Portland State
UNIVERSITY

DEPARTMENT OF
ELECTRICAL AND COMPUTER ENGINEERING

ECE331

LAB 4: NUMERICAL ELECTROMAGNETIC EXPERIMENTS

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Contents

Objective

The objective of this lab is to familiarize you with Matlab and using it as a tool to solve electromagnetic problems. Electromagnetic concepts can be difficult to visualize from the equations alone. Using the graphics from Matlab will help you gain intuition about Maxwell's equations and electromagnetics generally.

Concepts Covered

- Matlab workspace

- Matlab vector operations

- Matlab plotting

- Numerical integration

- Curl and divergence of vector fields

- Numerical solutions to electrostatic and magnetostatic problems

1   Pre-Lab Assignment

- Read through the Matlab getting started manual if you are not familiar with Matlab. You should know how to do simple things that are described in the lab introduction section. We will go over what you need during the lab but you should get somewhat familiar with the Matlab environment before coming to lab.
- Log into Matlab and run the publishing example mfile (provided to you) to do a gradient and plot the results. Use the publish button to reproduce the results shown in Fig. 4.
- Review example 4-4 in the book and bring the solution for the electric field along the axis of a ring of charge.
- Review problem 4.10 from the book (homework) and bring solution.
- Calculate the divergence and curl for following:

    1. Problem 3.44 (a) from book

    $$\vec{\mathbf{A}} = -\hat{\mathbf{x}} \cos x \sin y + \hat{\mathbf{y}} \sin x \cos y \tag{1}$$

    2. Problem 3.44 (c) from book

    $$\vec{\mathbf{A}} = -\hat{\mathbf{x}} xy + \hat{\mathbf{y}} y^2 \tag{2}$$

    3. Problem 3.44 (i) from book

    $$\vec{\mathbf{A}} = \hat{\mathbf{r}} r + \hat{\phi} r \cos \phi \tag{3}$$

    4. The magnetic field from a wire carrying a constant current:

    $$\vec{\mathbf{B}} = \hat{\phi} \frac{\mu_0 I}{2\pi r} \tag{4}$$

2   Introduction

Most of the material needed for this lab has been covered in Chapters 3 and 4 of Ulaby *et al.* However, in many cases, the vector fields we are working with are messy and finding exact solutions to electromagnetic problems is not possible. Usually exact solutions are possible when we can take advantage of a coordinate system or some symmetry. For example, for the ring of charge example (4-4 in Ulaby) the solution is possible analytically along the center of the ring. But, what if we want the field somewhere else? The math becomes difficult. With relatively simple programming we can find a solution to other locations and we will do this using Matlab for the ring of charge problem.

The first part of this lab will familiarize you with making vector plots as well as using Matlab's version of divergence and curl. From the vector plots you can begin to get an idea of what fields with divergence and curl look like. We will also do a real example for the

magnetic field from a DC current carrying wire. In this case, the ideal equation is given in Chapters 1 and 4 and this is fully derived in Chapter 5. Inside the wire the field decreases linearly and eventually is zero at the center of the wire. This is shown in example 5-4. We won't exactly simulate a wire with some radius but will assume the wire is a line but then we will set the magnetic field to zero at the origin.
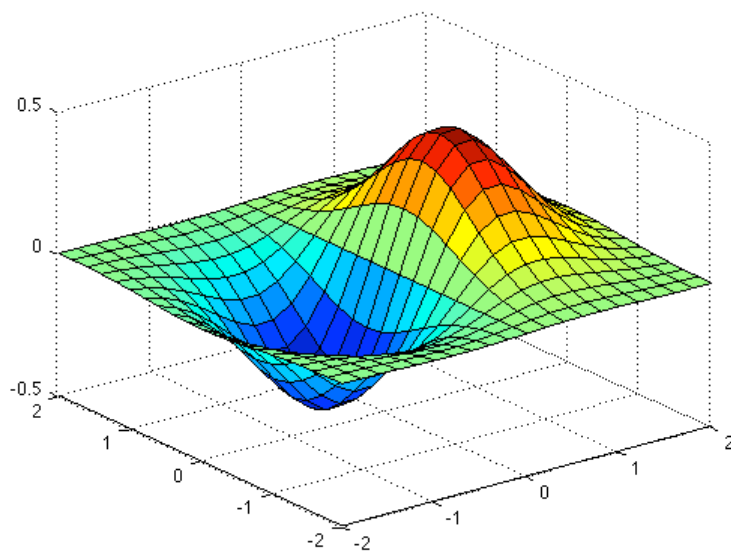
2.1   Matlab

We will be working in the EB325 labs so before coming to the lab you should know a bit about Matlab.

