Problem 2.27 At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)$ Ω. Find the input impedance.

Solution: Given a lossless transmission line, $Z_0 = 50$ Ω, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)$ Ω. Since the line is air filled, $u_p = c$ and therefore, from Eq. (2.48),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$

Since the line is lossless, Eq. (2.79) is valid:

$$\begin{aligned} Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) &= 50 \left[\frac{(40 + j20) + j50 \tan(2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan(2\pi \text{ rad/m} \times 2.5 \text{ m})} \right] \\ &= 50 \left[(40 + j20) + j50 \times 0 \right] 50 + j(40 + j20) \times 0 \\ &= (40 + j20) \ \Omega. \end{aligned}$$

Problem 2.28 A lossless transmission line of electrical length $l = 0.35\lambda$ is terminated in a load impedance as shown in Fig. P2.28. Find Γ, S, and Z_{in}. Verify your results using CD Modules 2.4 or 2.5. Include a printout of the screen's output display.

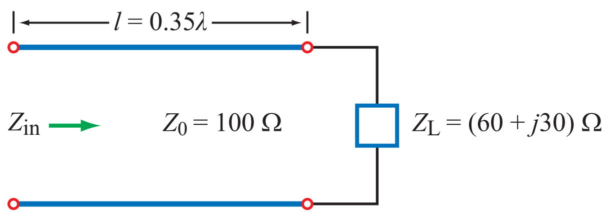


Figure P2.28: Circuit for Problem 2.28.

Solution: From Eq. (2.59),

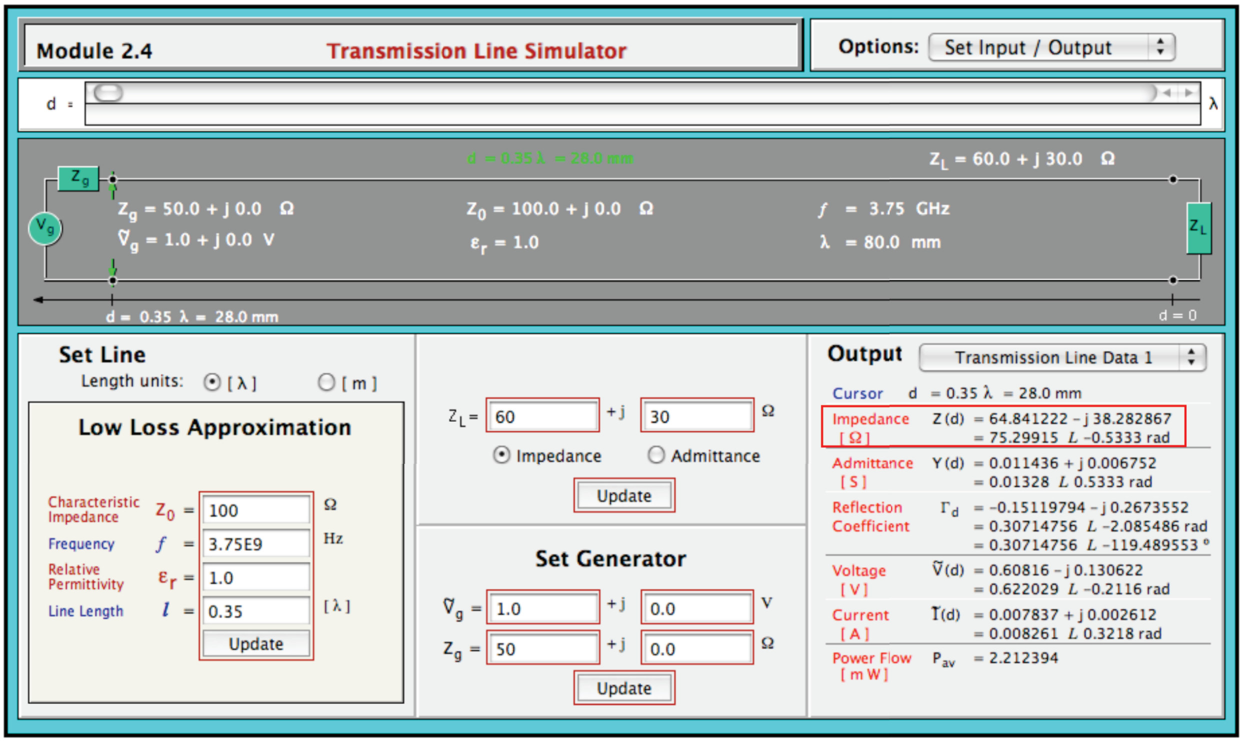
$$\Gamma = \frac{Z_{\text{L}} - Z_0}{Z_{\text{L}} + Z_0} = \frac{(60 + j30) - 100}{(60 + j30) + 100} = 0.307e^{j132.5^\circ}.$$

From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.307}{1 - 0.307} = 1.89.$$

