**Problem 5.1** An electron with a speed of $8 \times 10^6$ m/s is projected along the positive $x$-direction into a medium containing a uniform magnetic flux density $\mathbf{B} = (\hat{\mathbf{x}}4 - \hat{\mathbf{z}}3)$ T. Given that $e = 1.6 \times 10^{-19}$ C and the mass of an electron is $m_\text{e} = 9.1 \times 10^{-31}$ kg, determine the initial acceleration vector of the electron (at the moment it is projected into the medium).

**Solution:** The acceleration vector of a free particle is the net force vector divided by the particle mass. Neglecting gravity, and using Eq. (5.3), we have

$$\begin{aligned}\mathbf{a} = \frac{\mathbf{F}_\text{m}}{m_\text{e}} = \frac{-e}{m_\text{e}}\mathbf{u} \times \mathbf{B} &= \frac{-1.6 \times 10^{-19}}{9.1 \times 10^{-31}}(\hat{\mathbf{x}}8 \times 10^6) \times (\hat{\mathbf{x}}4 - \hat{\mathbf{z}}3) \\ &= -\hat{\mathbf{y}}4.22 \times 10^{18} \quad (\text{m/s}^2).\end{aligned}$$

**Problem 5.4** The rectangular loop shown in Fig. P5.4 consists of 20 closely wrapped turns and is hinged along the $z$-axis. The plane of the loop makes an angle of $30^\circ$ with the $y$-axis, and the current in the windings is 0.5 A. What is the magnitude of the torque exerted on the loop in the presence of a uniform field $\mathbf{B} = \hat{\mathbf{y}}2.4$ T? When viewed from above, is the expected direction of rotation clockwise or counterclockwise?

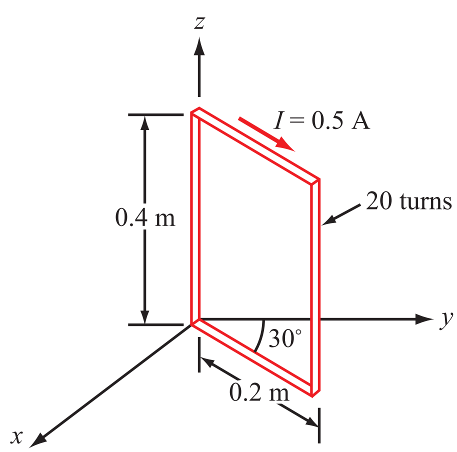


**Figure P5.4:** Hinged rectangular loop of Problem 5.4.

**Solution:** The magnetic torque on a loop is given by $\mathbf{T} = \mathbf{m} \times \mathbf{B}$ (Eq. (5.20)), where $\mathbf{m} = \hat{\mathbf{n}}NIA$ (Eq. (5.19)). For this problem, it is given that $I = 0.5$ A, $N = 20$ turns, and $A = 0.2 \text{ m} \times 0.4 \text{ m} = 0.08 \text{ m}^2$. From the figure, $\hat{\mathbf{n}} = -\hat{\mathbf{x}}\cos 30^\circ + \hat{\mathbf{y}}\sin 30^\circ$. Therefore, $\mathbf{m} = \hat{\mathbf{n}}0.8 \text{ (A} \cdot \text{m}^2)$ and $\mathbf{T} = \hat{\mathbf{n}}0.8 \text{ (A} \cdot \text{m}^2) \times \hat{\mathbf{y}}2.4 \text{ T} = -\hat{\mathbf{z}}1.66$ (N·m). As the torque is negative, the direction of rotation is clockwise, looking from above.

**Problem 5.12** Two infinitely long, parallel wires are carrying 6-A currents in opposite directions. Determine the magnetic flux density at point $P$ in Fig. P5.12.

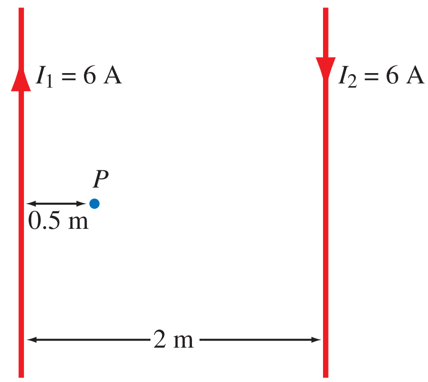


**Figure P5.12:** Arrangement for Problem 5.12.

**Solution:**

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi(0.5)} + \hat{\boldsymbol{\phi}} \frac{\mu_0 I_2}{2\pi(1.5)} = \hat{\boldsymbol{\phi}} \frac{\mu_0}{\pi}(6+2) = \hat{\boldsymbol{\phi}} \frac{8\mu_0}{\pi} \quad \text{(T)}.$$