- How to log in and bring up a Matlab session

- How to open and save a script file

- Be able to make a vector and/or a matrix and plot something. For example, x that goes from -10 to 10 and then plot x versus $\sin x$

- You should also know how to set up a for loop and how to index into arrays and matrices

- Know how to publish your results using the publish function in Matlab. An example m-file is provided and as a pre-lab assignment you should run this and make a pdf file of the published results.

Items we will cover in lab but it doesn't hurt to look ahead at these functions:

- MESHGRID: This is a convenient command to get x and y coordinates for a uniform grid for a numerical calculation. This command works as described below from the Matlab help in Fig. 1. The meshgrid command gives us two matrices X and Y that contain the x and y coordinates for each location on a uniform grid between -2 and 2 for x and y. We can then calculate a height function as given in the example. The example uses surf plot and we will use pcolor which is similar but for 2D plots.

- QUIVER: This command allows you to make vector plots. It is sometimes tricky to get the length of the arrows to come out looking nice in a plot but you can play with the scaling to help plots look better. The help and example given in Matlab shows how to use this function and is given in Fig. 2.

- PCOLOR: This is a color image plotting function. It is similar to imagesc but insures that the data point corresponds to the axis values. Can be particularly nice to use if the x and y data points are not on a uniform grid and this will plot them according to their actual location.

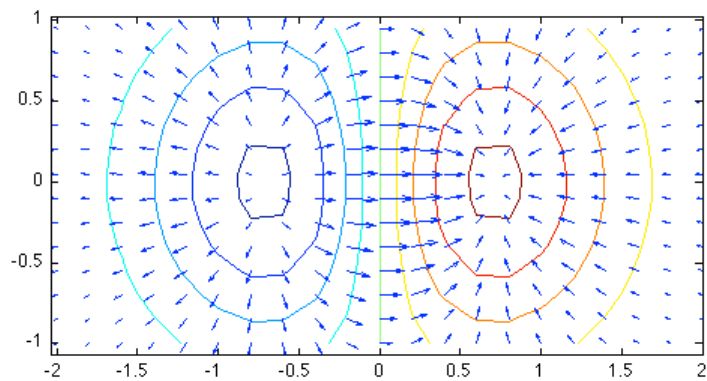- DIV: Numerical calculation of divergence.

```
% MESHGRID   X and Y arrays for 3-D plots.
%    [X,Y] = MESHGRID(x,y) transforms the domain specified by vectors
%    x and y into arrays X and Y that can be used for the evaluation
%    of functions of two variables and 3-D surface plots.
%    The rows of the output array X are copies of the vector x and
%    the columns of the output array Y are copies of the vector y.

[X,Y] = meshgrid(-2:.2:2, -2:.2:2);
Z = X .* exp(-X.^2 - Y.^2);
surf(X,Y,Z)
```


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Figure 1: Example of using the meshgrid command in Matlab (from Matlab help).

```
%  QUIVER Quiver plot.
%     QUIVER(X,Y,U,V) plots velocity vectors as arrows with components (u,v)
%     at the points (x,y).  The matrices X,Y,U,V must all be the same size
%     and contain corresponding position and velocity components (X and Y
%     can also be vectors to specify a uniform grid).  QUIVER automatically
%     scales the arrows to fit within the grid.
[x,y] = meshgrid(-2:.2:2,-1:.15:1);
z = x .* exp(-x.^2 - y.^2); [px,py] = gradient(z,.2,.15);
contour(x,y,z), hold on
quiver(x,y,px,py), hold off, axis image
```


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Figure 2: Example of using the quiver command in Matlab (from Matlab help).

- CURL: Numerical calculation of curl. NOTE: In Matlab you have to multiply the curl result by 2 to get it to agree with what we do analytically- no idea why.

