Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $$\frac{l}{\lambda} = \frac{lf}{u_\mathrm{p}} = \frac{(20 \times 10^{-2}\ \mathrm{m}) \times (20 \times 10^{3}\ \mathrm{Hz})}{3 \times 10^{8}\ \mathrm{m/s}} = 1.33 \times 10^{-5}\ \text{(negligible)}.$$

(b) $$\frac{l}{\lambda} = \frac{lf}{u_\mathrm{p}} = \frac{(50 \times 10^{3}\ \mathrm{m}) \times (60 \times 10^{0}\ \mathrm{Hz})}{3 \times 10^{8}\ \mathrm{m/s}} = 0.01\ \text{(borderline)}.$$

(c) $$\frac{l}{\lambda} = \frac{lf}{u_\mathrm{p}} = \frac{(20 \times 10^{-2}\ \mathrm{m}) \times (600 \times 10^{6}\ \mathrm{Hz})}{3 \times 10^{8}\ \mathrm{m/s}} = 0.40\ \text{(nonnegligible)}.$$

(d) $$\frac{l}{\lambda} = \frac{lf}{u_\mathrm{p}} = \frac{(1 \times 10^{-3}\ \mathrm{m}) \times (100 \times 10^{9}\ \mathrm{Hz})}{3 \times 10^{8}\ \mathrm{m/s}} = 0.33\ \text{(nonnegligible)}.$$

Problem 2.2 A two-wire copper transmission line is embedded in a dielectric material with $\varepsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

(a) Calculate the line parameters R', L', G', and C' at 2 GHz.

(b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:

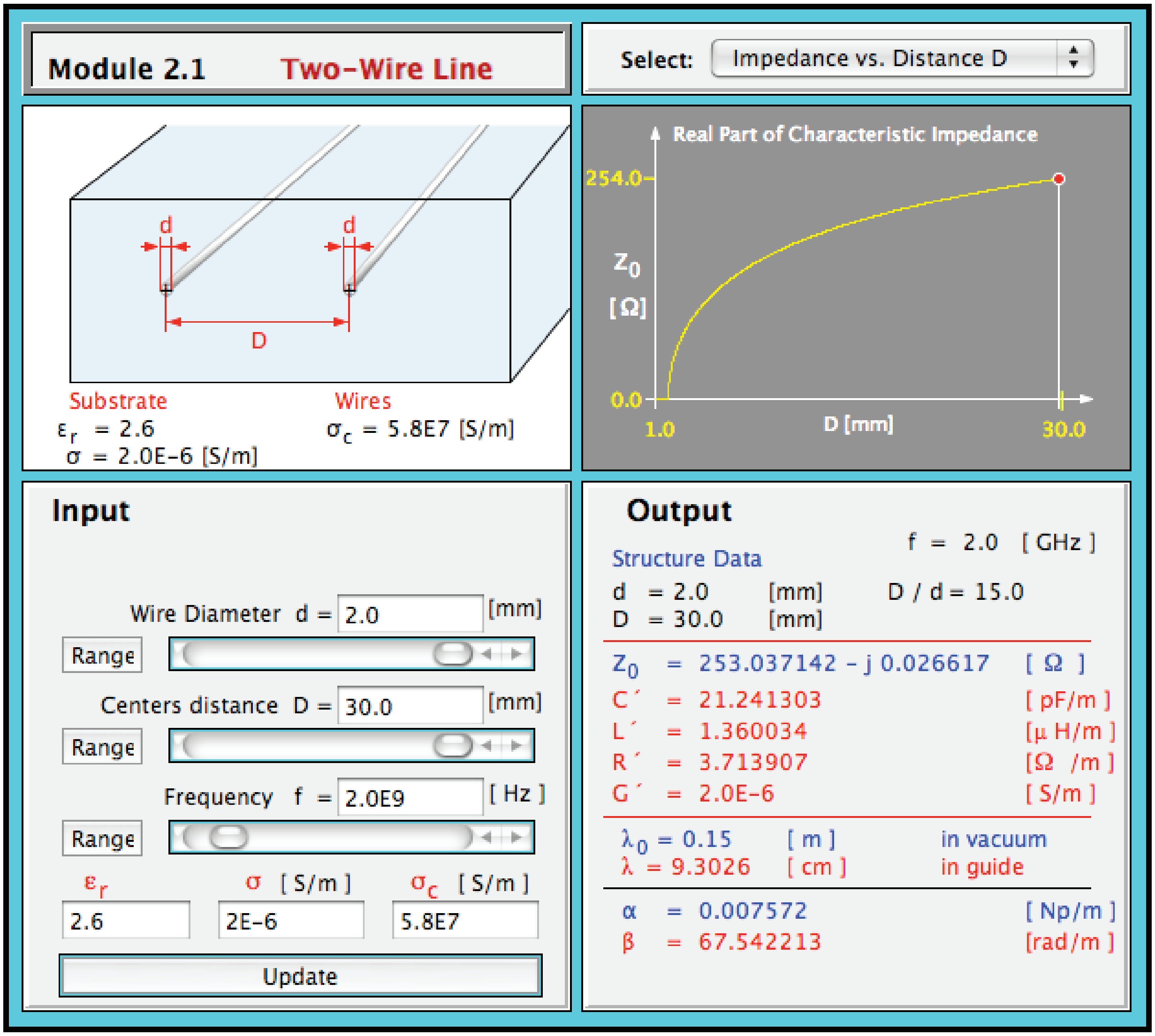
(a) Given:

$$
\begin{aligned}
f &= 2 \times 10^{9} \text{ Hz},\\
d &= 2 \times 10^{-3} \text{ m},\\
D &= 3 \times 10^{-2} \text{ m},\\
\sigma_c &= 5.8 \times 10^{7} \text{ S/m (copper)},\\
\varepsilon_r &= 2.6,\\
\sigma &= 2 \times 10^{-6} \text{ S/m},\\
\mu &= \mu_c = \mu_0.
\end{aligned}
$$

From Table 2-1:

$$
\begin{aligned}
R_s &= \sqrt{\pi f \mu_c / \sigma_c}\\
&= [\pi \times 2 \times 10^{9} \times 4\pi \times 10^{-7} / 5.8 \times 10^{7}]^{1/2}\\
&= 1.17 \times 10^{-2}\ \Omega,\\
R' &= \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2}}{2\pi \times 10^{-3}} = 3.71\ \Omega/\text{m},\\
L' &= \frac{\mu}{\pi} \ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right]\\
&= 1.36 \times 10^{-6} \text{ H/m},\\
G' &= \frac{\pi\sigma}{\ln[(D/d) + \sqrt{(D/d)^2 - 1}]}\\
&= 1.85 \times 10^{-6} \text{ S/m},\\
C' &= \frac{G'\varepsilon}{\sigma}\\
&= \frac{1.85 \times 10^{-6} \times 8.85 \times 10^{-12} \times 2.6}{2 \times 10^{-6}}\\
&= 2.13 \times 10^{-11} \text{ F/m}.
\end{aligned}
$$

(b) Solution via Module 2.1:


Module 2.1
Two-Wire Line
Select:
Impedance vs. Distance D
d
d
D
Substrate
εr = 2.6
σ = 2.0E-6 [S/m]
Wires
σc = 5.8E7 [S/m]
Real Part of Characteristic Impedance
254.0
Z0
[Ω]
0.0
1.0
D [mm]
30.0
Input
Wire Diameter d = 2.0 [mm]
Range
Centers distance D = 30.0 [mm]
Range
Frequency f = 2.0E9 [Hz]
Range
εr
σ [S/m]
σc [S/m]
2.6
2E-6
5.8E7
Update
Output
f = 2.0 [GHz]
Structure Data
d = 2.0 [mm]
D / d = 15.0
D = 30.0 [mm]
Z0 = 253.037142 - j 0.026617 [Ω]
C´ = 21.241303 [pF/m]
L´ = 1.360034 [μ H/m]
R´ = 3.713907 [Ω /m]
G´ = 2.0E-6 [S/m]
λ0 = 0.15 [m] in vacuum
λ = 9.3026 [cm] in guide
α = 0.007572 [Np/m]
β = 67.542213 [rad/m]

Problem 2.6 A coaxial line with inner and outer conductor diameters of 0.5 cm and 1 cm, respectively, is filled with an insulating material with $\varepsilon_r = 4.5$ and $\sigma = 10^{-3}$ S/m. The conductors are made of copper.

(a) Calculate the line parameters at 1 GHz.

(b) Compare your results with those based on CD Module 2.2. Include a printout of the screen display.

Solution: (a) Given

$$a = (0.5/2) \text{ cm} = 0.25 \times 10^{-2} \text{ m},$$
$$b = (1.0/2) \text{ cm} = 0.50 \times 10^{-2} \text{ m},$$

combining Eqs. (2.5) and (2.6) gives

$$\begin{aligned} R' &= \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right) \\ &= \frac{1}{2\pi}\sqrt{\frac{\pi(10^9 \text{ Hz})(4\pi\times 10^{-7} \text{ H/m})}{5.8\times 10^7 \text{ S/m}}}\left(\frac{1}{0.25\times 10^{-2} \text{ m}}+\frac{1}{0.50\times 10^{-2} \text{ m}}\right) \\ &= 0.788\ \Omega/\text{m}. \end{aligned}$$

