ECE241

HW #5

SOLUTION

Problems from "Introduction to Electric Circuits", Svoboda and Dorf, 9th ed. Pages 305-319.

1) P 7.2-7
2) P 7.4-5
3) P 7.5-19
4) P 7.7-5
5) P 7.8-6
6) P 7.8-7

SOLUTIONS:

P 7.2-7 The voltage across a 40-μF capacitor is 25 V at $t_0 = 0$. If the current through the capacitor as a function of time is given by $i(t) = 6e^{-6t}$ mA for $t < 0$, find $v(t)$ for $t > 0$.
Answer: $v(t) = 50 - 25e^{-6t}$ V

Solution:

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(\tau)\,d\tau = 25 + 2.5\times10^4\int_0^t \left(6\times10^{-3}\right)e^{-6\tau}\,d\tau$$

$$= 25 + 150\int_0^t e^{-6\tau}\,d\tau$$

$$= 25 + 150\left[-\frac{1}{6}e^{-6\tau}\right]_0^t = \underline{50 - 25e^{-6t}\ \text{V}}$$

P7.4-5 Determine the value of the capacitance C in the circuit shown in Figure P 7.4-5, given that $C_{eq} = 8$ F.

Answer: $C = 20$ F

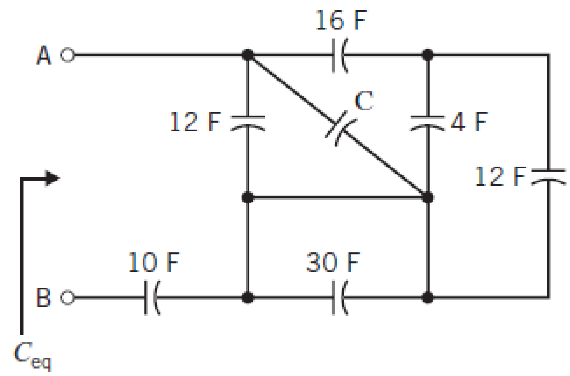
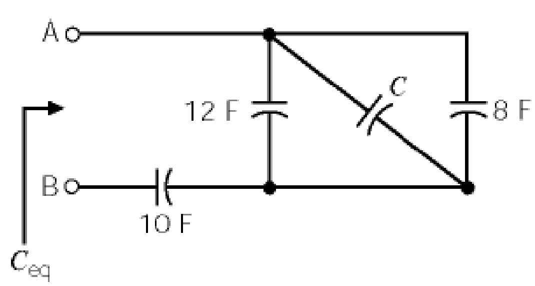


Figure P 7.4-5

Solution: The 16 F capacitor is in series with a parallel combination of 4 F and 12 F capacitors. The capacitance of the equivalent capacitor is

$$\frac{16(4+12)}{16+(4+12)} = 8 \text{ F}$$

The 30 F capacitor is in parallel with a short circuit, which is equivalent to a short circuit. After making these simplifications, we have



Then

$$8 = C_{eq} = \frac{10(12+C+8)}{10+(12+C+8)} \quad \Rightarrow \quad C = 20 \text{ F}$$

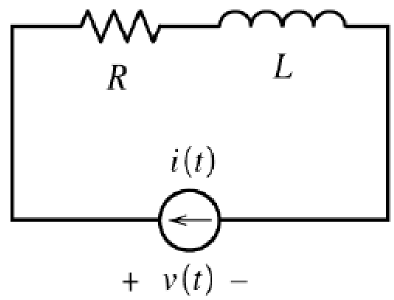


Figure P7.5-19

P7.5-19 . The input to the circuit shown in Figure P7.5-19 is the current

$$i(t) = 5 + 2e^{-7t} \text{ A} \quad \text{for } t > 0$$

The output is the voltage: $v(t) = 75 - 82e^{-7t}$ V for $t > 0$

Determine the values of the resistance and inductance.

Solution:

$$v(t) = 75 - 82e^{-7t} = R\left(5 + 2e^{-7t}\right) + L\frac{d}{dt}\left(5 + 2e^{-7t}\right)$$
$$= R\left(5 + 2e^{-7t}\right) + L\left((-7)2e^{-7t}\right) = 5R + (2R - 14L)e^{-7t}$$

Equating coefficients gives $75 = 5R \quad \Rightarrow \quad R = 15\ \Omega$ and

and $-82 = 2R - 14L = 30 - 14L \quad \Rightarrow \quad L = \dfrac{82 + 30}{14} = 8 \text{ H}$

P 7.7-5 Determine the value of the inductance L in the circuit shown in Figure P 7.7-5, given that L_{eq} = 18 H.
Answer: L = 20 H

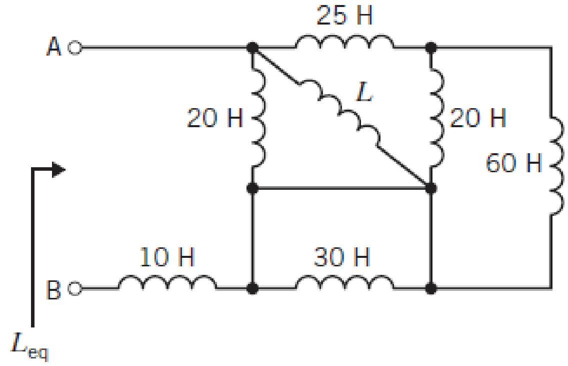
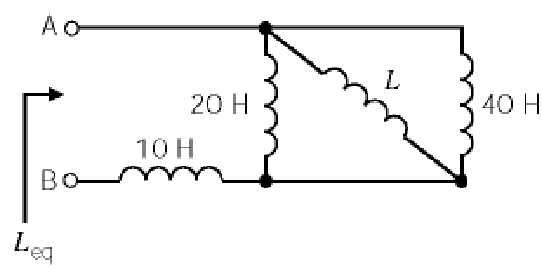