- DOT: The dot product of two vectors (or for the magnitude of a vector can use as $\sqrt{\text{dot}(R, R)}$).

- Numerical integration: We will do simple numerical integration by summing up values (also sometimes known as rectangular method). It often works well as long as the steps are reasonably small (how small depends on the problem). You will see from these exercises it is plenty good for what we are doing. The numerical integration is approximated as the below:

$$\int_a^b f(x)dx \approx \Delta x \sum_{n=0}^{N-1} f(x_n) \tag{5}$$

where $x_n = a + n\Delta x$ and $\Delta x = (b-a)/N$. You can think of Δx as the approximation to the differential element dx. When we work in cylindrical coordinates we will have to keep in mind the differential element. If a line integral is in ϕ direction then the differential element is $rd\phi$ so the numerical Δ will need to be $r\Delta\phi$.

- Publishing results in Matlab: An easy way to organize your Matlab code and plots is to use the built in Matlab publishing capability. If you put your commands to run these numerical experiments you can use the publishing button to make a pdf of your results. An example is given in Fig. 3 and Fig. 4. The m-file is provided to you so you can run this and then publish to duplicate the results shown in Fig. 4.

```matlab
%% Example of publishing
% Text can go here
%
%% Gradient of scalar fields and examples plotting
% Code here, for example
[x,y] = meshgrid([-10:1:10],[-10:1:10]);
z = -(x.^2 + y.^2);
figure
pcolor(x,y,z);shading('flat');colorbar
set(gca,'Fontsize',14)
title('Sclar Field Example')
xlabel('X-distance')
ylabel('Y-distance')
%note, you can also use surf if you want a 3D looking plot
figure
surf(x,y,z);
set(gca,'Fontsize',14)
title('Scalar Field Example')
xlabel('X-distance')
ylabel('Y-distance')
[px,py] = gradient(z,1,1);
figure
quiver(x,y,px,py)
set(gca,'Fontsize',14)
title('Gradient Example')
xlabel('X-distance')
ylabel('Y-distance')

%% Questions for Curl and divergence of vector fields
% Question:
%% Questions for Curl and divergence of vector fields
% Question:
%
%% Questions for Curl and divergence of magnetic field from a wire
% Question:
%% Questions for electric field from a ring of charge
% Question:
```

Figure 3: Example m-file to take a gradient and publish results.
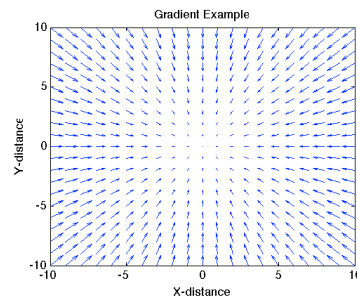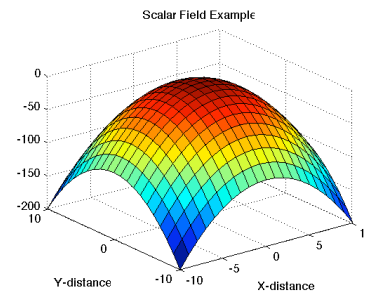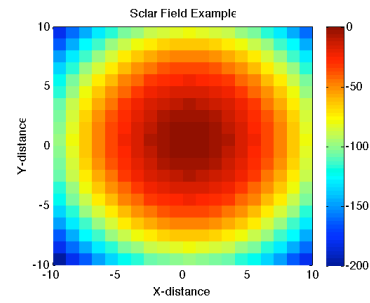
Example of publishing

Text can go here

Contents

Gradient of scalar fields and examples plotting

Code here, for example

```
[x,y] = meshgrid([-10:1:10],[-10:1:10]);
z = -(x.^2 + y.^2);
figure
pcolor(x,y,z);shading('flat');colorbar
set(gca,'Fontsize',14)
title('Sclar Field Example')
xlabel('X-distance')
ylabel('Y-distance')
%note, you can also use surf if you want a 3D looking plot
figure
surf(x,y,z);
set(gca,'Fontsize',14)
title('Scalar Field Example')
xlabel('X-distance')
ylabel('Y-distance')
[px,py] = gradient(z,1,1);
figure
quiver(x,y,px,py)
set(gca,'Fontsize',14)
title('Gradient Example')
xlabel('X-distance')
ylabel('Y-distance')
```


Questions for Curl and divergence of vector fields

Question:

Questions for Curl and divergence of vector fields

Question:

Questions for Curl and divergence of magnetic field from a wire

Question:

Questions for electric field from a ring of charge

Question:

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Figure 4: Example showing publishing results.