From Eq. (2.7),

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times 10^{-7} \text{ H/m}}{2\pi}\ln 2 = 139 \text{ nH/m}.$$

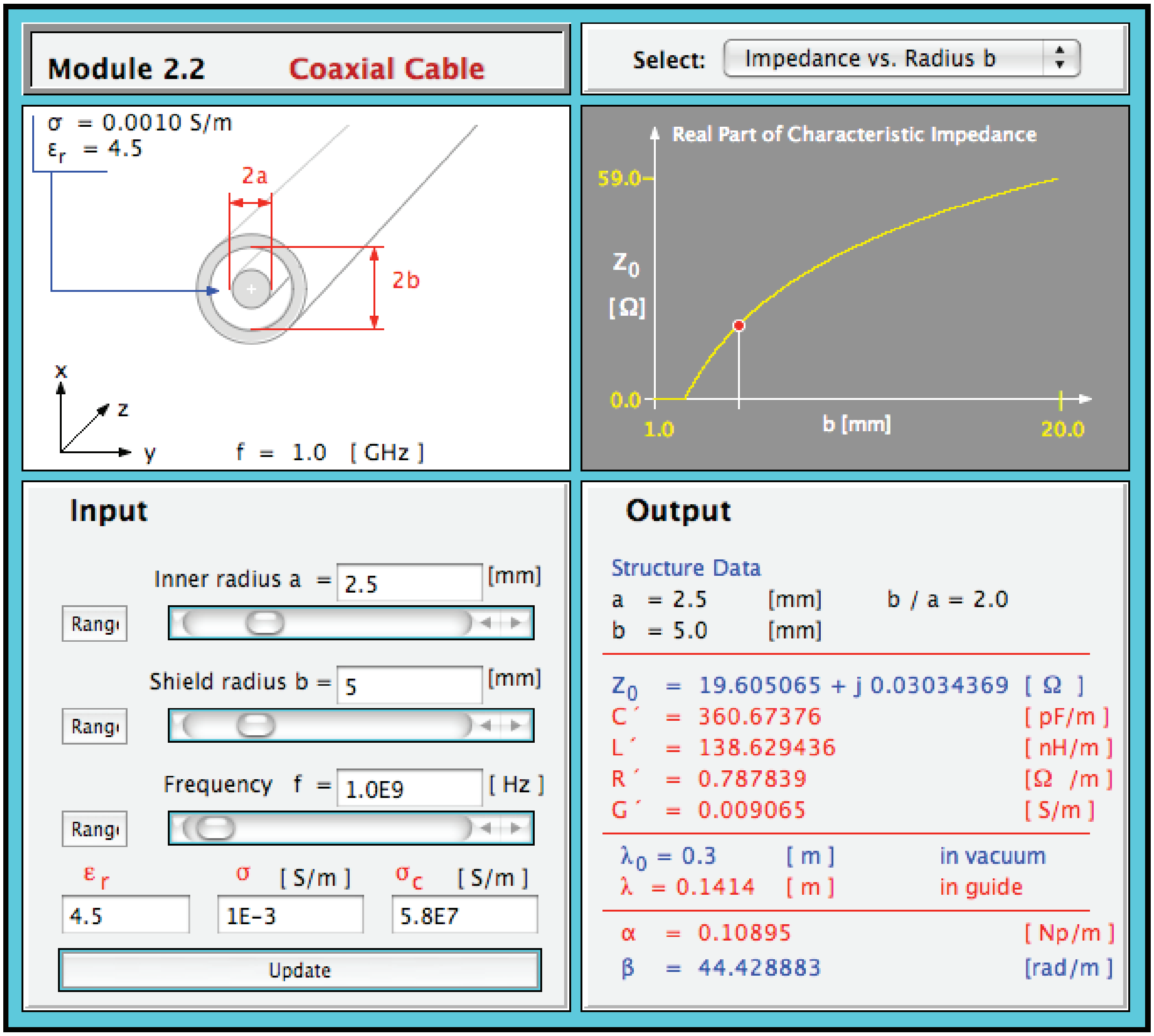
From Eq. (2.8),

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi\times 10^{-3} \text{ S/m}}{\ln 2} = 9.1 \text{ mS/m}.$$

From Eq. (2.9),

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln(b/a)} = \frac{2\pi\times 4.5\times(8.854\times 10^{-12} \text{ F/m})}{\ln 2} = 362 \text{ pF/m}.$$

(b) Solution via Module 2.2:



Module 2.2
Coaxial Cable
Select: Impedance vs. Radius b
σ = 0.0010 S/m
εr = 4.5
2a
2b
x
z
y
f = 1.0 [GHz]
Real Part of Characteristic Impedance
59.0
Z0
[Ω]
0.0
1.0
b [mm]
20.0
Input
Inner radius a = 2.5 [mm]
Shield radius b = 5 [mm]
Frequency f = 1.0E9 [Hz]
εr
σ [S/m]
σc [S/m]
4.5
1E-3
5.8E7
Update
Output
Structure Data
a = 2.5 [mm] b / a = 2.0
b = 5.0 [mm]
Z0 = 19.605065 + j 0.03034369 [Ω]
C´ = 360.67376 [pF/m]
L´ = 138.629436 [nH/m]
R´ = 0.787839 [Ω /m]
G´ = 0.009065 [S/m]
λ0 = 0.3 [m] in vacuum
λ = 0.1414 [m] in guide
α = 0.10895 [Np/m]
β = 44.428883 [rad/m]

Problem 2.7 Find α, β, u_{p}, and Z_0 for the two-wire line of Problem 2.2. Compare results with those based on CD Module 2.1. Include a printout of the screen display.

Solution: From Problem 2.2:

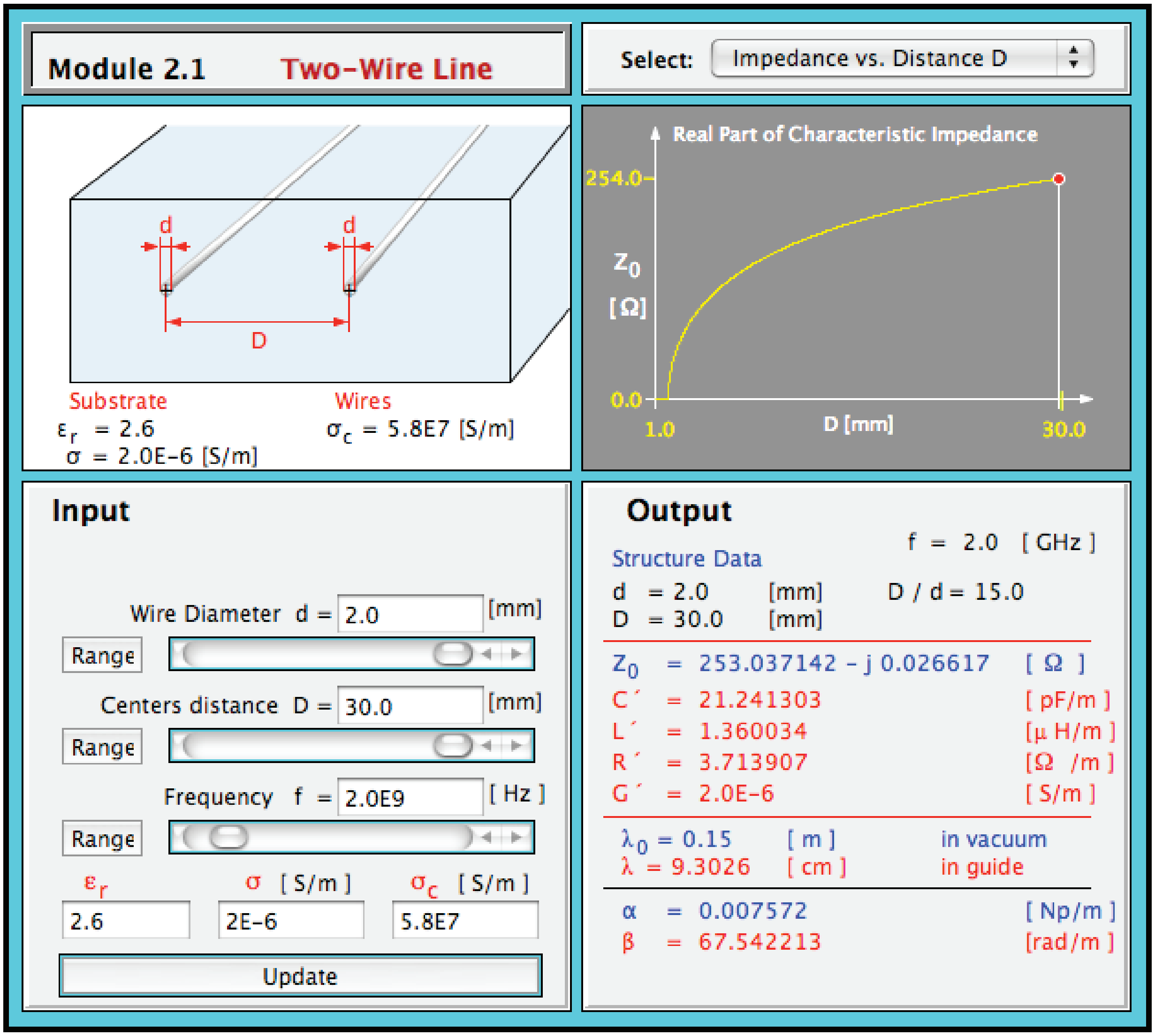
$$
\begin{aligned}
R' &= 3.71\ \Omega/\mathrm{m}, \\
L' &= 1.36 \times 10^{-6}\ \mathrm{H/m}, \\
G' &= 1.85 \times 10^{-6}\ \mathrm{S/m}, \\
C' &= 2.13 \times 10^{-11}\ \mathrm{F/m}.
\end{aligned}
$$