**Problem 5.14** Two parallel, circular loops carrying a current of 40 A each are arranged as shown in Fig. P5.14. The first loop is situated in the $x$–$y$ plane with its center at the origin, and the second loop's center is at $z = 2$ m. If the two loops have the same radius $a = 3$ m, determine the magnetic field at:

**(a)** $z = 0$

**(b)** $z = 1$ m

**(c)** $z = 2$ m

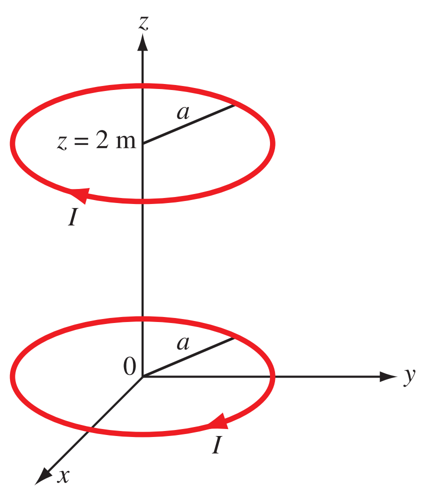


**Figure P5.14:** Parallel circular loops of Problem 5.14.

**Solution:** The magnetic field due to a circular loop is given by (5.34) for a loop in the $x$–$y$ plane carrying a current $I$ in the $+\hat{\boldsymbol{\phi}}$-direction. Considering that the bottom loop in Fig. is in the $x$–$y$ plane, but the current direction is along $-\hat{\boldsymbol{\phi}}$,

$$\mathbf{H}_1 = -\hat{\mathbf{z}} \frac{Ia^2}{2(a^2+z^2)^{3/2}},$$

where $z$ is the observation point along the $z$-axis. For the second loop, which is at a height of 2 m, we can use the same expression but $z$ should be replaced with $(z-2)$. Hence,

$$\mathbf{H}_2 = -\hat{\mathbf{z}} \frac{Ia^2}{2[a^2+(z-2)^2]^{3/2}}.$$

The total field is

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 = -\hat{\mathbf{z}} \frac{Ia^2}{2} \left[ \frac{1}{(a^2+z^2)^{3/2}} + \frac{1}{[a^2+(z-2)^2]^{3/2}} \right] \text{ A/m}.$$

**(a)** At $z = 0$, and with $a = 3$ m and $I = 40$ A,

$$\mathbf{H} = -\hat{\mathbf{z}}\,\frac{40 \times 9}{2}\left[\frac{1}{3^3} + \frac{1}{(9+4)^{3/2}}\right] = -\hat{\mathbf{z}}\,10.5 \text{ A/m}.$$

**(b)** At $z = 1$ m (midway between the loops):

$$\mathbf{H} = -\hat{\mathbf{z}}\,\frac{40 \times 9}{2}\left[\frac{1}{(9+1)^{3/2}} + \frac{1}{(9+1)^{3/2}}\right] = -\hat{\mathbf{z}}\,11.38 \text{ A/m}.$$

**(c)** At $z = 2$ m, **H** should be the same as at $z = 0$. Thus,

$$\mathbf{H} = -\hat{\mathbf{z}}\,10.5 \text{ A/m}.$$

**Problem 5.22** A long cylindrical conductor whose axis is coincident with the $z$-axis has a radius $a$ and carries a current characterized by a current density $\mathbf{J} = \hat{\mathbf{z}} J_0/r$, where $J_0$ is a constant and $r$ is the radial distance from the cylinder's axis. Obtain an expression for the magnetic field $\mathbf{H}$ for

**(a)** $0 \le r \le a$

**(b)** $r > a$

**Solution:** This problem is very similar to Example 5-5.

**(a)** For $0 \le r_1 \le a$, the total current flowing within the contour $C_1$ is

$$I_1 = \iint \mathbf{J} \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{r=0}^{r_1} \left( \frac{\hat{\mathbf{z}} J_0}{r} \right) \cdot (\hat{\mathbf{z}} r\, dr\, d\phi) = 2\pi \int_{r=0}^{r_1} J_0\, dr = 2\pi r_1 J_0.$$

Therefore, since $I_1 = 2\pi r_1 H_1$, $H_1 = J_0$ within the wire and $\mathbf{H}_1 = \hat{\boldsymbol{\phi}} J_0$.

**(b)** For $r \ge a$, the total current flowing within the contour is the total current flowing within the wire:

$$I = \iint \mathbf{J} \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{r=0}^{a} \left( \frac{\hat{\mathbf{z}} J_0}{r} \right) \cdot (\hat{\mathbf{z}} r\, dr\, d\phi) = 2\pi \int_{r=0}^{a} J_0\, dr = 2\pi a J_0.$$

Therefore, since $I = 2\pi r H_2$, $H_2 = J_0 a/r$ within the wire and $\mathbf{H}_2 = \hat{\boldsymbol{\phi}} J_0 (a/r)$.