3   Experiments

3.1   Numerical Computation of Divergence and Curl

1. Make an x and y grid of data points where $-\pi \leq x \leq \pi$ and $-\pi \leq y \leq \pi$ m. Use meshgrid and the distance between grid points should be 0.25 m.

2. Make x and y components of the vector field, given in problem 3.44 (a): $\vec{\mathbf{A}} = -\hat{\mathbf{x}} \cos x \sin y + \hat{\mathbf{y}} \sin x \cos y$.

3. Plot $\vec{\mathbf{A}}$ using Matlab quiver command. Make sure to label axes.

4. Plot the divergence of $\vec{\mathbf{A}}$ that you calculated analytically (by hand). Use the pcolor command and label axes.

5. Compute the divergence of $\vec{\mathbf{A}}$ using Matlab div command. Plot the results using pcolor command and be sure to label axes.

6. Plot the curl of $\vec{\mathbf{A}}$ that you calculated analytically (by hand). Use the pcolor command and label axes.

7. Compute the curl of $\vec{\mathbf{A}}$ using Matlab curl command. Plot the results using pcolor command and be sure to label axes.

8. Repeat steps 1-7 for problem 3.44 (c) with $\vec{\mathbf{A}} = -\hat{\mathbf{x}} xy + \hat{\mathbf{y}} y^2$ and with $-10i \leq x \leq 10$ and $10 \leq y \leq 10$ m with the distance between grid points of 0.5 m.

9. Repeat the above plots for problem 3.44 (i) with $\vec{\mathbf{A}} = \hat{\mathbf{r}} r + \hat{\boldsymbol{\phi}} r \cos \phi$ and with $-10i \leq x \leq 10$ and $10 \leq y \leq 10$ m with the distance between grid points of 0.5 m. For this problem you will need to convert the given cylindrical coordinates to Cartesian coordinates.

Use the colorbar command to put a colorbar on the pcolor plots. Also, be sure to set the color scale to the same value for the exact and numerical solutions so that the colors can be compared between plots (e.g., use caxis([minvalue maxvalue]).

3.2   Numerical Computation of Divergence and Curl for a Current Carrying Wire

1. Make an x and y grid of data points with $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ m. Use meshgrid and the distance between grid points should be 0.05 m.

2. Numerically calculate the cylindrical r and ϕ values that correspond to each of the x and y grid point values.

3. Numerically calculate the x and y vector components of the magnetic field using the expression, $\vec{\mathbf{B}} = \hat{\phi}\frac{\mu_0 I}{2\pi r}$ (using the r and ϕ values computed in step 2) and you need to write $\hat{\phi}$ in Cartesian vector coordinates. Take the current to be $10,000$ Amps.

4. Set the value of $\vec{\mathbf{B}}$ to be zero at the origin.

5. Plot $\vec{\mathbf{B}}$ using Matlab quiver command. Make sure to label axes.

6. Plot the divergence of $\vec{\mathbf{B}}$ that you calculated analytically (by hand). Use the pcolor command and label axes.

7. Compute the divergence of $\vec{\mathbf{B}}$ using Matlab div command. Plot the results using pcolor command and be sure to label axes.

8. Plot the curl of $\vec{\mathbf{B}}$ that you calculated analytically (by hand). Use the pcolor command and label axes.

9. Compute the curl of $\vec{\mathbf{B}}$ using Matlab curl command. Plot the results using pcolor command and be sure to label axes.

Use the colorbar command to put a colorbar on the pcolor plots. Also, be sure to set the color scale to the same value for the exact and numerical solutions so that the colors can be compared between plots (e.g., use caxis([minvalue maxvalue]).

3.3   Numerical Integration and Calculating the Electric Field from a Ring of Charge

Here we will set up the grid for the numerical integration and calculation of the electric field due to a ring of charge and a half-ring of charge. Example of these steps in Matlab code is given in Fig. 5.