At 2 GHz:

$$
\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= 0.0076 + j67.54.
\end{aligned}
$$

Hence

$$
\begin{aligned}
\alpha &= 0.0076\ \mathrm{Np/m}, \\
\beta &= 67.54\ \mathrm{rad/m}. \\
u_{\mathrm{p}} &= \frac{\omega}{\beta} = \frac{2\pi \times 2 \times 10^9}{67.54} = 1.86 \times 10^8\ \mathrm{m/s}, \\
Z_0 &= \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = 253\ \Omega.
\end{aligned}
$$


Module 2.1
Two-Wire Line
Select:
Impedance vs. Distance D
d
d
D
Substrate
εr = 2.6
σ = 2.0E-6 [S/m]
Wires
σc = 5.8E7 [S/m]
Real Part of Characteristic Impedance
254.0
Z0
[Ω]
0.0
1.0
D [mm]
30.0
Input
Wire Diameter d = 2.0 [mm]
Range
Centers distance D = 30.0 [mm]
Range
Frequency f = 2.0E9 [Hz]
Range
εr
σ [S/m]
σc [S/m]
2.6
2E-6
5.8E7
Update
Output
f = 2.0 [GHz]
Structure Data
d = 2.0 [mm]
D / d = 15.0
D = 30.0 [mm]
Z0 = 253.037142 - j 0.026617 [Ω]
C´ = 21.241303 [pF/m]
L´ = 1.360034 [μ H/m]
R´ = 3.713907 [Ω /m]
G´ = 2.0E-6 [S/m]
λ0 = 0.15 [m] in vacuum
λ = 9.3026 [cm] in guide
α = 0.007572 [Np/m]
β = 67.542213 [rad/m]

Problem 2.9 A lossless microstrip line uses a 1-mm–wide conducting strip over a 1-cm–thick substrate with $\varepsilon_{\mathrm{r}} = 2.5$. Determine the line parameters, $\varepsilon_{\mathrm{eff}}$, Z_0, and β at 10 GHz. Compare your results with those obtained by using CD Module 2.3. Include a printout of the screen display.

Solution: Given

$$\begin{aligned} w &= 10^{-3}\ \mathrm{m}, \\ h &= 10^{-2}\ \mathrm{m}, \\ \varepsilon_{\mathrm{r}} &= 2.5, \\ f &= 1 \times 10^{10}\ \mathrm{Hz}, \\ s &= \frac{w}{h} = 0.1. \end{aligned}$$

From Eq. (2.36),

$$\varepsilon_{\mathrm{eff}} = \frac{\varepsilon_{\mathrm{r}} + 1}{2} + \left(\frac{\varepsilon_{\mathrm{r}} - 1}{2}\right)\left(1 + \frac{10}{s}\right)^{-xy}$$

with

$$\begin{aligned} x &= 0.56 \left[\frac{\varepsilon_{\mathrm{r}} - 0.9}{\varepsilon_{\mathrm{r}} + 3}\right]^{0.05} \\ &= 0.56 \left[\frac{2.5 - 0.9}{2.5 + 3}\right]^{0.05} = 0.526, \\ y &= 1 + 0.02 \ln\left(\frac{s^4 + 3.7 \times 10^{-4} s^2}{s^4 + 0.43}\right) \\ &\quad + 0.05 \ln(1 + 1.7 \times 10^{-4} s^3) \\ &\approx 0.83, \end{aligned}$$

which leads to

$$\varepsilon_{\mathrm{eff}} \approx 1.85.$$

By Eq. (2.39),

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\mathrm{eff}}}} \ln\left\{\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right\}$$