From Eq. (2.79)

$$\begin{aligned} Z_{\text{in}} &= Z_0 \left(\frac{Z_{\text{L}} + jZ_0 \tan\beta l}{Z_0 + jZ_{\text{L}} \tan\beta l} \right) \\ &= 100 \left[\frac{(60 + j30) + j100 \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)}{100 + j(60 + j30) \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)} \right] = (64.8 - j38.3)\ \Omega. \end{aligned}$$



Module 2.4
Transmission Line Simulator
Options: Set Input / Output
d = 0.35 λ = 28.0 mm
Z_L = 60.0 + j 30.0 Ω
Z_g = 50.0 + j 0.0 Ω
Ṽ_g = 1.0 + j 0.0 V
Z_0 = 100.0 + j 0.0 Ω
ε_r = 1.0
f = 3.75 GHz
λ = 80.0 mm
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 100 Ω
Frequency f = 3.75E9 Hz
Relative Permittivity ε_r = 1.0
Line Length l = 0.35 [λ]
Update
Z_L = 60 + j 30 Ω
Impedance
Admittance
Update
Set Generator
Ṽ_g = 1.0 + j 0.0 V
Z_g = 50 + j 0.0 Ω
Update
Output
Transmission Line Data 1
Cursor d = 0.35 λ = 28.0 mm
Impedance [Ω] Z(d) = 64.841222 – j 38.282867 = 75.29915 ∠ –0.5333 rad
Admittance [S] Y(d) = 0.011436 + j 0.006752 = 0.01328 ∠ 0.5333 rad
Reflection Coefficient Γ_d = –0.15119794 – j 0.2673552 = 0.30714756 ∠ –2.085486 rad = 0.30714756 ∠ –119.489553 °
Voltage [V] Ṽ(d) = 0.60816 – j 0.130622 = 0.622029 ∠ –0.2116 rad
Current [A] Ĩ(d) = 0.007837 + j 0.002612 = 0.008261 ∠ 0.3218 rad
Power Flow [m W] P_av = 2.212394

Problem 2.32 A 6-m section of 150-Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \quad \text{(V)}$$

and $Z_g = 150$ Ω. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)$ Ω, determine:

(a) λ on the line.

(b) The reflection coefficient at the load.

(c) The input impedance.

(d) The input voltage $\widetilde{V}_i$.

(e) The time-domain input voltage $v_i(t)$.

(f) Quantities in (a) to (d) using CD Modules 2.4 or 2.5.

Solution:

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \text{ V},$$

$$\widetilde{V}_g = 5e^{-j30°} \text{ V}.$$

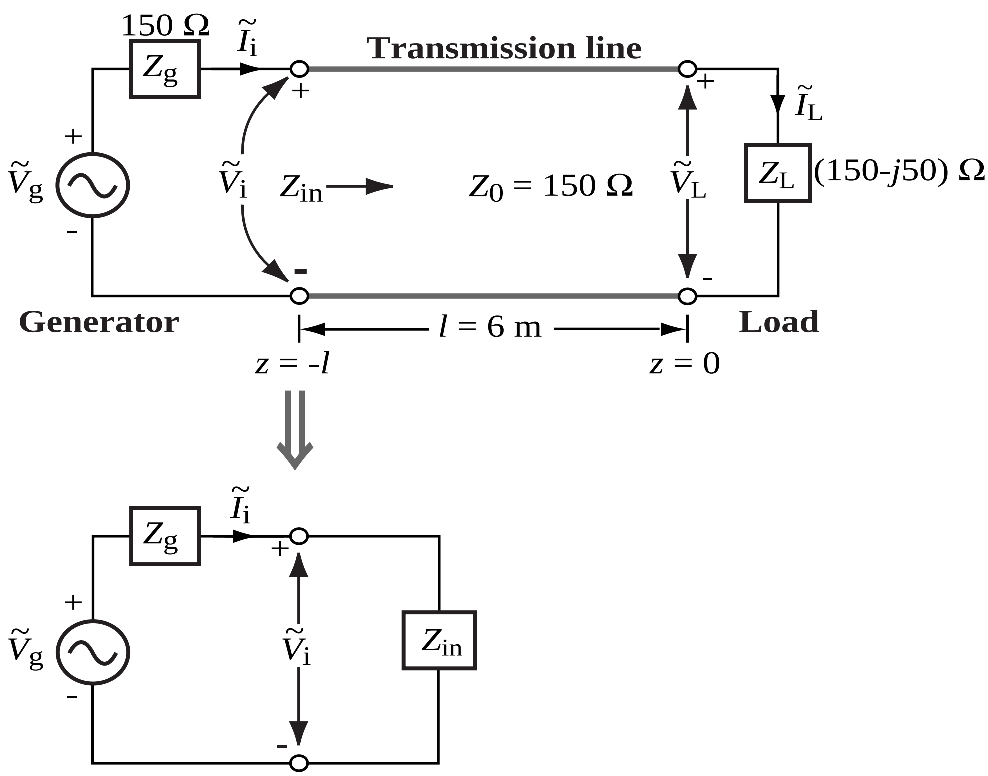


Figure P2.32: Circuit for Problem 2.32.

