac{di_1}{dt} = 0 \Rightarrow \frac{di_1}{dt} = \frac{V_g}{L_1}$$

$$-V_g + L_1 \frac{di_1}{dt} + v_1 = 0 \Rightarrow \frac{di_1}{dt} = \frac{-v_1}{L_1} + \frac{V_g}{L_1}$$

$$v_2 + L_2 \frac{di_2}{dt} - v_1 = 0 \Rightarrow \frac{di_2}{dt} = \frac{v_1}{L_2} - \frac{v_2}{L_2}$$

$$v_2 + L_2 \frac{di_2}{dt} = 0 \Rightarrow \frac{di_2}{dt} = -\frac{v_2}{L_2}$$

$$-i_2 = C_1 \frac{dv_1}{dt} \Rightarrow \frac{dv_1}{dt} = \frac{-i_2}{C_1}$$

$$i_1 = C_1 \frac{dv_1}{dt} \Rightarrow \frac{dv_1}{dt} = \frac{i_1}{C_1}$$

$$i_2 = C_2 \frac{dv_2}{dt} + \frac{v_2}{R} \Rightarrow \frac{dv_2}{dt} = \frac{i_2}{C_2} - \frac{v_2}{RC_2}$$

$$i_2 = C_2 \frac{dv_2}{dt} + \frac{v_2}{R} \Rightarrow \frac{dv_2}{dt} = \frac{i_2}{C_2} - \frac{v_2}{RC_2}$$

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} V_g$$

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0. \end{bmatrix} V_g$$

$$A = \begin{bmatrix} 0 & 0 & -\frac{D'}{L_1} & 0 \\ 0 & 0 & \frac{D}{L_2} & -\frac{1}{L_2} \\ \frac{D'}{C_1} & -\frac{D}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|sI - A| = \begin{vmatrix} s & 0 & \frac{D'}{L_1} & 0 \\ 0 & s & -\frac{D}{L_2} & \frac{1}{L_2} \\ -\frac{D'}{C_1} & \frac{D}{C_1} & s & 0 \\ 0 & -\frac{1}{C_2} & 0 & s + \frac{1}{RC_2} \end{vmatrix}$$

Let $|A| = \begin{vmatrix} B & C \\ D & E \end{vmatrix}$ then $|A| = |B| \; |E - DB^{-1}C|$

$$\Rightarrow |sI - A| = s^2 \left| \begin{bmatrix} s & 0 \\ 0 & s + \frac{1}{RC_2} \end{bmatrix} - \begin{bmatrix} -\frac{D'}{C_1} & \frac{D}{C_1} \\ 0 & -\frac{1}{C_2} \end{bmatrix} \underbrace{\begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} \frac{D'}{L_1} & 0 \\ -\frac{D}{L_2} & \frac{1}{L_2} \end{bmatrix}} \right|$$

$$\underbrace{\begin{bmatrix} -\frac{D'}{C_1} & \frac{D}{C_1} \\ 0 & -\frac{1}{C_2} \end{bmatrix} \begin{bmatrix} \frac{D'}{sL_1} & 0 \\ -\frac{D}{sL_2} & \frac{1}{sL_2} \end{bmatrix}}$$

$$\begin{bmatrix} -\frac{D'^2}{sL_1C_1} - \frac{D^2}{sL_2C_1} & \frac{D}{sL_2C_1} \\ \frac{D}{sL_2C_2} & -\frac{1}{sL_2C_2} \end{bmatrix}$$

