Problem 4.41 A cylindrical bar of silicon has a radius of 4 mm and a length of 8 cm. If a voltage of 5 V is applied between the ends of the bar and $\mu_e = 0.13$ (m^2/V·s), $\mu_h = 0.05$ (m^2/V·s), $N_e = 1.5 \times 10^{16}$ electrons/m^3, and $N_h = N_e$, find the following:

(a) The conductivity of silicon.

(b) The current I flowing in the bar.

(c) The drift velocities $\mathbf{u}_e$ and $\mathbf{u}_h$.

(d) The resistance of the bar.

(e) The power dissipated in the bar.

Solution:

(a) Conductivity is given in Eq. (4.65),

$$\begin{aligned}\sigma &= (N_e\mu_e + N_h\mu_h)e \\ &= (1.5\times10^{16})(0.13+0.05)(1.6\times10^{-19}) = 4.32\times10^{-4} \quad \text{(S/m)}.\end{aligned}$$

(b) Similarly to Example 4.8, parts b and c,

$$I = JA = \sigma EA = (4.32\times10^{-4})\left(\frac{5\text{V}}{0.08}\right)(\pi(4\times10^{-3})^2) = 1.36 \quad (\mu\text{A}).$$

(c) From Eqs. (4.62a) and (4.62b),

$$\begin{aligned}\mathbf{u}_e &= -\mu_e\mathbf{E} = -(0.13)\left(\frac{5}{0.08}\right)\frac{\mathbf{E}}{|\mathbf{E}|} = -8.125\frac{\mathbf{E}}{|\mathbf{E}|} \quad \text{(m/s)},\\ \mathbf{u}_h &= \mu_h\mathbf{E} = +(0.05)\left(\frac{5}{0.08}\right)\frac{\mathbf{E}}{|\mathbf{E}|} = 3.125\frac{\mathbf{E}}{|\mathbf{E}|} \quad \text{(m/s)}.\end{aligned}$$

(d) To find the resistance, we use what we calculated above,

$$R = \frac{V}{I} = \frac{5\text{V}}{1.36\ \mu\text{A}} = 3.68 \quad (\text{M}\Omega).$$

(e) Power dissipated in the bar is $P = IV = (5\text{V})(1.36\ \mu\text{A}) = 6.8\ (\mu\text{W})$.

Problem 4.43 A 100-m-long conductor of uniform cross-section has a voltage drop of 4 V between its ends. If the density of the current flowing through it is 1.4×10^6 (A/m^2), identify the material of the conductor.

Solution: We know that conductivity characterizes a material:

$$\mathbf{J} = \sigma \mathbf{E}, \qquad 1.4 \times 10^6 \text{ (A/m}^2\text{)} = \sigma \left(\frac{4 \text{ (V)}}{100 \text{ (m)}} \right), \qquad \sigma = 3.5 \times 10^7 \quad \text{(S/m)}.$$

From Table B-2, we find that aluminum has $\sigma = 3.5 \times 10^7$ (S/m).

Problem 4.46 A 2×10^{-3}-mm-thick square sheet of aluminum has 5 cm $\times$ 5 cm faces. Find the following:

(a) The resistance between opposite edges on a square face.

(b) The resistance between the two square faces. (See Appendix B for the electrical constants of materials.)

Solution:

(a)

$$R = \frac{l}{\sigma A}.$$

For aluminum, $\sigma = 3.5 \times 10^7$ (S/m) [Appendix B].

$$l = 5 \text{ cm}, \qquad A = 5 \text{ cm} \times 2 \times 10^{-3} \text{ mm} = 10 \times 10^{-2} \times 10^{-6} = 1 \times 10^{-7} \text{ m}^2,$$

$$R = \frac{5 \times 10^{-2}}{3.5 \times 10^7 \times 1 \times 10^{-7}} = 14 \quad (\text{m}\Omega).$$

(b) Now, $l = 2 \times 10^{-3}$ mm and $A = 5 \text{ cm} \times 5 \text{ cm} = 2.5 \times 10^{-3} \text{ m}^2$.

$$R = \frac{2 \times 10^{-6}}{3.5 \times 10^7 \times 2.5 \times 10^{-3}} = 22.8 \text{ p}\Omega.$$

Problem 4.48 With reference to Fig. 4-19, find $\mathbf{E}_1$ if $\mathbf{E}_2 = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}2$ (V/m), $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 18\varepsilon_0$, and the boundary has a surface charge density $\rho_s = 3.54 \times 10^{-11}$ (C/m^2). What angle does $\mathbf{E}_2$ make with the z-axis?

Solution: We know that $\mathbf{E}_{1t} = \mathbf{E}_{2t}$ for any 2 media. Hence, $\mathbf{E}_{1t} = \mathbf{E}_{2t} = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2$. Also, $(\mathbf{D}_1 - \mathbf{D}_2) \cdot \hat{\mathbf{n}} = \rho_s$ (from Table 4.3). Hence, $\varepsilon_1(\mathbf{E}_1 \cdot \hat{\mathbf{n}}) - \varepsilon_2(\mathbf{E}_2 \cdot \hat{\mathbf{n}}) = \rho_s$, which gives

$$E_{1z} = \frac{\rho_s + \varepsilon_2 E_{2z}}{\varepsilon_1} = \frac{3.54 \times 10^{-11}}{2\varepsilon_0} + \frac{18(2)}{2} = \frac{3.54 \times 10^{-11}}{2 \times 8.85 \times 10^{-12}} + 18 = 20 \quad \text{(V/m)}.$$