(a)

$$\begin{aligned}
u_{\rm p} &= \frac{c}{\sqrt{\varepsilon_{\rm r}}} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8 \quad \text{(m/s)},\\
\lambda &= \frac{u_{\rm p}}{f} = \frac{2\pi u_{\rm p}}{\omega} = \frac{2\pi\times2\times10^8}{8\pi\times10^7} = 5 \ \text{ m},\\
\beta &= \frac{\omega}{u_{\rm p}} = \frac{8\pi\times10^7}{2\times10^8} = 0.4\pi \quad \text{(rad/m)},\\
\beta l &= 0.4\pi\times6 = 2.4\pi \quad \text{(rad)}.
\end{aligned}$$

Since this exceeds 2π (rad), we can subtract 2π, which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \dfrac{Z_{\rm L}-Z_0}{Z_{\rm L}+Z_0} = \dfrac{150-j50-150}{150-j50+150} = \dfrac{-j50}{300-j50} = 0.16\,e^{-j80.54^\circ}$.

(c)

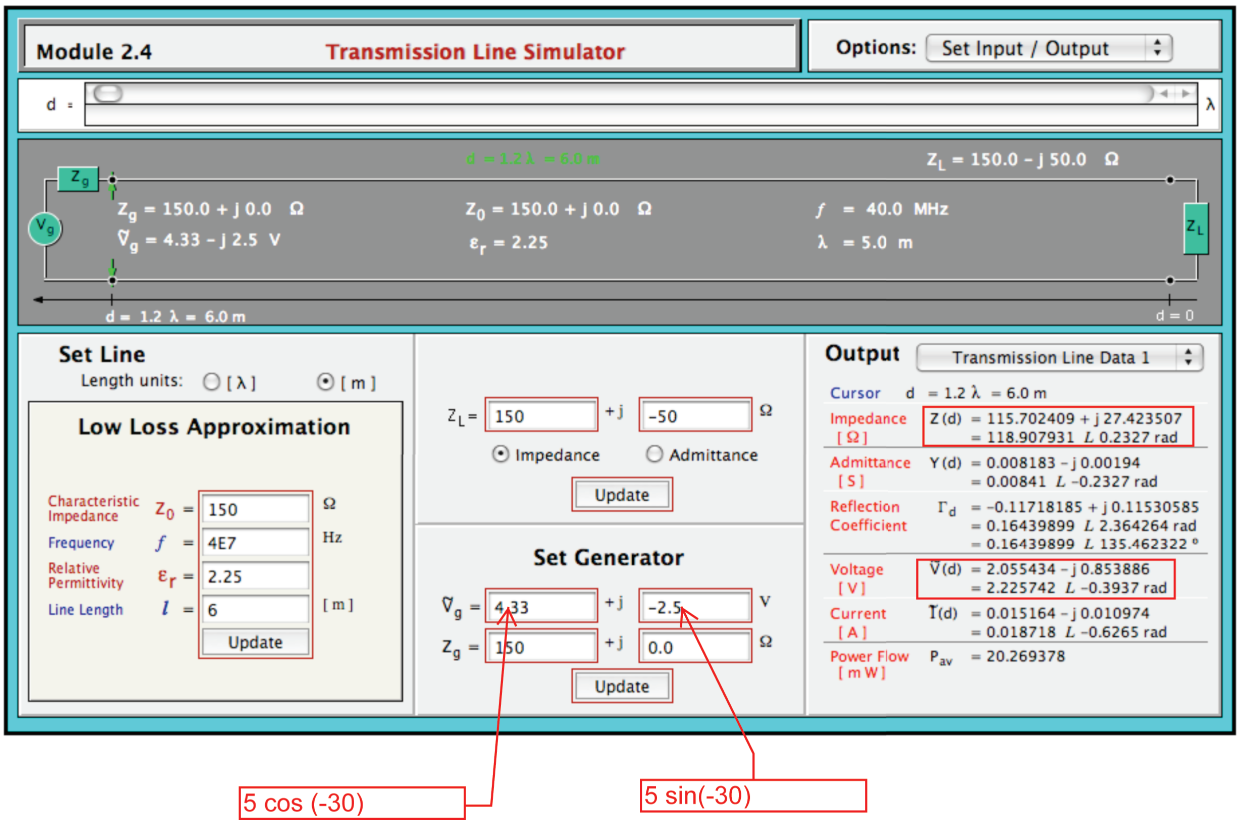
$$\begin{aligned}
Z_{\rm in} &= Z_0\left[\frac{Z_{\rm L}+jZ_0\tan\beta l}{Z_0+jZ_L\tan\beta l}\right]\\
&= 150\left[\frac{(150-j50)+j150\tan(0.4\pi)}{150+j(150-j50)\tan(0.4\pi)}\right] = (115.70+j27.42)\ \Omega.
\end{aligned}$$