$s+a)\ (s+b)$

$s^2 + (a+b)s + ab$

$$= s^2 \begin{vmatrix} s + \frac{D'^2}{sL_1C_1} + \frac{D^2}{sL_2C_1} & -\frac{D}{sL_2C_1} \\ -\frac{D}{sL_2C_2} & s + \frac{1}{RC_2} + \frac{1}{sL_2C_2} \end{vmatrix}$$

$$= s^2 \left[s^2 + s\left(\frac{1}{RC_2} + \frac{1}{sL_2C_2} + \frac{D'^2}{sL_1C_1} + \frac{D^2}{sL_2C_1}\right) + \left(\frac{D'^2}{sL_1C_1} + \frac{D^2}{sL_2C_1}\right)\left(\frac{1}{RC_2} + \frac{1}{sL_2C_2}\right) - \frac{D^2}{s^2L_2^2C_1C_2} \right]$$

$$= s^2 \left[s^2 + \frac{s}{RC_2} + \frac{1}{L_2C_2} + \frac{D'^2}{L_1C_1} + \frac{D^2}{L_2C_1} + \frac{D'^2}{sL_1C_1RC_2} + \frac{D'^2}{s^2L_1C_1L_2C_2} + \frac{D^2}{sL_2C_1RC_2} + \frac{D^2}{s^2L_2^2C_1C_2} - \frac{D^2}{s^2L_2^2C_1C_2} \right]$$

$$= s^4 + \frac{s^3}{RC_2} + s^2\left(\frac{1}{L_2C_2} + \frac{D'^2}{L_1C_1} + \frac{D^2}{L_2C_1}\right) + s\left(\frac{D'^2}{L_1C_1RC_2} + \frac{D^2}{L_2C_1RC_2}\right) + \frac{D'^2}{L_1C_1L_2C_2}$$

$$= \frac{D'^2}{L_1C_1L_2C_2}\left[s^4 \frac{L_1C_1L_2C_2}{D'^2} + \frac{s^3}{RC_2}\,\frac{L_1C_1L_2C_2}{D'^2} + s^2\,\frac{L_1C_1L_2C_2}{D'^2}\left(\frac{1}{L_2C_2} + \frac{D'^2}{L_1C_1} + \frac{D^2}{L_2C_1}\right)\right.$$

$$\left. + \; s\,\frac{L_1C_1L_2C_2}{D'^2}\left(\frac{D'^2}{L_1C_1RC_2} + \frac{D^2}{L_2C_1RC_2}\right) + 1 \right]$$

$$= \frac{D'^2}{L_1C_1L_2C_2}\left[s^4 \frac{L_1C_1L_2C_2}{D'^2} + s^3\,\frac{L_1C_1L_2}{D'^2 R} + s^2\left(\frac{L_1C_1}{D'^2} + L_2C_2 + \frac{D^2}{D'^2}L_1C_2\right)\right.$$

$$\left. + \; s\left(\frac{L_2}{R} + \frac{D^2}{D'^2}\,\frac{L_1}{R}\right) + 1 \right]$$

$$B_d = (A_1 - A_2)X + (B_1 - B_2)V$$

$$X = \begin{bmatrix} \frac{D^2}{D'^2}\frac{V_g}{R} \\ \frac{D}{D'}\frac{V_g}{R} \\ \frac{1}{D'}V_g \\ \frac{D}{D'}V_g \end{bmatrix} = \begin{bmatrix} \frac{D^2}{D'R} \\ \frac{D}{R} \\ 1 \\ D \end{bmatrix} \frac{V_g}{D'}$$

$$B_d = \begin{bmatrix} 0 & 0 & \frac{1}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & 0 \\ -\frac{1}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{D^2}{D'R} \\ \frac{D}{R} \\ 1 \\ D \end{bmatrix} \frac{V_g}{D'}$$

$$= \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \\ -\frac{1}{C_1}\left(\frac{D^2}{D'R} + \frac{D}{R}\right) \\ 0 \end{bmatrix} \frac{V_g}{D'}$$

$$\frac{D^2 + DD'}{D'} = \frac{D^2 + D(1-D)}{D'}$$

$$= \frac{\cancel{D^2} + D - D^2}{D'}$$

$$= \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \\ -\frac{D}{D'RC_1} \\ 0 \end{bmatrix} \frac{V_g}{D'}$$

