Problem 1.1 A 2-kHz sound wave traveling in the x-direction in air was observed to have a differential pressure $p(x,t) = 10$ N/m^2 at $x = 0$ and $t = 50$ μs. If the reference phase of $p(x,t)$ is 36°, find a complete expression for $p(x,t)$. The velocity of sound in air is 330 m/s.

Solution: The general form is given by Eq. (1.17),

$$p(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right),$$

where it is given that $\phi_0 = 36°$. From Eq. (1.26), $T = 1/f = 1/(2\times10^3) = 0.5$ ms. From Eq. (1.27),

$$\lambda = \frac{u_{\rm p}}{f} = \frac{330}{2\times10^3} = 0.165 \text{ m}.$$

Also, since

$$\begin{aligned} p(x=0,\ t=50\ \mu\text{s}) = 10\ (\text{N/m}^2) &= A\cos\left(\frac{2\pi\times50\times10^{-6}}{5\times10^{-4}} + 36°\frac{\pi\text{ rad}}{180°}\right) \\ &= A\cos(1.26\text{ rad}) = 0.31A, \end{aligned}$$

it follows that $A = 10/0.31 = 32.36$ N/m^2. So, with t in (s) and x in (m),

$$\begin{aligned} p(x,t) &= 32.36\cos\left(2\pi\times10^6\frac{t}{500} - 2\pi\times10^3\frac{x}{165} + 36°\right) \quad (\text{N/m}^2) \\ &= 32.36\cos(4\pi\times10^3 t - 12.12\pi x + 36°) \quad (\text{N/m}^2). \end{aligned}$$

Problem 1.4 A wave traveling along a string is given by

$$y(x,t) = 2\sin(4\pi t + 10\pi x) \quad \text{(cm)},$$

where x is the distance along the string in meters and y is the vertical displacement. Determine: (a) the direction of wave travel, (b) the reference phase ϕ_0, (c) the frequency, (d) the wavelength, and (e) the phase velocity.

Solution:

(a) We start by converting the given expression into a cosine function of the form given by (1.17):

$$y(x,t) = 2\cos\left(4\pi t + 10\pi x - \frac{\pi}{2}\right) \quad \text{(cm)}.$$

Since the coefficients of t and x both have the same sign, the wave is traveling in the negative x-direction.

(b) From the cosine expression, $\phi_0 = -\pi/2$.

(c) $\omega = 2\pi f = 4\pi$,

$$f = 4\pi/2\pi = 2 \text{ Hz}.$$

(d) $2\pi/\lambda = 10\pi$,

$$\lambda = 2\pi/10\pi = 0.2 \text{ m}.$$

(e) $u_p = f\lambda = 2 \times 0.2 = 0.4$ (m/s).

Problem 1.14 A certain electromagnetic wave traveling in seawater was observed to have an amplitude of 98.02 (V/m) at a depth of 10 m, and an amplitude of 81.87 (V/m) at a depth of 100 m. What is the attenuation constant of seawater?

Solution: The amplitude has the form $Ae^{\alpha z}$. At $z = 10$ m,

$$Ae^{-10\alpha} = 98.02$$

and at $z = 100$ m,

$$Ae^{-100\alpha} = 81.87$$

The ratio gives

$$\frac{e^{-10\alpha}}{e^{-100\alpha}} = \frac{98.02}{81.87} = 1.20$$

or

$$e^{-10\alpha} = 1.2e^{-100\alpha}.$$

Taking the natural log of both sides gives

$$\begin{aligned} \ln(e^{-10\alpha}) &= \ln(1.2e^{-100\alpha}), \\ -10\alpha &= \ln(1.2) - 100\alpha, \\ 90\alpha &= \ln(1.2) = 0.18. \end{aligned}$$

Hence,

$$\alpha = \frac{0.18}{90} = 2 \times 10^{-3} \quad \text{(Np/m)}.$$

Problem 1.16 Evaluate each of the following complex numbers and express the result in rectangular form:

(a) $z_1 = 8e^{j\pi/3}$

(b) $z_2 = \sqrt{3}\, e^{j3\pi/4}$

(c) $z_3 = 2e^{-j\pi/2}$

(d) $z_4 = j^3$

(e) $z_5 = j^{-4}$

(f) $z_6 = (1-j)^3$

(g) $z_7 = (1-j)^{1/2}$

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a) $z_1 = 8e^{j\pi/3} = 8(\cos\pi/3 + j\sin\pi/3) = 4.0 + j6.93.$

(b)

$$z_2 = \sqrt{3}e^{j3\pi/4} = \sqrt{3}\left[\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right] = -1.22 + j1.22 = 1.22(-1+j).$$

(c) $z_3 = 2e^{-j\pi/2} = 2[\cos(-\pi/2) + j\sin(-\pi/2)] = -j2.$

(d) $z_4 = j^3 = j \cdot j^2 = -j$, or

$$z_4 = j^3 = (e^{j\pi/2})^3 = e^{j3\pi/2} = \cos(3\pi/2) + j\sin(3\pi/2) = -j.$$

(e) $z_5 = j^{-4} = (e^{j\pi/2})^{-4} = e^{-j2\pi} = 1.$

(f)

$$\begin{aligned} z_6 = (1-j)^3 = (\sqrt{2}e^{-j\pi/4})^3 &= (\sqrt{2})^3 e^{-j3\pi/4} \\ &= (\sqrt{2})^3[\cos(3\pi/4) - j\sin(3\pi/4)] \\ &= -2 - j2 = -2(1+j). \end{aligned}$$

(g)

$$\begin{aligned} z_7 = (1-j)^{1/2} = (\sqrt{2}e^{-j\pi/4})^{1/2} = \pm 2^{1/4}e^{-j\pi/8} &= \pm 1.19(0.92 - j0.38) \\ &= \pm(1.10 - j0.45). \end{aligned}$$

Problem 1.19 If $z = -2 + j4$, determine the following quantities in polar form:

(a) $1/z$,

(b) z^3,

(c) $|z|^2$,

(d) $\mathfrak{Im}\{z\}$,

(e) $\mathfrak{Im}\{z^*\}$.

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a)

$$\frac{1}{z} = \frac{1}{-2+j4} = (-2+j4)^{-1} = (4.47\,e^{j116.6^\circ})^{-1} = (4.47)^{-1}e^{-j116.6^\circ} = 0.22\,e^{-j116.6^\circ}.$$

(b) $z^3 = (-2+j4)^3 = (4.47\,e^{j116.6^\circ})^3 = (4.47)^3 e^{j350.0^\circ} = 89.44\,e^{-j10^\circ}$.

(c) $|z|^2 = z \cdot z^* = (-2+j4)(-2-j4) = 4+16 = 20$.

(d) $\mathfrak{Im}\{z\} = \mathfrak{Im}\{-2+j4\} = 4$.

(e) $\mathfrak{Im}\{z^*\} = \mathfrak{Im}\{-2-j4\} = -4 = 4e^{j\pi}$.

Problem 1.20 Find complex numbers $t = z_1 + z_2$ and $s = z_1 - z_2$, both in polar form, for each of the following pairs:

(a) $z_1 = 2 + j3$ and $z_2 = 1 - j2$,

(b) $z_1 = 3$ and $z_2 = -j3$,

(c) $z_1 = 3\angle 30^\circ$ and $z_2 = 3\angle -30^\circ$,

(d) $z_1 = 3\angle 30^\circ$ and $z_2 = 3\angle -150^\circ$.