with

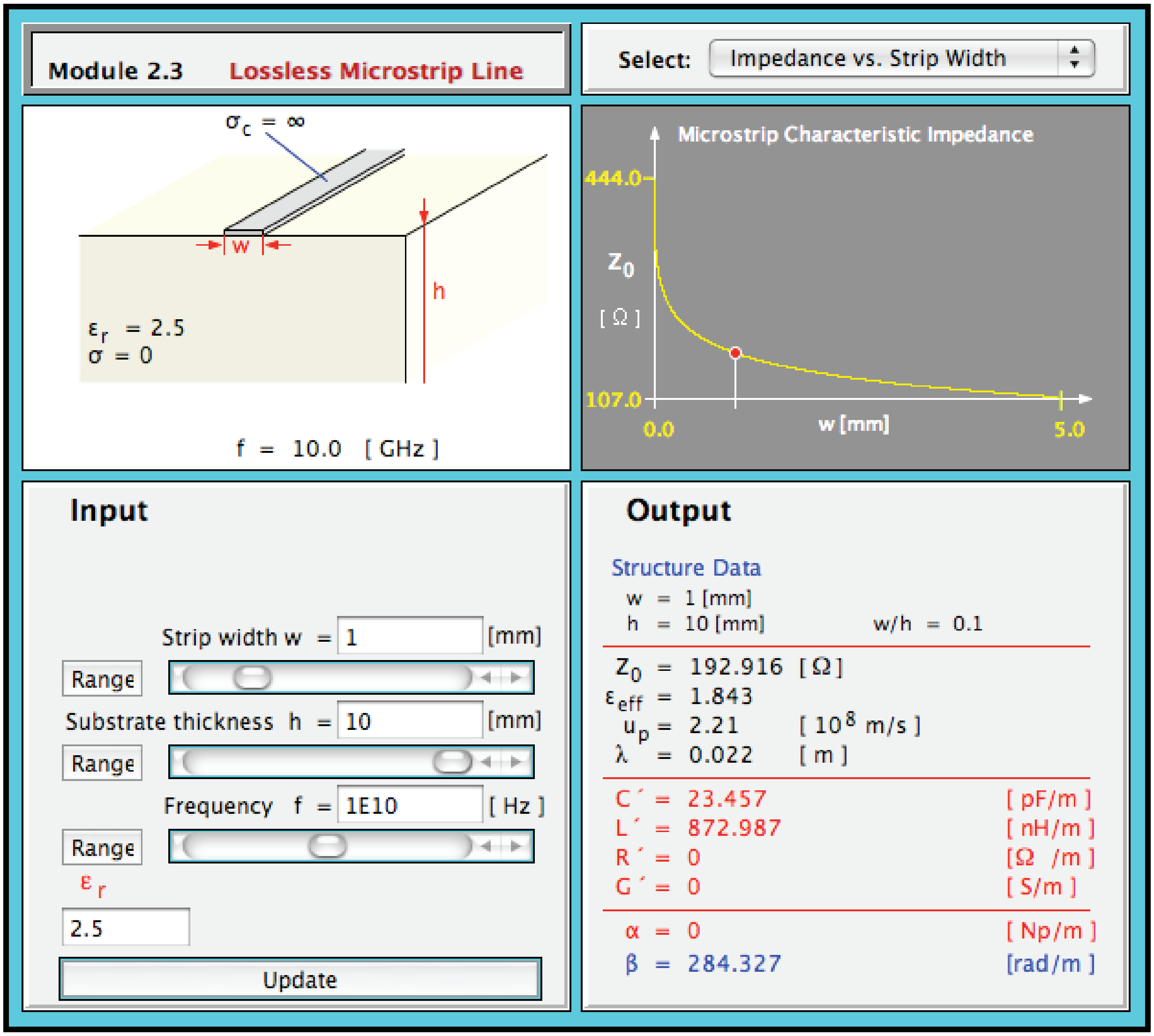
$$t = \left(\frac{30.67}{s}\right)^{0.75} = \left(\frac{30.67}{0.1}\right)^{0.75} = 73.29.$$

Hence,

$$Z_0 = 193.3\ \Omega.$$

Also,

$$\beta = \frac{\omega}{c}\sqrt{\varepsilon_{\text{eff}}} = \frac{2\pi \times 10^{10}}{3 \times 10^{8}}\sqrt{1.85} = 284.87 \text{ rad/m}.$$



Problem 2.16 A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R', L', G', and C'.

Solution: Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40\ \Omega$, $\alpha = 0.02$ Np/m, and $\beta = 0.75$ rad/m. Since Z_0 is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'}\sqrt{\frac{R'}{G'}} = \sqrt{R'G'}\sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \text{ Np/m} \times 40\ \Omega = 0.6\ \Omega\text{/m}$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \text{ Np/m})^2}{0.8\ \Omega\text{/m}} = 0.5 \text{ mS/m}.$$

Problem 2.18 Polyethylene with $\varepsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with characteristic impedance of 50 Ω. The radius of the inner conductor is 1.2 mm.

(a) What is the radius of the outer conductor?

(b) What is the phase velocity of the line?

