ECE331 Formula Sheet

Mathematical and Physical Models

TEM Transmission Lines

$$L'C' = \mu\epsilon$$

$$\frac{G'}{C'} = \frac{\sigma}{\epsilon}$$

$$\alpha = \Re\mathfrak{e}(\gamma) = \Re\mathfrak{e}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(Np/m)}$$

$$\beta = \Im\mathfrak{m}(\gamma) = \Im\mathfrak{m}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega)$$

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

Step Function Transient Response

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}$$

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

Lossless Line

$$\alpha = 0$$

$$\beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

$$u_p = \frac{1}{\sqrt{\mu\epsilon}} \quad \text{(m/s)}$$

$$\lambda = \frac{u_p}{f} = \frac{c}{f}\frac{1}{\sqrt{\epsilon_r}} = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}$$

$$d_{min} = \frac{\theta_r \lambda}{4\pi} + \frac{(2n+1)\lambda}{4}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}[1 - |\Gamma|^2]$$

Table 2-1 Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right]}$	$\frac{\epsilon w}{h}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ϵ, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c / \sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right] \approx \ln(2D/d)$.

Table 2-2 Characteristic parameters of transmission lines.

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(60/\sqrt{\epsilon_r}\right)\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(120/\sqrt{\epsilon_r}\right) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \approx \left(120/\sqrt{\epsilon_r}\right)\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(120\pi/\sqrt{\epsilon_r}\right)(h/w)$

Notes: (1) $\mu = \mu_0$, $\epsilon = \epsilon_r\epsilon_0$, $c = 1/\sqrt{\mu_0\epsilon_0}$, and $\sqrt{\mu_0/\epsilon_0} \approx (120\pi)\ \Omega$, where ϵ_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Table 2-4 Properties of standing waves on a lossless transmission line.

Voltage maximum **Voltage minimum**	$\lvert\widetilde{V}\rvert_{max} = \lvert V_0^+\rvert[1 + \lvert\Gamma\rvert]$ $\lvert\widetilde{V}\rvert_{min} = \lvert V_0^+\rvert[1 - \lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima)	$d_{max} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$
Position of first maximum (also position of first current minimum)	$d_{max} = \begin{cases} \frac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \frac{\theta_r\lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of current maxima)	$d_{min} = \frac{\theta_r\lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$
Position of first minimum (also position of first current maximum)	$d_{min} = \frac{\lambda}{4}\left(1 + \frac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0\left(\frac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l}\right) = Z_0\left(\frac{1 + \Gamma_l}{1 - \Gamma_l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\frac{1 + \lvert\Gamma\rvert}{1 - \lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\frac{1 - \lvert\Gamma\rvert}{1 + \lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of matched line	$Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave; $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.

$e = 1.6\ x\ 10^{-19}\ C$ [charge of single electron] $F = R\ \left(\frac{q1q2}{4\pi\varepsilon_0 R^2}\right) N$ [Coulomb's Law]

$\varepsilon_0 = 8.854\ x\ 10^{-12}$ F/m [Electrical permittivity of free space] ε = $\varepsilon_r \varepsilon_0$

E = R ($\frac{q}{4\pi\varepsilon_0 R^2}$) V/m [Electric Field Intensity] D = εE C/m^2 [electric flux density]

c = $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ = 3 x 10^8 m/s [velocity of light in free space] **B = μH** [Magnetic field intensity **H**]

$\mu_0 = 4\pi\ x\ 10^{-7}$ H/m [Magnetic permeability]

y(x,t) = Acos ($\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0$) m

y(x,t) = Acosϕ_0(x,t) where ϕ_0(x,t) = ($\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0$) rad

where A = amplitude, T = time period, λ = special wavelength, ϕ_0 = reference phase

f = $\frac{1}{T}$ Hz [frequency] $u_p = \frac{\lambda}{T} = f\lambda = \frac{\omega}{\beta}$ m/s [phase velocity]

ω = 2πf rad/s [angular velocity] β = $\frac{2\pi}{\lambda}$ rad/m [phase constant]

$\lambda = \frac{c}{f}$ [wavelength in vacuum]

y(x,t) = A$e^{-\alpha x}$cos($\omega t - \beta x + \phi_0$)[Lossy Medium] p(x,t) = Acos($\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \phi_0$) N/$m^2$

z = x+jy [rectangular form] z = |z| < θ [polar form]

