

ECE241
HW #6
SOLUTION

Problems from “Introduction to Electric Circuits”, Svoboda and Dorf, 9th ed. Pages 363-364.

- 1) P 8.3-1
- 2) P 8.3-2
- 3) P 8.3-10
- 4) P 8.3-11

SOLUTIONS:

P 8.3-1 The circuit shown in Figure P 8.3-1 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Determine $v(t)$ for $t > 0$.

Answer: $v(t) = 6 - 2e^{-1.33t}$ V for $t > 0$

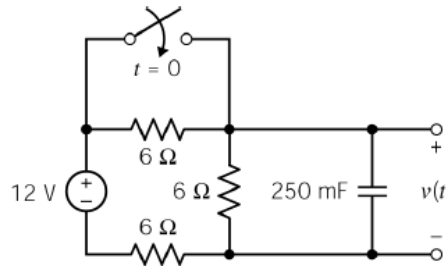
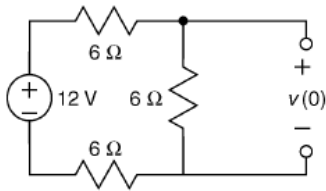


Figure P 8.3-1

Solution:

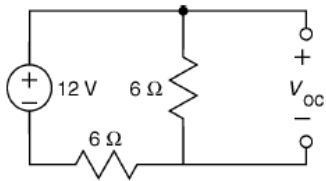


Here is the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit.

A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the initial capacitor voltage, $v(0)$.

By voltage division

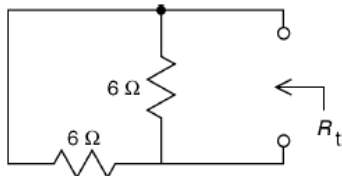
$$v(0) = \frac{6}{6+6+6}(12) = 4 \text{ V}$$



Next, consider the circuit after the switch closes. The closed switch is modeled as a short circuit.

We need to find the Thevenin equivalent of the part of the circuit connected to the capacitor. Here's the circuit used to calculate the open circuit voltage, V_{∞} .

$$V_{\infty} = \frac{6}{6+6}(12) = 6 \text{ V}$$



Here is the circuit that is used to determine R_t . A short circuit has replaced the closed switch. Independent sources are set to zero when calculating R_t , so the voltage source has been replaced by a short circuit.

$$R_t = \frac{(6)(6)}{6+6} = 3 \text{ } \Omega$$

Then $\tau = R_t C = 3(0.25) = 0.75 \text{ s}$

Finally, $v(t) = V_{\infty} + (v(0) - V_{\infty})e^{-t/\tau} = 6 - 2e^{-1.33t}$ V for $t > 0$

P 8.3-2 The circuit shown in Figure P 8.3-2 is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the current in the inductor, $i(t)$. Determine $i(t)$ for $t > 0$.

Answer: $i(t) = 1 + e^{-0.5t}$ A for $t > 0$

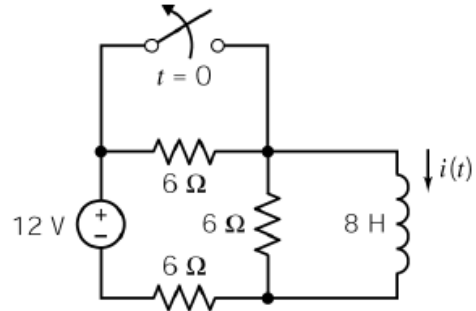
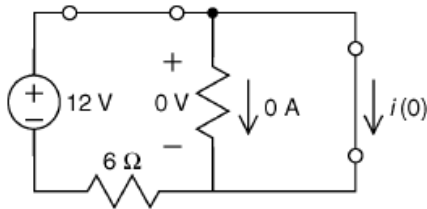


Figure P 8.3-2

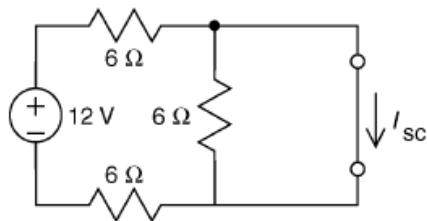
Solution:



Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit.

An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the initial inductor current, $i(0)$.

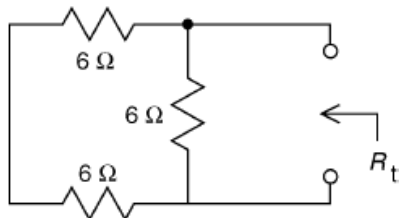
$$i(0) = \frac{12}{6} = 2 \text{ A}$$



Next, consider the circuit after the switch opens. The open switch is modeled as an open circuit.