(d)

$$\begin{aligned}
\widetilde{V}_{\rm i} = \frac{\widetilde{V}_{\rm g}Z_{\rm in}}{Z_{\rm g}+Z_{\rm in}} &= \frac{5e^{-j30^\circ}(115.7+j27.42)}{150+115.7+j27.42}\\
&= 5e^{-j30^\circ}\left(\frac{115.7+j27.42}{265.7+j27.42}\right)\\
&= 5e^{-j30^\circ}\times0.44\,e^{j7.44^\circ} = 2.2\,e^{-j22.56^\circ} \quad \text{(V)}.
\end{aligned}$$

(e)

$$v_{\rm i}(t) = \mathfrak{Re}[\widetilde{V}_{\rm i}e^{j\omega t}] = \mathfrak{Re}[2.2\,e^{-j22.56^\circ}e^{j\omega t}] = 2.2\cos(8\pi\times10^7t-22.56^\circ)\ \text{ V}.$$


Module 2.4
Transmission Line Simulator
Options: Set Input / Output
d = 1.2 λ = 6.0 m
Z_L = 150.0 − j 50.0 Ω
Z_g = 150.0 + j 0.0 Ω
V_g = 4.33 − j 2.5 V
Z_0 = 150.0 + j 0.0 Ω
ε_r = 2.25
f = 40.0 MHz
λ = 5.0 m
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 150 Ω
Frequency f = 4E7 Hz
Relative Permittivity ε_r = 2.25
Line Length l = 6 [m]
Update
Z_L = 150 + j -50 Ω
Impedance
Admittance
Update
Set Generator
V_g = 4.33 + j -2.5 V
Z_g = 150 + j 0.0 Ω
Update
Output
Transmission Line Data 1
Cursor d = 1.2 λ = 6.0 m
Impedance [Ω] Z(d) = 115.702409 + j 27.423507 = 118.907931 ∠ 0.2327 rad
Admittance [S] Y(d) = 0.008183 − j 0.00194 = 0.00841 ∠ -0.2327 rad
Reflection Coefficient Γ_d = -0.11718185 + j 0.11530585 = 0.16439899 ∠ 2.364264 rad = 0.16439899 ∠ 135.462322 °
Voltage [V] V(d) = 2.055434 − j 0.853886 = 2.225742 ∠ -0.3937 rad
Current [A] I(d) = 0.015164 − j 0.010974 = 0.018718 ∠ -0.6265 rad
Power Flow [m W] P_av = 20.269378
5 cos (-30)
5 sin(-30)

Problem 2.41 A 50-Ω lossless line of length $l = 0.375\lambda$ connects a 300-MHz generator with $\widetilde{V}_g = 300$ V and $Z_g = 50$ Ω to a load Z_L. Determine the time-domain current through the load for:

(a) $Z_L = (50 - j50)$ Ω

(b) $Z_L = 50$ Ω

(c) $Z_L = 0$ (short circuit)

For (a), verify your results by deducing the information you need from the output products generated by CD Module 2.4.

Solution:

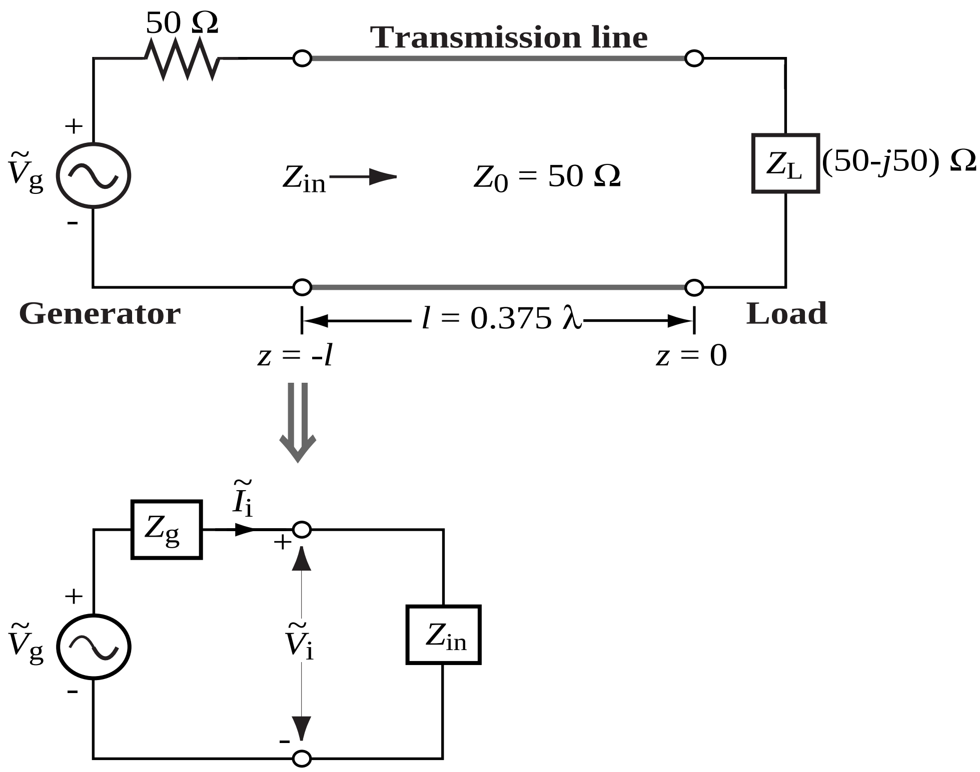


Figure P2.41: Circuit for Problem 2.41(a).