**Problem 5.24** In a certain conducting region, the magnetic field is given in cylindrical coordinates by

$$\mathbf{H} = \hat{\boldsymbol{\phi}}\frac{4}{r}[1-(1+3r)e^{-3r}]$$

Find the current density **J**.

**Solution:**

$$\mathbf{J} = \nabla\times\mathbf{H} = \hat{\mathbf{z}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{4}{r}[1-(1+3r)e^{-3r}]\right)$$

$$= \hat{\mathbf{z}}\frac{1}{r}[12e^{-2r}(1+2r)-12e^{-2r}] = \hat{\mathbf{z}}24e^{-3r}\ \text{A/m}^2.$$

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$$= \hat{z}\,\frac{1}{r}\left[12e^{-3r}(1+3r) - 12e^{-3r}\right]$$

$$= \hat{z}\,36e^{-3r}\ \text{A/m}^2$$

**Problem 5.27** In a given region of space, the vector magnetic potential is given by $\mathbf{A} = \hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x)$ (Wb/m).

**(a)** Determine **B**.

**(b)** Use Eq. (5.66) to calculate the magnetic flux passing through a square loop with 0.25-m-long edges if the loop is in the $x$–$y$ plane, its center is at the origin, and its edges are parallel to the $x$- and $y$-axes.

**(c)** Calculate $\Phi$ again using Eq. (5.67).

**Solution:**

**(a)** From Eq. (5.53), $\mathbf{B} = \nabla\times\mathbf{A} = \hat{\mathbf{z}}5\pi\sin\pi y - \hat{\mathbf{y}}\pi\cos\pi x$.

**(b)** From Eq. (5.66),

$$\begin{aligned}
\Phi = \iint \mathbf{B}\cdot d\mathbf{s} &= \int_{y=-0.125\text{ m}}^{0.125\text{ m}}\int_{x=-0.125\text{ m}}^{0.125\text{ m}} (\hat{\mathbf{z}}5\pi\sin\pi y - \hat{\mathbf{y}}\pi\cos\pi x)\cdot(\hat{\mathbf{z}}\,dx\,dy) \\
&= \left(\left(-5\pi x\frac{\cos\pi y}{\pi}\right)\Big|_{x=-0.125}^{0.125}\right)\Big|_{y=-0.125}^{0.125} \\
&= \frac{-5}{4}\left(\cos\left(\frac{\pi}{8}\right) - \cos\left(\frac{-\pi}{8}\right)\right) = 0.
\end{aligned}$$

**(c)** From Eq. (5.67), $\Phi = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$, where $C$ is the square loop in the $x$-$y$ plane with sides of length 0.25 m centered at the origin. Thus, the integral can be written as

$$\Phi = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell} = S_{\text{front}} + S_{\text{back}} + S_{\text{left}} + S_{\text{right}},$$

where $S_{\text{front}}$, $S_{\text{back}}$, $S_{\text{left}}$, and $S_{\text{right}}$ are the sides of the loop.

$$\begin{aligned}
S_{\text{front}} &= \int_{x=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))|_{y=-0.125}\cdot(\hat{\mathbf{x}}\,dx) \\
&= \int_{x=-0.125}^{0.125} 5\cos\pi y|_{y=-0.125}\,dx \\
&= \left((5x\cos\pi y)|_{y=-0.125}\right)\Big|_{x=-0.125}^{0.125} = \frac{5}{4}\cos\left(\frac{-\pi}{8}\right) = \frac{5}{4}\cos\left(\frac{\pi}{8}\right), \\
S_{\text{back}} &= \int_{x=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))|_{y=0.125}\cdot(-\hat{\mathbf{x}}\,dx) \\
&= -\int_{x=-0.125}^{0.125} 5\cos\pi y|_{y=0.125}\,dx \\
&= \left((-5x\cos\pi y)|_{y=0.125}\right)\Big|_{x=-0.125}^{0.125} = -\frac{5}{4}\cos\left(\frac{\pi}{8}\right),
\end{aligned}$$

$$
\begin{aligned}
S_{\text{left}} &= \int_{y=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))|_{x=-0.125} \cdot (-\hat{\mathbf{y}}\,dy) \\
&= -\int_{y=-0.125}^{0.125} 0|_{x=-0.125}\,dy = 0, \\
S_{\text{right}} &= \int_{y=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))|_{x=0.125} \cdot (\hat{\mathbf{y}}\,dy) \\
&= \int_{y=-0.125}^{0.125} 0|_{x=0.125}\,dy = 0.
\end{aligned}
$$

Thus,

$$
\Phi = \oint_{c} \mathbf{A}\cdot d\boldsymbol{\ell} = S_{\text{front}} + S_{\text{back}} + S_{\text{left}} + S_{\text{right}} = \frac{5}{4}\cos\left(\frac{\pi}{8}\right) - \frac{5}{4}\cos\left(\frac{\pi}{8}\right) + 0 + 0 = 0.
$$