$e^{j\theta} = cos\theta + sin\theta$ [Euler's identity] z = |z|$e^{j\theta}$ = |z|cosθ+sinθ [Polar to rectangular]

x=|z|cosθ, y=|z|sinθ, |z|=+$\sqrt{x^2 + y^2}$, θ= $tan^{-1}(\frac{y}{x})$ [Rectangular-polar relations]

z* = (x+jy)* = x-jy = z = |z|$e^{j\theta}$ = |z| < θ [complex conjugate] |z| = +$\sqrt{z\ z*}$

z1 + z2 = (x1+x2)+j(y1+y2) [addition] z1z2 = (x1x2-y1y2)+j(x1y2+x2y1) [multiplication]

z1/z2 = $\frac{(x1x2+y1y2)+j(x2y1-x1y2)}{x2^{2}+y2^{2}}$ [division] Ri(t) + $\frac{1}{C}\int i(t)dt = v_s(t)$

vs(t) = V0cos(ωt+$\phi_0 - \frac{\pi}{2}$) vs(t) = Re[**Vs**e^{jwt}]

Vs = V0$e^{j(\phi_0 - \frac{\pi}{2})}$ i(t) = Re(**I**e^{jwt})

$\int i\ dt = Re\left(\frac{I}{jw}e^{jwt}\right)$ Re{[R+1/jwC)**I-Vs**] e^{jwt}} = 0

Im{[R+1/jwC)**I-Vs**] e^{jwt}} = 0 **I** (R+$\frac{1}{j\omega C}$) = $\boldsymbol{Vs}$ [phasor domain]

I = $\frac{\boldsymbol{Vs}}{R+1/(jwC)}$ i(t) = $\frac{V0wC}{+\sqrt{1+w^2R^2C^2}}\cos(wt + \phi 0 - \phi 1)$

$$Rs = \sqrt{\pi f \frac{\mu_c}{\sigma_c}}, \quad \phi_0 = \frac{\varpi l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda}\ for\ \mu_p = c\ (if\ \frac{l}{\lambda} < .01\ ignore)$$

All TEM lines: L'C' = με and $\frac{G\prime}{C\prime} = \frac{\sigma}{\varepsilon}$

Telegraphers equations: Phasor Form

$$-\frac{dV(z)}{dz} = (r' + jwL')(I(z)\ , \quad -\frac{dI(z)}{z} = (G' + jwC')V(z)$$

$$\frac{d^2V(z)}{dz^2} = -\gamma^2 V(z) = 0, \quad \frac{d^2I(z)}{dz^2} = -\gamma^2 I(z) = 0$$

$$\gamma = \sqrt{(R' + jwL')(G' + jwC')}\ , \gamma = \alpha + j\beta, \alpha = \mathbb{R}\ , \ \beta = \Im$$

$$Z_o = \frac{R' + jwL\prime}{\gamma} = \sqrt{\frac{R' + jwL\prime}{G' + jwC\prime}}\,\Omega$$

$$\varepsilon eff = \frac{\varepsilon r + 1}{2} + \frac{\varepsilon r - 1}{2} + (1 + \frac{10}{s})^{-xy}$$

$$s = \frac{width}{thick}, x = .56(\frac{\varepsilon r - 0.9}{\varepsilon r + 3})^{0.05}, y = 1 + .02\ln(\frac{s^4 + 3.7 * 10^{-4}s^2}{s^4 + 0.43}) + .05\ln(1 + 1.7 * 10^{-4}s^3)$$

$$Zo(micro) = \frac{60}{\sqrt{\varepsilon eff}}\ln\left(\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right), t = (\frac{30.67}{s})^{.75}, \beta = \frac{\varpi}{c}\sqrt{\varepsilon eff}$$

(Lossless Zo's(R'=G'=0): $Coax: Zo = \frac{60}{\sqrt{\varepsilon r}}\ln\left(\frac{b}{a}\right)$, $2wire: Zo = \frac{120}{\sqrt{\varepsilon r}} * \ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$,

$$ParPlate: Zo = \frac{120\pi}{\sqrt{\varepsilon r}} * \frac{width}{separation})^{**}$$

** Notes for above line: $\mu = \mu o$, $\varepsilon = \varepsilon r \varepsilon o$, $c = \frac{1}{\sqrt{\mu o \varepsilon o}}$, $\sqrt{\mu o / \varepsilon o}\ approx = 120\pi$

$\Gamma = \frac{Vo^-}{Vo^+} = \frac{zL - 1}{zL + 1} = -\frac{Io^-}{Io^+}$, $zL = \frac{Z_L}{Z_o}$, $\Gamma = \|\Gamma\| e^{j\theta_r}$ Standing: $S = \frac{1 + \|\Gamma\|}{1 - \|\Gamma\|} = \frac{\|V\|max}{\|V\|min}$

Transient Responses to Step:

$$V_1^+ = \frac{VgZo}{Rg + Zo}\ ,\ V_\infty = \frac{VgR_L}{Rg + R_L}\ ,\ \Gamma_g = \frac{Rg - Zo}{Rg + Z0}\ , \Gamma_L = \frac{R_L - Zo}{R_L + Zo}$$

$$P_{avg} = \frac{|V_o^+|^2}{2Zo}(1 - |\Gamma|^2)$$