(a) $Z_L = (50 - j50)$ Ω, $\beta l = \frac{2\pi}{\lambda} \times 0.375\lambda = 2.36$ (rad) $= 135°$.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j50 - 50}{50 - j50 + 50} = \frac{-j50}{100 - j50} = 0.45e^{-j63.43°}.$$

Application of Eq. (2.79) gives:

$$Z_{\text{in}} = Z_0 \left[\frac{Z_{\text{L}} + jZ_0 \tan\beta l}{Z_0 + jZ_{\text{L}} \tan\beta l}\right] = 50 \left[\frac{(50 - j50) + j50 \tan 135^\circ}{50 + j(50 - j50)\tan 135^\circ}\right] = (100 + j50)\ \Omega.$$

Using Eq. (2.82) gives

$$\begin{aligned}
V_0^+ &= \left(\frac{\widetilde{V}_{\text{g}} Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) \\
&= \frac{300(100 + j50)}{50 + (100 + j50)}\left(\frac{1}{e^{j135^\circ} + 0.45\, e^{-j63.43^\circ} e^{-j135^\circ}}\right) \\
&= 150\, e^{-j135^\circ} \quad \text{(V)}, \\
\widetilde{I}_{\text{L}} &= \frac{V_0^+}{Z_0}(1 - \Gamma) = \frac{150\, e^{-j135^\circ}}{50}(1 - 0.45\, e^{-j63.43^\circ}) = 2.68\, e^{-j108.44^\circ} \quad \text{(A)}, \\
i_{\text{L}}(t) &= \mathfrak{Re}[\widetilde{I}_{\text{L}} e^{j\omega t}] \\
&= \mathfrak{Re}[2.68\, e^{-j108.44^\circ} e^{j6\pi \times 10^8 t}] \\
&= 2.68 \cos(6\pi \times 10^8 t - 108.44^\circ) \quad \text{(A)}.
\end{aligned}$$

(b)

$$\begin{aligned}
Z_{\text{L}} &= 50\ \Omega, \\
\Gamma &= 0, \\
Z_{\text{in}} &= Z_0 = 50\ \Omega, \\
V_0^+ &= \frac{300 \times 50}{50 + 50}\left(\frac{1}{e^{j135^\circ} + 0}\right) = 150\, e^{-j135^\circ} \quad \text{(V)}, \\
\widetilde{I}_{\text{L}} &= \frac{V_0^+}{Z_0} = \frac{150}{50}\, e^{-j135^\circ} = 3\, e^{-j135^\circ} \quad \text{(A)}, \\
i_{\text{L}}(t) &= \mathfrak{Re}[3\, e^{-j135^\circ} e^{j6\pi \times 10^8 t}] = 3 \cos(6\pi \times 10^8 t - 135^\circ) \quad \text{(A)}.
\end{aligned}$$

(c)

$$\begin{aligned}
Z_{\text{L}} &= 0, \\
\Gamma &= -1, \\
Z_{\text{in}} &= Z_0 \left(\frac{0 + jZ_0 \tan 135^\circ}{Z_0 + 0}\right) = jZ_0 \tan 135^\circ = -j50 \quad (\Omega), \\
V_0^+ &= \frac{300(-j50)}{50 - j50}\left(\frac{1}{e^{j135^\circ} - e^{-j135^\circ}}\right) = 150\, e^{-j135^\circ} \quad \text{(V)},
\end{aligned}$$

$$\widetilde{I}_L = \frac{V_0^+}{Z_0}[1-\Gamma] = \frac{150\,e^{-j135^\circ}}{50}[1+1] = 6e^{-j135^\circ} \quad \text{(A)},$$

$$i_L(t) = 6\cos(6\pi\times 10^8 t - 135^\circ) \quad \text{(A)}.$$

From output of Module 2.4, at $d = 0$ (load)