Numerator

$$\dot{\hat{x}} = A\hat{x} + B_d \hat{d}$$

$$\hat{x} = \begin{bmatrix} \hat{i}_1 \\ \hat{i}_2 \\ \hat{v}_1 \\ \hat{v}_2 \end{bmatrix}$$

$$(sI - A)\hat{x} = B_d \hat{d} \qquad \hat{x}_4 = \hat{v}_2$$

Cramer's rule $\Rightarrow$

$$\text{Numerator} = \begin{vmatrix} s & 0 & \frac{D'}{L_1} & \frac{V_g}{D'L_1} \\ 0 & s & -\frac{D}{L_2} & \frac{V_g}{D'L_2} \\ -\frac{D'}{C_1} & \frac{D}{C_1} & s & \frac{-D V_g}{D'^2 R C_1} \\ 0 & -\frac{1}{C_2} & 0 & 0 \end{vmatrix}$$

$$= s^2 \left| \begin{bmatrix} s & -\frac{D V_g}{D'^2 R C_1} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -\frac{D'}{C_1} & \frac{D}{C_1} \\ 0 & -\frac{1}{C_2} \end{bmatrix} \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} \frac{D'}{L_1} & \frac{V_g}{D'L_1} \\ -\frac{D}{L_2} & \frac{V_g}{D'L_2} \end{bmatrix} \right|$$

$$\begin{bmatrix} -\frac{D'}{C_1} & \frac{D}{C_1} \\ 0 & -\frac{1}{C_2} \end{bmatrix} \begin{bmatrix} \frac{D'}{sL_1} & \frac{V_g}{sD'L_1} \\ -\frac{D}{sL_2} & \frac{V_g}{sD'L_2} \end{bmatrix}$$

$$\begin{bmatrix} -\frac{D'^2}{sL_1C_1} - \frac{D^2}{sL_2C_1} & -\frac{D'V_g}{sD'L_1C_1} + \frac{DV_g}{sD'L_2C_1} \\ \frac{D}{sL_2C_2} & -\frac{V_g}{sD'L_2C_2} \end{bmatrix}$$

$$= s^2 \begin{vmatrix} s + \frac{D'^2}{sL_1C_1} + \frac{D^2}{sL_2C_1} & -\frac{DV_g}{D'^2RC_1} + \frac{D'V_g}{sD'L_1C_1} - \frac{DV_g}{sD'L_2C_1} \\ -\frac{D}{sL_2C_2} & \frac{V_g}{sD'L_2C_2} \end{vmatrix}$$

$$D'^2 + DD' = D'(D'+D) = D'$$

$$= s^2 \left[\frac{V_g}{D' L_2 C_2} + \frac{D'^2 V_g}{s^2 D' L_1 C_1 L_2 C_2} + \cancel{\frac{D^2 V_g}{s^2 D' L_2^2 C_1 C_2}} - \frac{D^2 V_g}{s D'^2 L_2 C_2 R C_1} + \frac{D D' V_g}{s^2 D' L_1 C_1 L_2 C_2} - \cancel{\frac{D^2 V_g}{s^2 D' L_2^2 C_1 C_2}} \right]$$

$$= V_g \left[\frac{s^2}{D' L_2 C_2} - s \frac{D^2}{D'^2 L_2 C_2 R C_1} + \frac{1}{L_1 C_1 L_2 C_2} \right]$$

$$= \frac{V_g}{L_1 C_1 L_2 C_2} \left[s^2 \frac{L_1 C_1}{D'} - s \frac{D^2 L_1}{D'^2 R} + 1 \right]$$

$$\Rightarrow \frac{\hat{v}_2}{\hat{d}} = \frac{V_g}{D'^2} \frac{\left(s^2 \frac{L_1 C_1}{D'} - s \frac{D^2 L_1}{D'^2 R} + 1 \right)}{s^4 \frac{L_1 C_1 L_2 C_2}{D'^2} + s^3 \frac{L_1 C_1 L_2}{D'^2 R} + s^2 \left(\frac{L_1 C_1}{D'^2} + L_2 C_2 + \frac{D^2}{D'^2} L_1 C_2 \right) + s \left(\frac{L_2}{R} + \frac{D^2}{D'^2} \frac{L_1}{R} \right) + 1}$$