Solution:

(a)

$$t = z_1 + z_2 = (2 + j3) + (1 - j2) = 3 + j1,$$
$$s = z_1 - z_2 = (2 + j3) - (1 - j2) = 1 + j5 = 5.1\, e^{j78.7^\circ}.$$

(b)

$$t = z_1 + z_2 = 3 - j3 = 4.24\, e^{-j45^\circ},$$
$$s = z_1 - z_2 = 3 + j3 = 4.24\, e^{j45^\circ}.$$

(c)

$$\begin{aligned} t = z_1 + z_2 &= 3\angle 30^\circ + 3\angle -30^\circ \\ &= 3e^{j30^\circ} + 3e^{-j30^\circ} = (2.6 + j1.5) + (2.6 - j1.5) = 5.2, \\ s = z_1 - z_2 &= 3e^{j30^\circ} - 3e^{-j30^\circ} = (2.6 + j1.5) - (2.6 - j1.5) = j3 = 3e^{j90^\circ}. \end{aligned}$$

(d)

$$t = z_1 + z_2 = 3\angle 30^\circ + 3\angle -150^\circ = (2.6 + j1.5) + (-2.6 - j1.5) = 0,$$
$$s = z_1 - z_2 = (2.6 + j1.5) - (-2.6 - j1.5) = 5.2 + j3 = 6e^{j30^\circ}.$$

Problem 1.23 If $z = 3 - j4$, find the value of e^z.

Solution:

$$e^z = e^{3-j4} = e^3 \cdot e^{-j4} = e^3(\cos 4 - j\sin 4),$$
$$e^3 = 20.09, \qquad \text{and} \qquad 4 \text{ rad} = \frac{4}{\pi} \times 180^\circ = 229.18^\circ.$$

Hence, $e^z = 20.08(\cos 229.18^\circ - j\sin 229.18^\circ) = -13.13 + j15.20$.

Problem 1.26 Find the phasors of the following time functions:

(a) $\upsilon(t) = 9\cos(\omega t - \pi/3)$ (V)

(b) $\upsilon(t) = 12\sin(\omega t + \pi/4)$ (V)

(c) $i(x,t) = 5e^{-3x}\sin(\omega t + \pi/6)$ (A)

(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A)

(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A)

Solution:

(a) $\widetilde{V} = 9e^{-j\pi/3}$ V.

(b) $v(t) = 12\sin(\omega t + \pi/4) = 12\cos(\pi/2 - (\omega t + \pi/4)) = 12\cos(\omega t - \pi/4)$ V, $\widetilde{V} = 12e^{-j\pi/4}$ V.

(c)

$$
\begin{aligned}
i(t) &= 5e^{-3x}\sin(\omega t + \pi/6) \text{ A} = 5e^{-3x}\cos[\pi/2 - (\omega t + \pi/6)] \text{ A} \\
&= 5e^{-3x}\cos(\omega t - \pi/3) \text{ A}, \\
\widetilde{I} &= 5e^{-3x}e^{-j\pi/3} \text{ A}.
\end{aligned}
$$

(d)

$$
\begin{aligned}
i(t) &= -2\cos(\omega t + 3\pi/4), \\
\widetilde{I} &= -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}.
\end{aligned}
$$

(e)

$$
\begin{aligned}
i(t) &= 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6) \\
&= 4\cos[\pi/2 - (\omega t + \pi/3)] + 3\cos(\omega t - \pi/6) \\
&= 4\cos(-\omega t + \pi/6) + 3\cos(\omega t - \pi/6) \\
&= 4\cos(\omega t - \pi/6) + 3\cos(\omega t - \pi/6) = 7\cos(\omega t - \pi/6), \\
\widetilde{I} &= 7e^{-j\pi/6} \text{ A}.
\end{aligned}
$$

Problem 1.27 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -5e^{j\pi/3}$ (V)
(b) $\widetilde{V} = j6e^{-j\pi/4}$ (V)
(c) $\widetilde{I} = (6 + j8)$ (A)
(d) $\tilde{I} = -3 + j2$ (A)
(e) $\tilde{I} = j$ (A)
(f) $\tilde{I} = 2e^{j\pi/6}$ (A)

Solution:

(a)

$$\widetilde{V} = -5e^{j\pi/3} \text{ V} = 5e^{j(\pi/3-\pi)} \text{ V} = 5e^{-j2\pi/3} \text{ V},$$
$$v(t) = 5\cos(\omega t - 2\pi/3) \text{ V}.$$