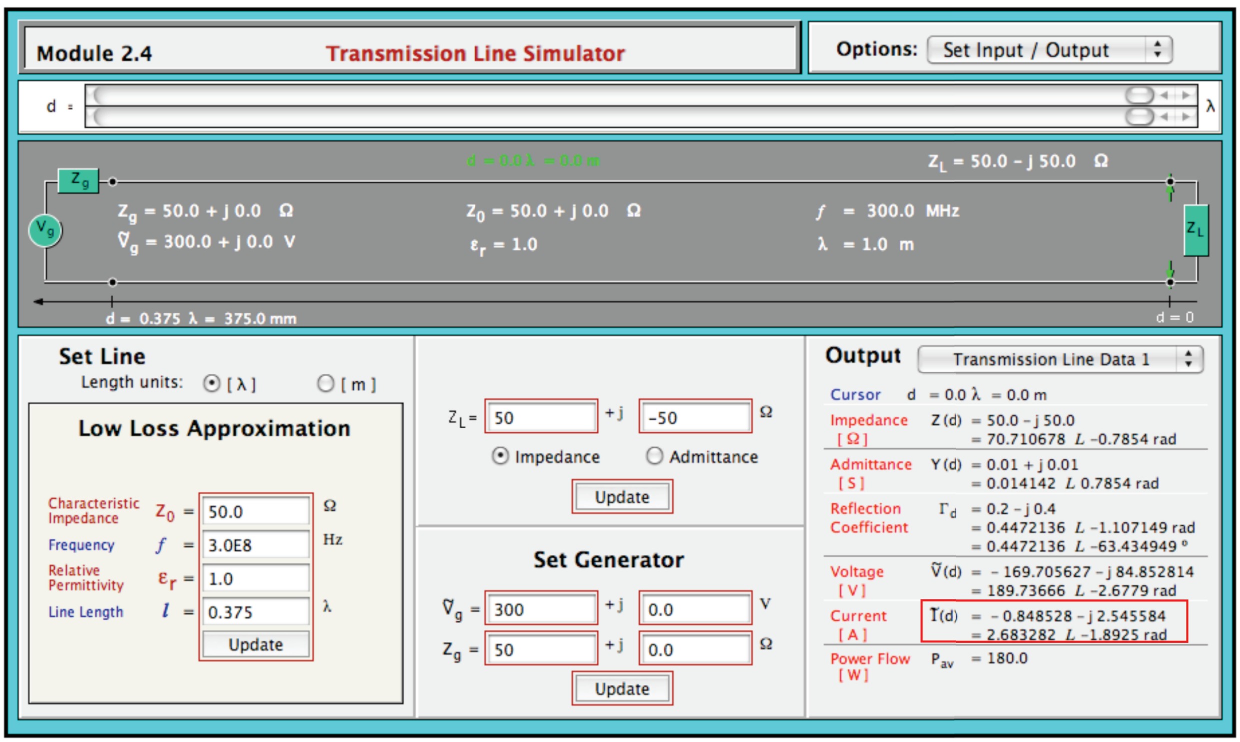
$$\widetilde{I}(d) = 2.68\angle{-1.89 \text{ rad}}\,,$$

which corresponds to

$$\widetilde{I}(d) = 2.68\angle{-108.29^\circ}\,.$$

The equivalent time-domain current at $f = 300$ MHz is

$$i_L(t) = 2.68\cos(6\pi\times 10^8 t - 108.29^\circ) \quad \text{(A)}.$$



Problem 2.42 A generator with $\widetilde{V}_g = 300$ V and $Z_g = 50\ \Omega$ is connected to a load $Z_L = 75\ \Omega$ through a 50-Ω lossless line of length $l = 0.15\lambda$.

(a) Compute Z_{in}, the input impedance of the line at the generator end.

(b) Compute $\widetilde{I}_i$ and $\widetilde{V}_i$.

(c) Compute the time-average power delivered to the line, $P_{in} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_i \widetilde{I}_i^*]$.

(d) Compute $\widetilde{V}_L$, $\widetilde{I}_L$, and the time-average power delivered to the load, $P_L = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_L \widetilde{I}_L^*]$. How does P_{in} compare to P_L? Explain.

(e) Compute the time-average power delivered by the generator, P_g, and the time-average power dissipated in Z_g. Is conservation of power satisfied?

Solution:

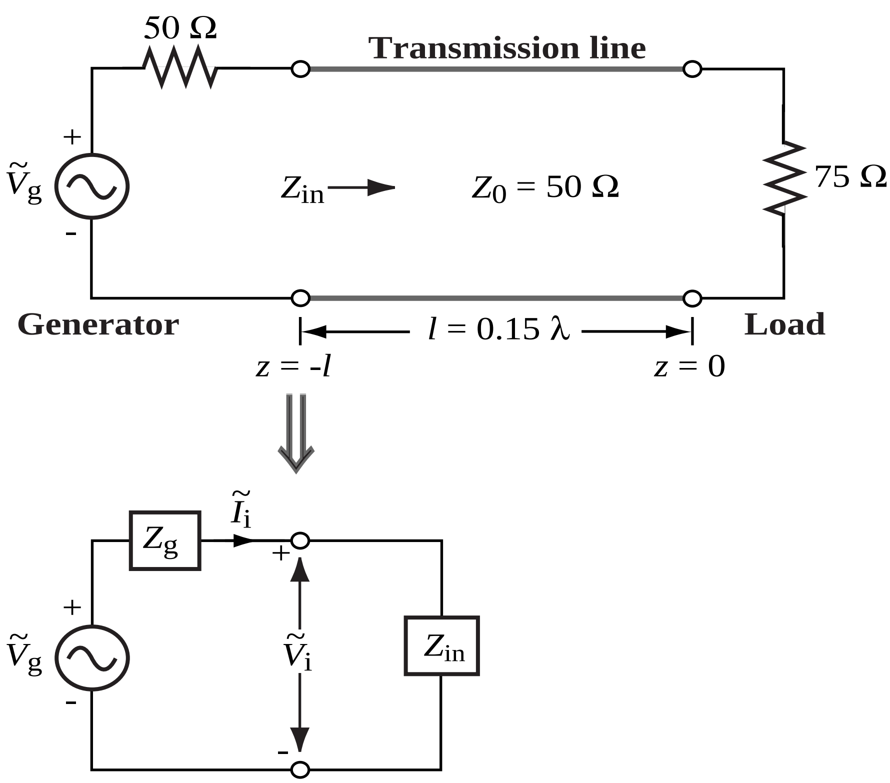


Figure P2.42: Circuit for Problem 2.42.

(a)

$$\beta l = \frac{2\pi}{\lambda} \times 0.15\lambda = 54^\circ,$$

$$Z_{\rm in} = Z_0 \left[\frac{Z_{\rm L} + jZ_0 \tan\beta l}{Z_0 + jZ_{\rm L} \tan\beta l}\right] = 50 \left[\frac{75 + j50 \tan 54^\circ}{50 + j75 \tan 54^\circ}\right] = (41.25 - j16.35)\ \Omega.$$

(b)

$$\widetilde{I}_{\rm i} = \frac{\widetilde{V}_{\rm g}}{Z_{\rm g} + Z_{\rm in}} = \frac{300}{50 + (41.25 - j16.35)} = 3.24\,e^{j10.16^\circ} \quad \text{(A)},$$
$$\widetilde{V}_{\rm i} = \widetilde{I}_{\rm i} Z_{\rm in} = 3.24\,e^{j10.16^\circ}(41.25 - j16.35) = 143.6\,e^{-j11.46^\circ} \quad \text{(V)}.$$

(c)

$$P_{\rm in} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\rm i}\widetilde{I}_{\rm i}^*] = \frac{1}{2}\mathfrak{Re}[143.6\,e^{-j11.46^\circ} \times 3.24\,e^{-j10.16^\circ}]$$
$$= \frac{143.6 \times 3.24}{2}\cos(21.62^\circ) = 216 \quad \text{(W)}.$$

(d)

$$\Gamma = \frac{Z_{\rm L} - Z_0}{Z_{\rm L} + Z_0} = \frac{75 - 50}{75 + 50} = 0.2,$$
$$V_0^+ = \widetilde{V}_{\rm i}\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \frac{143.6\,e^{-j11.46^\circ}}{e^{j54^\circ} + 0.2\,e^{-j54^\circ}} = 150e^{-j54^\circ} \quad \text{(V)},$$
$$\widetilde{V}_{\rm L} = V_0^+(1 + \Gamma) = 150e^{-j54^\circ}(1 + 0.2) = 180e^{-j54^\circ} \quad \text{(V)},$$
$$\widetilde{I}_{\rm L} = \frac{V_0^+}{Z_0}(1 - \Gamma) = \frac{150e^{-j54^\circ}}{50}(1 - 0.2) = 2.4\,e^{-j54^\circ} \quad \text{(A)},$$
$$P_{\rm L} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\rm L}\widetilde{I}_{\rm L}^*] = \frac{1}{2}\mathfrak{Re}[180e^{-j54^\circ} \times 2.4\,e^{j54^\circ}] = 216 \quad \text{(W)}.$$

$P_{\rm L} = P_{\rm in}$, which is as expected because the line is lossless; power input to the line ends up in the load.

(e)

Power delivered by generator:

$$P_{\rm g} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\rm g}\widetilde{I}_{\rm i}] = \frac{1}{2}\mathfrak{Re}[300 \times 3.24\,e^{j10.16^\circ}] = 486\cos(10.16^\circ) = 478.4 \quad \text{(W)}.$$

Power dissipated in $Z_{\rm g}$*:*

$$P_{Z_{\rm g}} = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_{\rm i}\widetilde{V}_{Z_{\rm g}}] = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_{\rm i}\widetilde{I}_{\rm i}^* Z_{\rm g}] = \frac{1}{2}|\widetilde{I}_{\rm i}|^2 Z_{\rm g} = \frac{1}{2}(3.24)^2 \times 50 = 262.4 \quad \text{(W)}.$$

Note 1: $P_{\rm g} = P_{Z_{\rm g}} + P_{\rm in} = 478.4$ W.