1. We will only calculate the E-field values in the $y - z$ plane (i.e., $x = 0$).

2. Make a y and z grid of data points with $-5 \leq y \leq 5$ and $0 \leq z \leq 5$ m. Use meshgrid and the distance between grid points should be 0.5 m for the y variable and distance of 0.25 m for the z variable. (y and z will be matrices)

3. Take the ring radius to be $b = 2$ m.

4. Take the line charge density to be $\rho_l = 1$ C/m^3.

5. Define the ϕ variable around the ring as going from 0 to 2π with step size of $\pi/100$.

6. Calculate the value of the differential element for ϕ line integration.

7. Pre-allocate the matrices for E_x, E_y and E_z (use the zeros command to put zeros into these matrices). There will be a value for E_x, E_y and E_z at each point in the $y - z$ grid.

8. Set up a double for loop over every index of y and z matrices.

9. Inside the above loops set up a third loop over ϕ indices. In this loop we will do the numerical integration of $\vec{\mathbf{E}} = \frac{\rho_l}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\hat{\mathbf{R}}b\mathrm{d}\phi}{R^2}$:

   - Get the x coordinate of the ring ($R\cos\phi(n)$ where n is the index of the innermost loop).

   - Get the y coordinate of the ring ($R\sin\phi(n)$ where n is the index of the innermost loop).

   - Get the $\mathbf{R}$ vector. This is the vector from a point on the ring to the observation point. You can find by taking $\mathbf{R_{obs}} - \mathbf{R_{ring}}$.

   - Calculate R^2 as the dot product, $\mathbf{R} \cdot \mathbf{R}$.

   - Calculate the $\hat{\mathbf{R}}$ unit vector as $\mathbf{R}/R$.

   - Keep a running sum of the total of all contributions from the loop as we index around the loop. Example code is given in Fig. 5.

10. **Problem 4-10 Half Ring of Charge**: Now modify your code to get the electric field if there is only a half-ring of charge as in problem 4-10 in Ulaby.

Figure 5: Example code for the electric field from a ring charge problem.

Lab Report Items to Turn in and Questions

Numerical Computation of Divergence and Curl

Items to Turn in

- Hand computed divergence and curl for problems assigned (5 pts)

- Publish your Matlab results with code and plots in a pdf file.

- You should have 5 plots for 3.44 (a), (b) and (c) for a total of 15 plots (each problem has a quiver plot, an exact divergence plot, a numerical divergence plot, an exact curl plot, a numerical curl plot). (15 pts)

Question

1. For the divergence and curl problems 3.44 (a), (b) and (c) were your numerical and exact solutions exactly the same? Explain why you think they were or were not. (5 pts)

Numerical Computation of Divergence and Curl for a Current Carrying Wire

Items to Turn in

- Hand computed divergence and curl of magnetic field from a current carrying wire. (5 pts)

- Publish your results with code and plots in a pdf file.

- You should have 5 plots (a quiver plot, an exact divergence plot, a numerical divergence plot, an exact curl plot, a numerical curl plot). (15 pts)

Question

1. For the magnetic field from a current carrying wire: Do your divergence and curl plots agree with Maxwell's equations? Write down the relevant Maxwell's equation and explain why your plots agree or not with that expected from these equations. Comment on differences between numerical and "exact".(5 pts)

Numerical Integration and Calculating the Electric Field from a Ring of Charge

Items to Turn in

- Publish your results as a pdf file with code and with plots for ring of charge.

- Plot of the exact field on the axis as well as the computed E_x, E_y and E_z curves. (10 pts)

- Quiver plot of the E_y and E_z components (plot in the $x = 0$ plane). (10 pts)

- Solution to problem 4.10 from book for a half-ring of charge. (5 pts)

- Published results for the half-ring of charge including your code and plots.

- Provide the numeric values of E_x, E_y and E_z at the origin for the half-ring of charge. (10 pts).

- Quiver plot of the E_y and E_z components (plot in the $x = 0$ plane) for the half-ring of charge. (10 pts)

Question

1. For the half-ring what happens to the values of E_x and E_y (at the origin) components if we had used a spacing for ϕ of 0.01 which is smaller than the value used of $\pi/100$? Are the results for E_x and E_y (at the origin) better or worse with the spacing for ϕ of 0.01 and why do you think you see those results? (5 pts)