(b)

$$\widetilde{V} = j6e^{-j\pi/4} \text{ V} = 6e^{j(-\pi/4+\pi/2)} \text{ V} = 6e^{j\pi/4} \text{ V},$$
$$v(t) = 6\cos(\omega t + \pi/4) \text{ V}.$$

(c)

$$\widetilde{I} = (6 + j8) \text{ A} = 10e^{j53.1^\circ} \text{ A},$$
$$i(t) = 10\cos(\omega t + 53.1^\circ) \text{ A}.$$

(d)

$$\widetilde{I} = -3 + j2 = 3.61\, e^{j146.31^\circ},$$
$$i(t) = \mathfrak{Re}\{3.61\, e^{j146.31^\circ} e^{j\omega t}\} = 3.61\cos(\omega t + 146.31^\circ) \text{ A}.$$

(e)

$$\widetilde{I} = j = e^{j\pi/2},$$
$$i(t) = \mathfrak{Re}\{e^{j\pi/2} e^{j\omega t}\} = \cos(\omega t + \pi/2) = -\sin\omega t \text{ A}.$$

(f)

$$\widetilde{I} = 2e^{j\pi/6},$$
$$i(t) = \mathfrak{Re}\{2e^{j\pi/6} e^{j\omega t}\} = 2\cos(\omega t + \pi/6) \text{ A}.$$

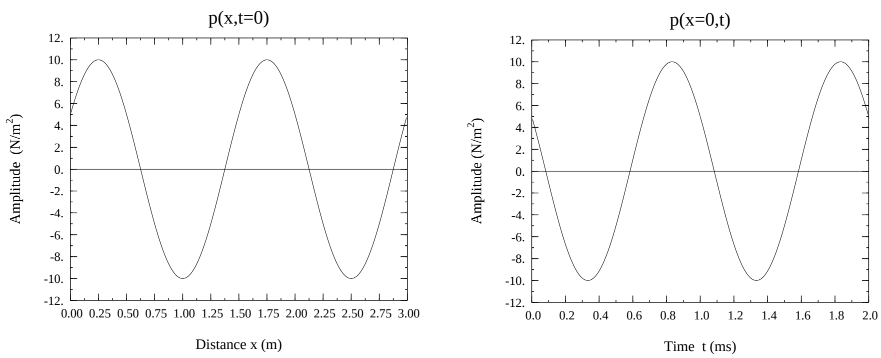
Problem 1.2 For the pressure wave described in Example 1-1, plot

(a) $p(x,t)$ versus x at $t = 0$,

(b) $p(x,t)$ versus t at $x = 0$.

Be sure to use appropriate scales for x and t so that each of your plots covers at least two cycles.

Solution: Refer to Fig. P1.2(a) and Fig. P1.2(b).



(a) (b)

Figure P1.2: (a) Pressure wave as a function of distance at $t = 0$ and (b) pressure wave as a function of time at $x = 0$.

Problem 1.3 A harmonic wave traveling along a string is generated by an oscillator that completes 180 vibrations per minute. If it is observed that a given crest, or maximum, travels 300 cm in 10 s, what is the wavelength?

Solution:

$$f = \frac{180}{60} = 3 \text{ Hz}.$$

$$u_{\text{p}} = \frac{300 \text{ cm}}{10 \text{ s}} = 0.3 \text{ m/s}.$$

$$\lambda = \frac{u_{\text{p}}}{f} = \frac{0.3}{3} = 0.1 \text{ m} = 10 \text{ cm}.$$

Problem 1.5 Two waves, $y_1(t)$ and $y_2(t)$, have identical amplitudes and oscillate at the same frequency, but $y_2(t)$ leads $y_1(t)$ by a phase angle of 60°. If

$$y_1(t) = 4\cos(2\pi \times 10^3 t),$$

write the expression appropriate for $y_2(t)$ and plot both functions over the time span from 0 to 2 ms.