Problem 2.75 Generate a bounce diagram for the voltage $V(z,t)$ for a 1-m–long lossless line characterized by $Z_0 = 50\ \Omega$ and $u_p = 2c/3$ (where c is the velocity of light) if the line is fed by a step voltage applied at $t = 0$ by a generator circuit with $V_g = 60$ V and $R_g = 100\ \Omega$. The line is terminated in a load $R_L = 25\ \Omega$. Use the bounce diagram to plot $V(t)$ at a point midway along the length of the line from $t = 0$ to $t = 25$ ns.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3},$$
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25-50}{25+50} = \frac{-25}{75} = \frac{-1}{3}.$$

From Eq. (2.149b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100+50} = 20 \text{ V}.$$

Also,

$$T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5 \text{ ns}.$$

The bounce diagram is shown in Fig. P2.75(a) and the plot of $V(t)$ in Fig. P2.75(b).

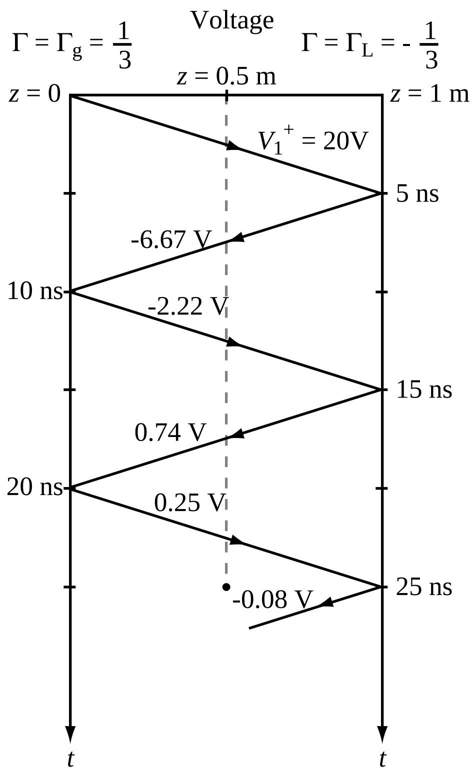


Figure P2.75: (a) Bounce diagram for Problem 2.75.

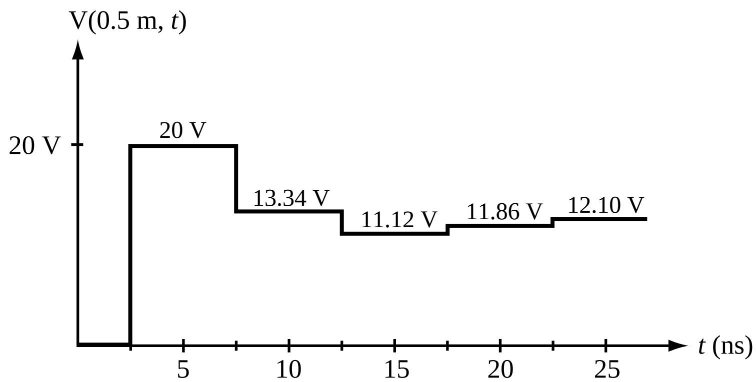


Figure P2.75: (b) Time response of voltage.

Problem 2.78 In response to a step voltage, the voltage waveform shown in Fig. P2.78 was observed at the sending end of a shorted line with $Z_0 = 50\ \Omega$ and $\varepsilon_r = 4$. Determine V_g, R_g, and the line length.

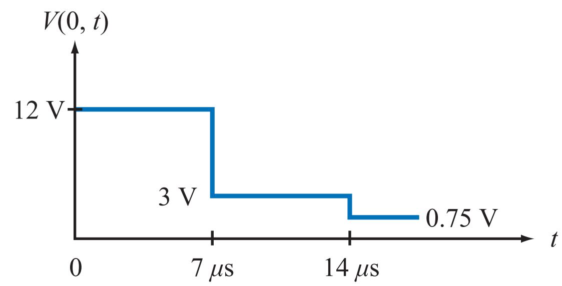


Figure P2.78: Voltage waveform of Problem 2.78.

Solution:

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{\sqrt{4}} = 1.5\times 10^8 \text{ m/s},$$

$$7\ \mu\text{s} = 7\times 10^{-6}\text{ s} = \frac{2l}{u_p} = \frac{2l}{1.5\times 10^8}.$$

Hence, $l = 525$ m.

From the voltage waveform, $V_1^+ = 12$ V. At $t = 7\mu$s, the voltage at the sending end is

$$V(z=0, t=7\mu\text{s}) = V_1^+ + \Gamma_L V_1^+ + \Gamma_g \Gamma_L V_1^+ = -\Gamma_g V_1^+ \qquad (\text{because } \Gamma_L = -1).$$

Hence, 3 V$= -\Gamma_g \times 12$ V, or $\Gamma_g = -0.25$. From Eq. (2.153),

$$R_g = Z_0 \left(\frac{1+\Gamma_g}{1-\Gamma_g}\right) = 50\left(\frac{1-0.25}{1+0.25}\right) = 30\ \Omega.$$

Also,

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}, \qquad \text{or} \quad 12 = \frac{V_g \times 50}{30+50},$$

which gives $V_g = 19.2$ V.