Hence, $\mathbf{E}_1 = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}20$ (V/m). Finding the angle $\mathbf{E}_2$ makes with the z-axis:

$$\mathbf{E}_2 \cdot \hat{\mathbf{z}} = |\mathbf{E}_2| \cos\theta, \qquad 2 = \sqrt{9+4+4}\cos\theta, \qquad \theta = \cos^{-1}\left(\frac{2}{\sqrt{17}}\right) = 61^\circ.$$

Problem 4.50 If $\mathbf{E} = \hat{\mathbf{R}}150$ (V/m) at the surface of a 5-cm conducting sphere centered at the origin, what is the total charge Q on the sphere's surface?

Solution: From Table 4-3, $\hat{\mathbf{n}} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$. $\mathbf{E}_2$ inside the sphere is zero, since we assume it is a perfect conductor. Hence, for a sphere with surface area $S = 4\pi a^2$,

$$D_{1R} = \rho_s, \qquad E_{1R} = \frac{\rho_s}{\varepsilon_0} = \frac{Q}{S\varepsilon_0},$$

$$Q = E_R S\varepsilon_0 = (150)4\pi(0.05)^2\varepsilon_0 = \frac{3\pi\varepsilon_0}{2} \quad \text{(C)}.$$

Problem 4.51 Figure P4.51 shows three planar dielectric slabs of equal thickness but with different dielectric constants. If $\mathbf{E}_0$ in air makes an angle of 45° with respect to the z-axis, find the angle of $\mathbf{E}$ in each of the other layers.

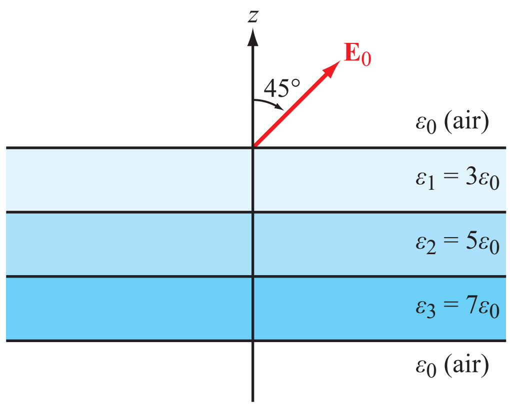

Figure P4.51: Dielectric slabs in Problem 4.51.

Solution: Labeling the upper air region as region 0 and using Eq. (4.99),

$$\theta_1 = \tan^{-1}\left(\frac{\varepsilon_1}{\varepsilon_0}\tan\theta_0\right) = \tan^{-1}(3\tan 45^\circ) = 71.6^\circ,$$

$$\theta_2 = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\theta_1\right) = \tan^{-1}\left(\frac{5}{3}\tan 71.6^\circ\right) = 78.7^\circ,$$

$$\theta_3 = \tan^{-1}\left(\frac{\varepsilon_3}{\varepsilon_2}\tan\theta_2\right) = \tan^{-1}\left(\frac{7}{5}\tan 78.7^\circ\right) = 81.9^\circ.$$

In the lower air region, the angle is again 45°.

Problem 4.52 Determine the force of attraction in a parallel-plate capacitor with $A = 5$ cm^2, $d = 2$ cm, and $\varepsilon_r = 4$ if the voltage across it is 50 V.

Solution: From Eq. (4.131),

$$\mathbf{F} = -\hat{\mathbf{z}}\,\frac{\varepsilon A|\mathbf{E}|^2}{2} = -\hat{\mathbf{z}}\,2\varepsilon_0(5\times10^{-4})\left(\frac{50}{0.02}\right)^2 = -\hat{\mathbf{z}}\,55.3\times10^{-9} \quad \text{(N)}.$$

Problem 4.53 Dielectric breakdown occurs in a material whenever the magnitude of the field $\mathbf{E}$ exceeds the dielectric strength anywhere in that material. In the coaxial capacitor of Example 4-12,

(a) At what value of r is $|E|$ maximum?

(b) What is the breakdown voltage if $a = 1$ cm, $b = 2$ cm, and the dielectric material is mica with $\varepsilon_r = 6$?

Solution:

(a) From Eq. (4.114), $\mathbf{E} = -\hat{\mathbf{r}}\rho_l/2\pi\varepsilon r$ for $a < r < b$. Thus, it is evident that $|\mathbf{E}|$ is maximum at $r = a$.

(b) The dielectric breaks down when $|\mathbf{E}| > 200$ (MV/m) (see Table 4-2), or

$$|\mathbf{E}| = \frac{\rho_l}{2\pi\varepsilon r} = \frac{\rho_l}{2\pi(6\varepsilon_0)(10^{-2})} = 200 \quad \text{(MV/m)},$$

which gives $\rho_l = (200 \text{ MV/m})(2\pi)6(8.854\times10^{-12})(0.01) = 667.6$ (μC/m).

From Eq. (4.115), we can find the voltage corresponding to that charge density,

$$V = \frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right) = \frac{(667.6\,\mu\text{C/m})}{12\pi(8.854\times10^{-12}\text{ F/m})}\ln(2) = 1.39 \quad \text{(MV)}.$$

Thus, $V = 1.39$ (MV) is the breakdown voltage for this capacitor.