Solution:

$$y_2(t) = 4\cos(2\pi \times 10^3 t + 60°).$$

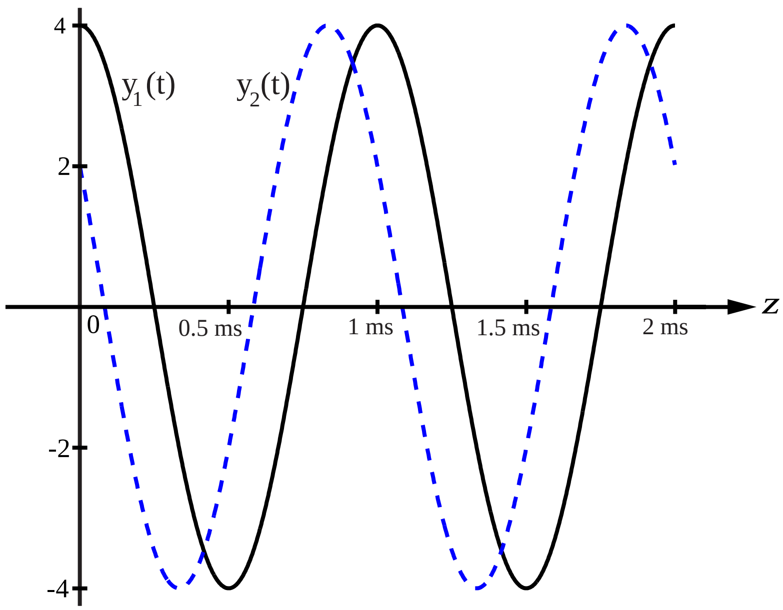


Figure P1.5: Plots of $y_1(t)$ and $y_2(t)$.

Problem 1.6 The height of an ocean wave is described by the function

$$y(x,t) = 1.5 \sin(0.5t - 0.6x) \quad \text{(m)}.$$

Determine the phase velocity and the wavelength, and then sketch $y(x,t)$ at $t = 2$ s over the range from $x = 0$ to $x = 2\lambda$.

Solution: The given wave may be rewritten as a cosine function:

$$y(x,t) = 1.5 \cos(0.5t - 0.6x - \pi/2).$$

By comparison of this wave with Eq. (1.32),

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0),$$

we deduce that

$$\omega = 2\pi f = 0.5 \text{ rad/s}, \qquad \beta = \frac{2\pi}{\lambda} = 0.6 \text{ rad/m},$$
$$u_{\text{p}} = \frac{\omega}{\beta} = \frac{0.5}{0.6} = 0.83 \text{ m/s}, \qquad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.6} = 10.47 \text{ m}.$$

At $t = 2$ s, $y(x,2) = 1.5 \sin(1 - 0.6x)$ (m), with the argument of the cosine function given in radians. Plot is shown in Fig. .

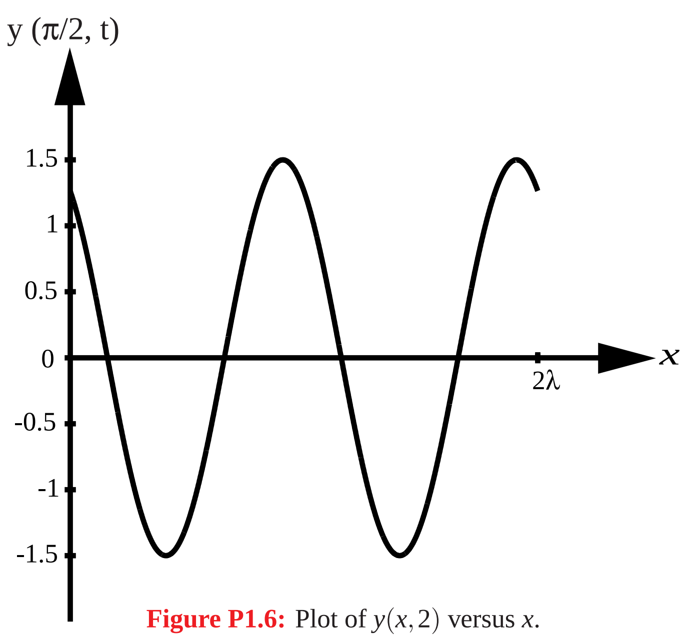


Figure P1.6: Plot of $y(x,2)$ versus x.