

ECE241
HW #7
SOLUTION

Problems from “Introduction to Electric Circuits”, Svoboda and Dorf, 9th ed. Pages 482-484.

- 1) P 10.2-2
- 2) P 10.2-4
- 3) P 10.3-2
- 4) P 10.3-5
- 5) P 10.3-7
- 6) P 10.3-10

SOLUTIONS:

P10.2-2 Given the sinusoids $v_1(t) = 8 \cos(100t - 54^\circ)$ V and $v_2(t) = 8 \cos(100t - 102^\circ)$ V, determine the time by which $v_2(t)$ is advanced or delayed with respect to $v_1(t)$.

Solution:

The period of both sinusoids is $T = \frac{2\pi}{100} = 62.8319$ ms

The difference in the phase angles is

$$\theta_2 - \theta_1 = -102^\circ - (-54^\circ) = -48^\circ$$

The delay time is $t_d = \frac{-48^\circ(62.8319)}{360^\circ} = -8.3776$ ms

(The minus sign indicates a delay.) The voltage $v_2(t)$ is delayed by 8.3776 ms with respect to $v_1(t)$.

P10.2-4 Express the voltage shown in Figure P10.2-4 in the general form

$$v(t) = A \cos(\omega t + \theta) \text{ V}$$

where $A \geq 0$ and $-180^\circ < \theta \leq 180^\circ$.

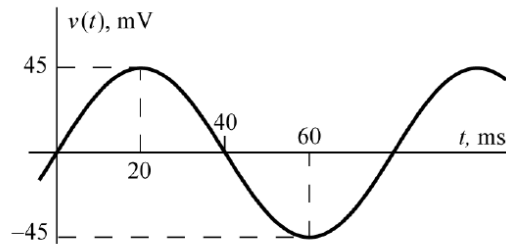


Figure P10.2-4

Solution: The amplitude is $A = 45$ mV and the period is given by $\frac{T}{2} = 60 - 20 = 40$ ms so the period is $T = 80$ ms. The frequency is given by $\omega = \frac{2\pi}{80 \times 10^{-3}} = 78.54$ rad/s. Noticing that $v(t)$ is 0 at time 0 and is increasing at time 0, we can write

$$v(t) = 45 \sin(78.54t) = 45 \cos(78.54t - 90^\circ) \text{ mV}$$

P10.3-2 Express the voltage

$$v(t) = 5\sqrt{2} \cos(8t) + 2 \sin(8t + 45^\circ) \text{ V}$$

In the general form

$$v(t) = A \cos(\omega t + \theta) \text{ V}$$

where $A \geq 0$ and $-180^\circ < \theta \leq 180^\circ$.

Solution:

$$\begin{aligned} v(t) &= 5\sqrt{2} \cos(8t) + 2 \sin(8t + 45^\circ) \\ &= 5\sqrt{2} \cos(8t) + 2 \cos(8t + 45^\circ - 90^\circ) = 5\sqrt{2} \cos(8t) + 2 \cos(8t - 45^\circ) \text{ V} \end{aligned}$$

Representing the sinusoids using phasors gives:

$$\begin{aligned} \mathbf{V}(\omega) &= 7.0711 + 10 \angle -45^\circ = 7.0711 + (7.0711 - j7.0711) \\ &= 14.1422 - j7.0711 = 15.811 \angle -26.6^\circ \text{ V} \end{aligned}$$

The corresponding sinusoid is:

$$v(t) = 15.811 \cos(8t - 26.6^\circ) \text{ V}$$

P 10.3-5 Determine the polar and rectangular form of the expression

$$\frac{(60 \angle 120^\circ)(-16 + j12 + 20 \angle 15^\circ)}{5 \angle -75^\circ}$$

Solution:

$$\begin{aligned} \frac{(60 \angle 120^\circ)(-16 + j12 + 20 \angle 15^\circ)}{5 \angle -75^\circ} &= \frac{(60 \angle 120^\circ)(-16 + j12 + 19.3185 + j5.1764)}{5 \angle -75^\circ} \\ &= \frac{(60 \angle 120^\circ)(3.3185 + j17.1764)}{5 \angle -75^\circ} \\ &= \frac{(60 \angle 120^\circ)(17.494 \angle 79.065^\circ)}{5 \angle -75^\circ} \\ &= \frac{1049.6 \angle -160.93^\circ}{5 \angle -75^\circ} = 139.95 \angle 109.07^\circ = 45.714 + j132.28 \end{aligned}$$

P10.3-7 The circuit shown in Figure 10.3-7 is at steady state. The inputs to this circuit are the current source current

$$i_1(t) = 0.12 \cos(100t + 45^\circ) \text{ A}$$

and the voltage source voltage

$$v_2(t) = 24 \cos(100t - 60^\circ) \text{ V}$$

Determine the current $i_2(t)$.

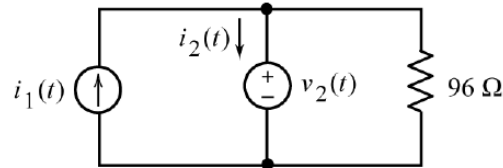


Figure P10.3-7

Solution: Using Ohm's and Kirchhoff's laws

$$\begin{aligned} i_2(t) &= i_1(t) - \frac{v_2(t)}{96} = 0.12 \cos(100t + 45^\circ) - \frac{24 \cos(100t - 60^\circ)}{96} \\ &= 0.12 \cos(100t + 45^\circ) - 0.25 \cos(100t - 60^\circ) \end{aligned}$$

Using phasors

$$\begin{aligned} \mathbf{I}_2(\omega) &= 0.12 \angle 45^\circ - 0.25 \angle 60^\circ = (0.0849 + j0.0849) - (0.1250 - j0.2165) \\ &= -0.0401 + j0.3014 = 0.3040 \angle 97.6^\circ \text{ A} \end{aligned}$$

The corresponding sinusoid is

$$i_2(t) = 0.3040 \cos(100t + 97.6^\circ) \text{ A}$$

P 10.3-10 The circuit shown in Figure P 10.3-10 is at steady state. The voltages $v_s(t)$ and $v_2(t)$ are given by

$$v_s(t) = 7.68 \cos(2t + 47^\circ) \text{ V}$$

and

$$v_2(t) = 1.59 \cos(2t + 125^\circ) \text{ V}$$

Find the steady-state voltage $v_1(t)$.

Answer: $v_1(t) = 7.51 \cos(2t + 35^\circ) \text{ V}$

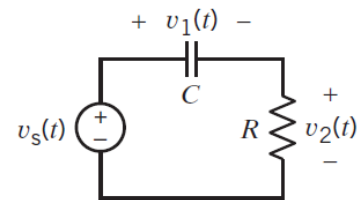


Figure P 10.3-10

Solution:

$$\begin{aligned} \mathbf{V}_1(\omega) &= \mathbf{V}_s(\omega) - \mathbf{V}_2(\omega) = 7.68 \angle 47^\circ - 1.59 \angle 125^\circ \\ &= (5.23 + j5.62) - (-0.91 + j1.30) \\ &= (5.23 + 0.91) + j(5.62 - 1.30) \\ &= 6.14 + j4.32 \\ &= 7.51 \angle 35^\circ \end{aligned}$$

$$v_1(t) = 7.51 \cos(2t + 35^\circ) \text{ V}$$