Solution: Given a lossless coaxial line, $Z_0 = 50\ \Omega$, $\varepsilon_r = 2.25$, $a = 1.2$ mm:

(a) From Table 2-2, $Z_0 = (60/\sqrt{\varepsilon_r}) \ln(b/a)$ which can be rearranged to give

$$b = ae^{Z_0\sqrt{\varepsilon_r}/60} = (1.2 \text{ mm})e^{50\sqrt{2.25}/60} = 4.2 \text{ mm}.$$

(b) Also from Table 2-2,

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times 10^8 \text{ m/s}}{\sqrt{2.25}} = 2.0\times 10^8 \text{ m/s}.$$

Problem 2.19 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)\ \Omega$. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,
(b) the standing-wave ratio on the line,
(c) the position of the voltage maximum nearest the load,
(d) the position of the current maximum nearest the load.
(e) Verify quantities in parts (a)–(d) using CD Module 2.4. Include a printout of the screen display.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57e^{-j79.8^\circ}.$$

(b) From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.70)

$$\begin{aligned} d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} &= \frac{-79.8^\circ \times 8 \text{ cm}}{4\pi} \frac{\pi \text{ rad}}{180^\circ} + \frac{n \times 8 \text{ cm}}{2} \\ &= -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}. \end{aligned}$$

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),

$$d_{min} = d_{max} - \lambda/4 = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$

(e) The problem statement does not specify the frequency, so in Module 2.4 we need to select the combination of f and ε_r such that $\lambda = 5$ cm. With ε_r chosen as 1,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-2}} = 3.75 \text{ GHz}.$$

The generator parameters are irrelevant to the problem.

The results listed in the output screens are very close to those given in parts (a) through (d).

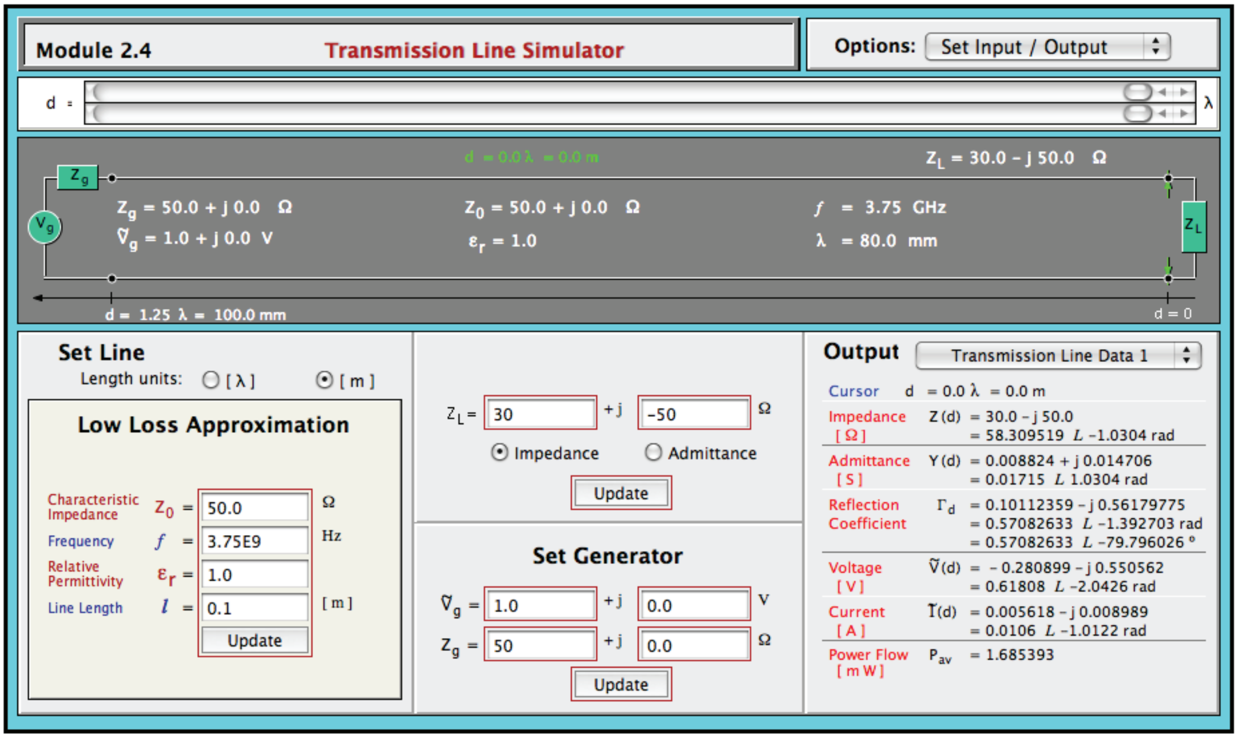


Figure P2.19(a)