Figure P 7.7-5

Solution: The 25 H inductor is in series with a parallel combination of 20 H and 60 H inductors. The inductance of the equivalent inductor is

$$25+\frac{60\times 20}{60+20}=40 \text{ H}$$

The 30 H inductor is in parallel with a short circuit, which is equivalent to a short circuit. After making these simplifications, we have



Then

$$18=L_{eq}=10+\frac{1}{\frac{1}{20}+\frac{1}{L}+\frac{1}{40}} \quad \Rightarrow \quad \frac{1}{20}+\frac{1}{L}+\frac{1}{40}=\frac{1}{8} \quad \Rightarrow \quad L=20 \text{ H}$$

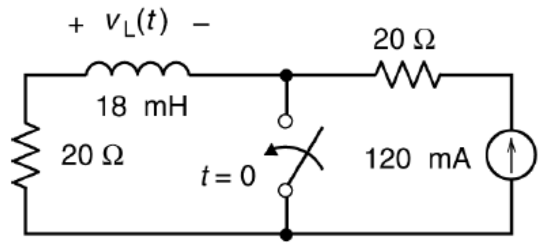
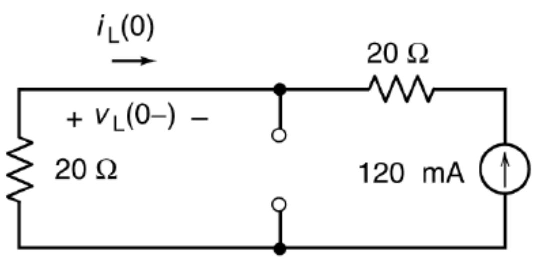


Figure P7.8-6

P7.8-6. The switch in the circuit shown in Figure P7.8-6 has been open for a long time before it closes at time $t = 0$. Determine the values of $v_L(0\text{-})$, the voltage across the inductor immediately before the switch closes and $v_L(0+)$, the voltage across the inductor immediately after the switch closes.

Solution:
The circuit is at steady state immediately before the switch closes. We have

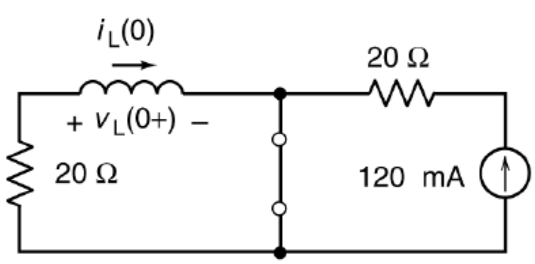


The inductor acts like a short circuit so $v_L(0-) = 0$.

The inductor current is the negative of the current source current:

$$i_L(0) = -120 \text{ mA}$$

The inductor current does not change instantaneously so $i_L(0+) = i_L(0-) \triangleq i_L(0)$. Immediately after the switch closes we have:



Applying KVL to the left mesh gives:

$$v_L(0+) + 20 i_L(0) = 0$$

$$v_L(0+) + 20(-0.12) = 0$$

$$v_L(0+) = 2.4$$

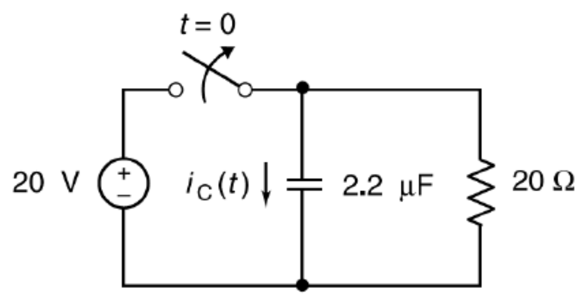
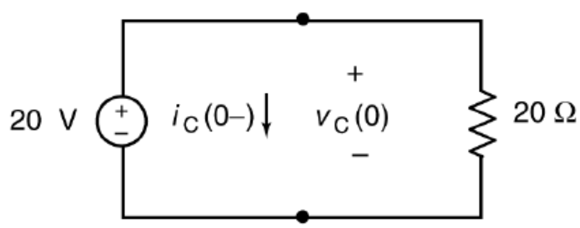


Figure P7.8-7

P7.8-7. The switch in the circuit shown in Figure P7.8-7 has been closed for a long time before it opens at time t = 0. Determine the values of i_C(0-), the current in the capacitor immediately before the switch opens and i_C(0+), the current in the capacitor immediately after the switch opens.

Solution:

The circuit is at steady state immediately before the switch opens. We have

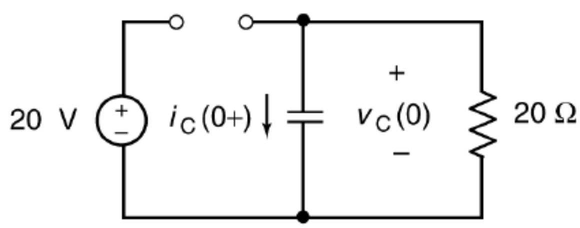


The capacitor acts like an open circuit so $i_C(0-)=0$.

The capacitor voltage is equal to the voltage source voltage:

$$v_C(0)=20\text{ V}$$

The capacitor does not change instantaneously so $v_C(0+)=v_C(0-)\triangleq v_C(0)$. Immediately after the switch opens we have:



Applying KCL at the top node of the capacitor, we see that:

$$i_C(0+)+\frac{v_C(0)}{20}=0$$

$$i_C(0+)=-\frac{v_C(0)}{20}=-1\text{ A}$$