ECE241
HW #3
SOLUTION

Problems from "Introduction to Electric Circuits", Svoboda and Dorf, 9th ed. Pages 155-164.

1) P 4.3-9
2) P 4.4-5
3) P 4.6-4
4) P 4.7-6
5) P 4.8-2

SOLUTION:

P 4.3-9 Determine the values of the node voltages of the circuit shown in Figure P 4.3-9.

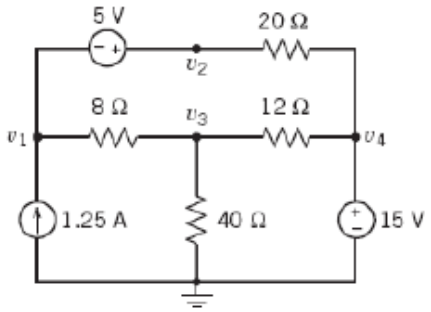


Figure P 4.3-9

Solution:
Express the voltage source voltages as functions of the node voltages to get

$$v_2 - v_1 = 5 \text{ and } v_4 = 15$$

Apply KCL to the supernode corresponding to the 5 V source to get

$$1.25 = \frac{v_1 - v_3}{8} + \frac{v_2 - 15}{20} = 0 \quad \Rightarrow \quad 80 = 5v_1 + 2v_2 - 5v_3$$

Apply KCL at node 3 to get

$$\frac{v_1 - v_3}{8} = \frac{v_3}{40} + \frac{v_3 - 15}{12} \quad \Rightarrow \quad -15v_1 + 28v_3 = 150$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} -1 & 1 & 0 \\ 5 & 2 & -5 \\ -15 & 0 & 28 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 80 \\ 150 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 22.4 \\ 27.4 \\ 17.4 \end{bmatrix}$$

So the node voltages are:

$$v_1 = 22.4 \text{ V}, v_2 = 27.4 \text{ V}, v_3 = 17.4 \text{ V}, \text{ and } v_4 = 15$$

P 4.4-5 Determine the value of the current i_x in the circuit of Figure P 4.4-5.

Answer: $i_x = 2.4$ A

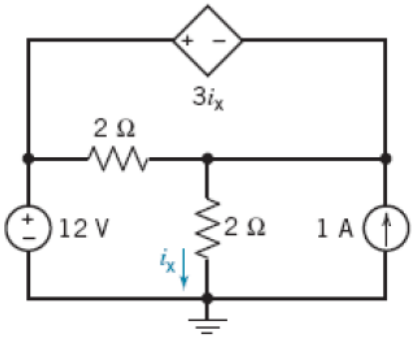
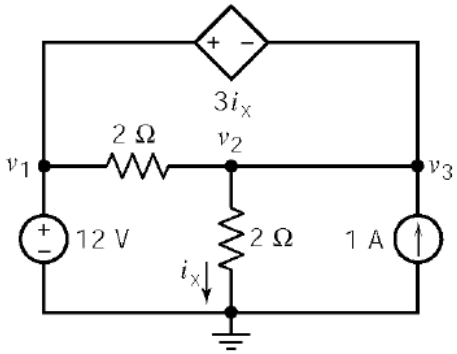


Figure P 4.4-5

Solution:



First, express the controlling current of the CCVS in terms of the node voltages: $i_x = \dfrac{v_2}{2}$

Next, express the controlled voltage in terms of the node voltages:

$$12 - v_2 = 3i_x = 3\frac{v_2}{2} \quad \Rightarrow v_2 = \frac{24}{5} \text{ V}$$

so $i_x = 12/5$ A $= 2.4$ A.

P 4.6-4 Find v_c for the circuit shown in Figure P 4.6-4.

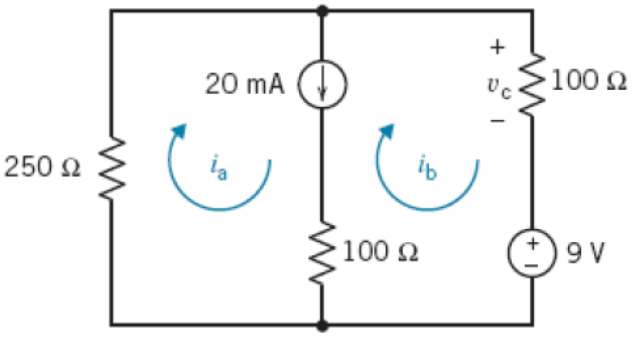


Figure P 4.6-4

Solution:

Express the current source current in terms of the mesh currents:

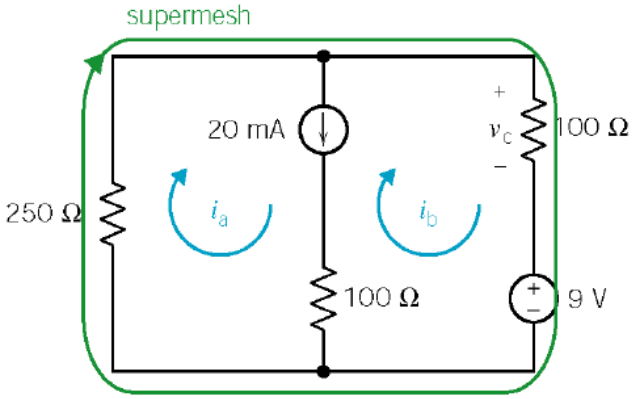
$$i_b = i_a - 0.02$$

Apply KVL to the supermesh:

$$250\, i_a + 100\,(i_a - 0.02) + 9 = 0$$

$$\therefore i_a = -.02 \text{ A} = -20 \text{ mA}$$

$$v_c = 100(i_a - 0.02) = \underline{-4 \text{ V}}$$



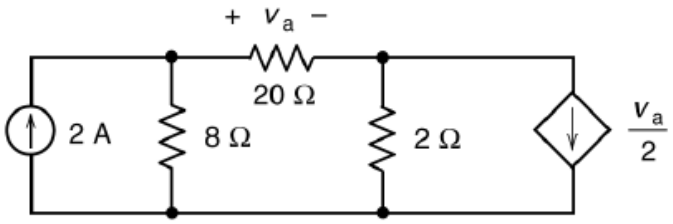
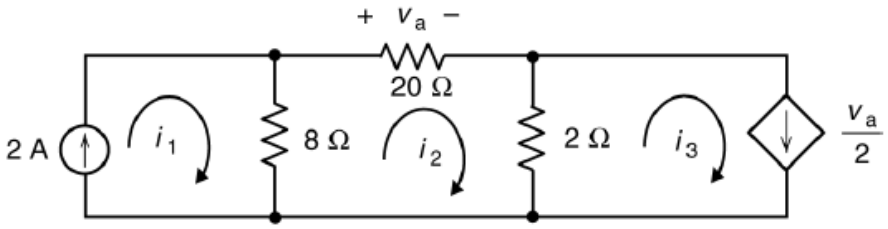


Figure P4.7-6

P4.7-6 Determine the value of the power supplied by the VCCS in the circuit shown Figure P4.7-6.

Solution: First, label the mesh currents.

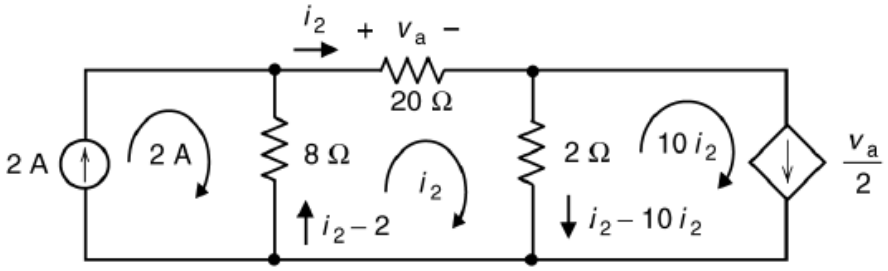


Next, express the controlling voltage of the VCCS in terms of the mesh currents:

$$v_a = 20 i_2$$

Notice that $i_1 = 2 \text{ A}$ and $i_3 = \frac{v_a}{2} = 10 i_2$

Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the middle mesh: $20 i_2 + 2\left(i_2 - 10 i_2\right) + 8\left(i_2 - 2\right) = 0 \;\Rightarrow\; i_2 = 1.6 \text{ A}$

Consequently $v_a = 20 i_2 = 20(1.6) = 32 \text{ V}$ and $i_3 = \frac{v_a}{2} = \frac{32}{2} = 16 \text{ A}$

The VCCS supplies $\frac{v_a}{2}\left[2\left(i_3 - i_2\right)\right] = \frac{32}{2}(2)(16 - 1.6) = 460.8 \text{ W}$

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = ai_s + bv_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

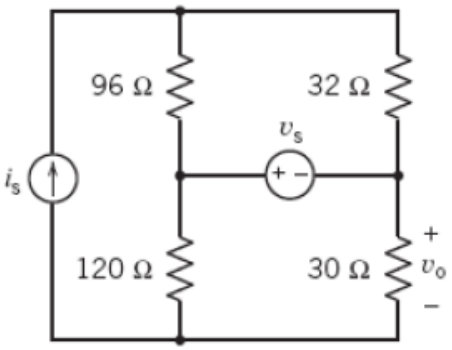
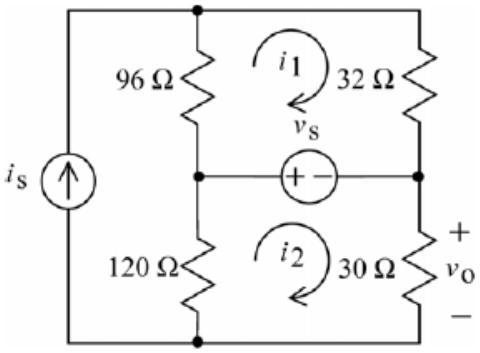


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32i_1 - v_s + 96\left(i_1 - i_s\right) = 0$$

$$v_s + 30i_2 + 120\left(i_2 - i_s\right) = 0$$

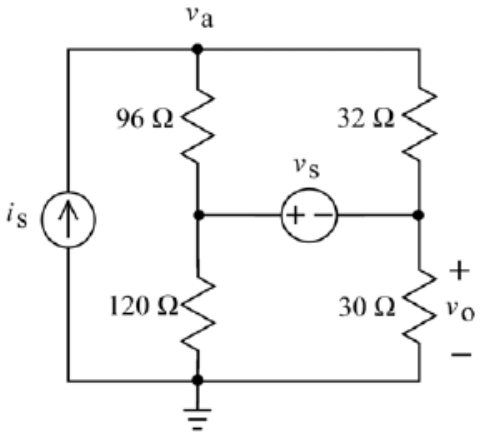
$$150i_2 = +120i_s - v_s$$

$$i_2 = \frac{4}{5}i_s - \frac{v_s}{150}$$

$$v_o = 30i_2 = 24i_s - \frac{1}{5}v_s$$

So $a = 24$ and $b = -.02$.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - \left(v_s + v_o\right)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24i_s - \frac{1}{5}v_s$$

So a = 24 and b = -0.2.