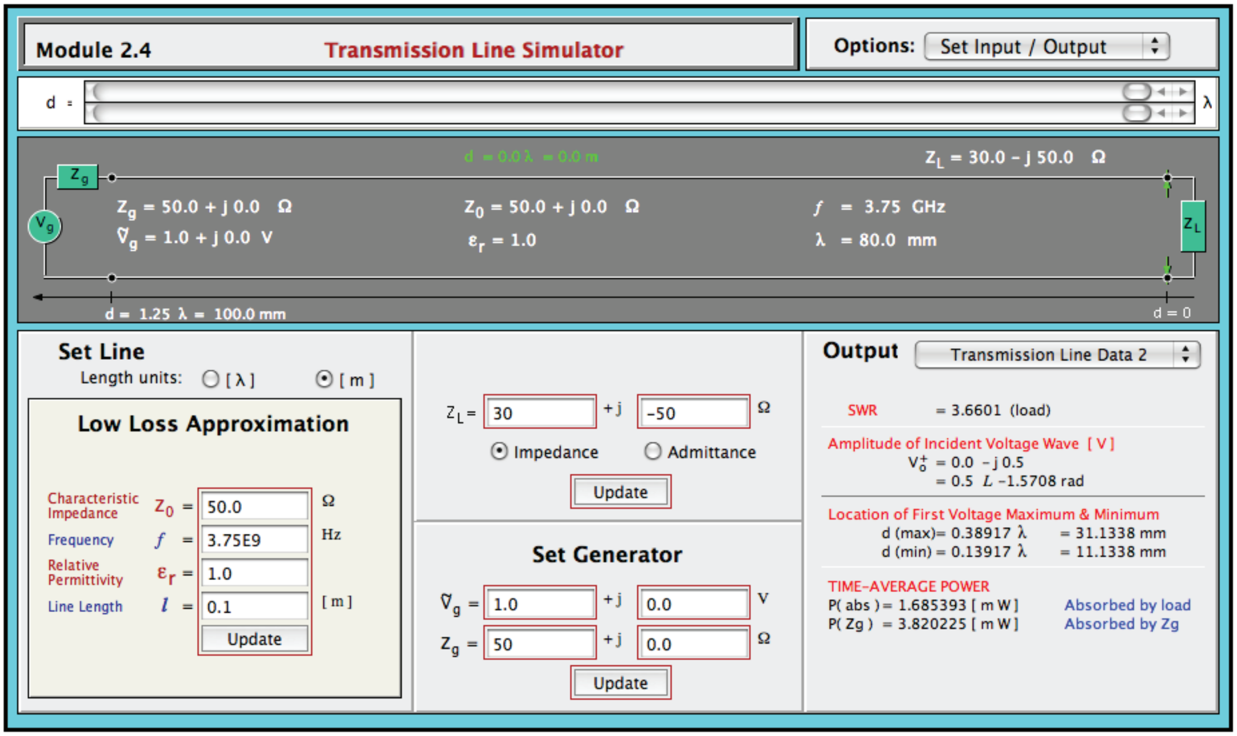


Figure P2.19(b)

Problem 2.21 On a 150-Ω lossless transmission line, the following observations were noted: distance of first voltage minimum from the load $= 3$ cm; distance of first voltage maximum from the load $= 9$ cm; $S = 3$. Find Z_L.

Solution: Distance between a minimum and an adjacent maximum $= \lambda/4$. Hence,

$$9 \text{ cm} - 3 \text{ cm} = 6 \text{ cm} = \lambda/4,$$

or $\lambda = 24$ cm. Accordingly, the first voltage minimum is at $d_{\text{min}} = 3 \text{ cm} = \frac{\lambda}{8}$. Application of Eq. (2.71) with $n = 0$ gives

$$\theta_r - 2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = -\pi,$$

which gives $\theta_r = -\pi/2$.

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5.$$

Hence, $\Gamma = 0.5e^{-j\pi/2} = -j0.5$.

Finally,

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 150 \left[\frac{1-j0.5}{1+j0.5}\right] = (90 - j120) \ \Omega.$$

Problem 2.24 A 50-Ω lossless line terminated in a purely resistive load has a voltage standing-wave ratio of 3. Find all possible values of Z_L.

Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

For a purely resistive load, $\theta_r = 0$ or π. For $\theta_r = 0$,

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 50 \left[\frac{1+0.5}{1-0.5}\right] = 150\ \Omega.$$

For $\theta_r = \pi,\ \Gamma = -0.5$ and

$$Z_L = 50 \left[\frac{1-0.5}{1+0.5}\right] = 15\ \Omega.$$