We need to find the Norton equivalent of the part of the circuit connected to the inductor. Here's the circuit used to calculate the short circuit current, I_{sc} .

$$I_{sc} = \frac{12}{6+6} = 1 \text{ A}$$



Here is the circuit that is used to determine R_t . An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t , so the voltage source has been replaced by a short circuit.

$$R_t = 6 \parallel (6+6) = \frac{(6+6)(6)}{(6+6)+6} = 4 \text{ } \Omega$$

Then
$$\tau = \frac{L}{R_t} = \frac{8}{4} = 2 \text{ s}$$

Finally,
$$i(t) = I_{sc} + (i(0) - I_{sc}) e^{-t/\tau} = 1 + e^{-0.5t} \text{ A for } t > 0$$

P 8.3-10 A security alarm for an office building door is modeled by the circuit of Figure P 8.3-10. The switch represents the door interlock, and v is the alarm indicator voltage. Find $v(t)$ for $t > 0$ for the circuit of Figure P 8.3-10. The switch has been closed for a long time at $t = 0^-$.

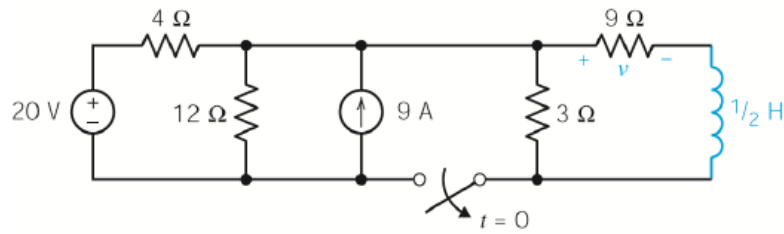
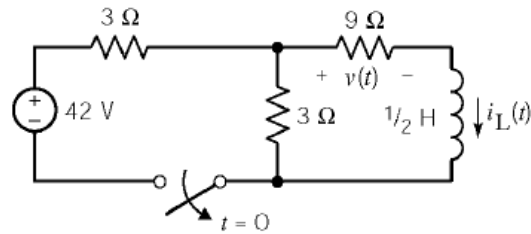
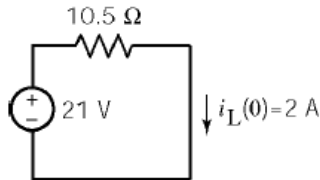


Figure P 8.3-10

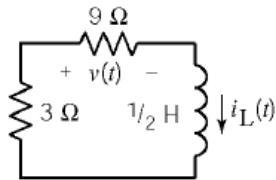
Solution: First, use source transformations to obtain the equivalent circuit



for $t < 0$:



for $t > 0$:



So $i_L(0) = 2 \text{ A}$, $I_{sc} = 0$, $R_t = 3 + 9 = 12 \text{ } \Omega$, $\tau = \frac{L}{R_t} = \frac{\frac{1}{2}}{12} = \frac{1}{24} \text{ s}$

and $i_L(t) = 2e^{-24t} \quad t > 0$

Finally $v(t) = 9 i_L(t) = 18 e^{-24t} \quad t > 0$

P 8.3-11 The voltage $v(t)$ in the circuit shown in Figure P 8.3-11 is given by

$$v(t) = 8 + 4e^{-2t} \text{ V for } t > 0$$

Determine the values of R_1 , R_2 , and C .

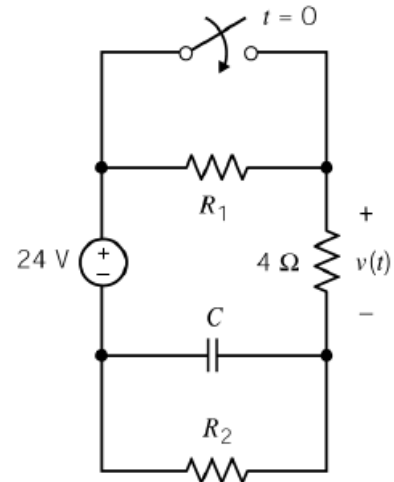


Figure P 8.3-11

Solution: As $t \rightarrow \infty$ the circuit reaches steady state and the capacitor acts like an open circuit. Also, from the given equation, $v(t) \rightarrow 8 \text{ V}$, as labeled on the drawing to the right, then

$$8 = \frac{4}{R_2 + 4} 24 \Rightarrow R_2 = 8 \Omega$$

After $t = 0$

$$v_C(t) = 24 - v(t) = 16 - 4e^{-2t}$$

Immediately after $t = 0$

$$v_C(0+) = 16 - 4 = 12 \text{ V}$$

The capacitor voltage cannot change instantaneously so

$$v(0-) = 12 \text{ V}$$

The circuit is at steady state just before the switch closes so the capacitor acts like an open circuit. Then

$$12 = \frac{8}{R_1 + 4 + 8} 24 \Rightarrow R_1 = 4 \Omega$$

After $t = 0$ the Thevenin resistance seen by the capacitor is

$$R_t = 8 \parallel 4 = \frac{8}{3} \Omega$$

so
$$2 = \frac{1}{\frac{8}{3}C} \Rightarrow C = \frac{3}{16} \text{ F}$$

