AC Steady State Problems and Solutions

Section 10.2 Sinusoidal Sources

P10.2-1 Given the sinusoids $v_1(t)=8\cos(250t+15°)$ V and $v_2(t)=6\cos(250t-45°)$ V, determine the time by which $v_2(t)$ is advanced or delayed with respect to $v_1(t)$.

Solution:

The period of both sinusoids is $T=\dfrac{2\pi}{250}=25.1327\ \text{ms}$

The difference in the phase angles is

$$\theta_2-\theta_1=-45°-15°=-60°$$

The delay time is $t_d=\dfrac{-60(25.1327)}{2\pi}=-4.188\ \text{ms}$

(The minus sign indicates a delay.) The voltage $v_2(t)$ is delayed by 4.188 ms with respect to $v_1(t)$.

P10.2-2 Given the sinusoids $v_1(t)=8\cos(100t-54°)$ V and $v_2(t)=8\cos(100t-102°)$ V, determine the time by which $v_2(t)$ is advanced or delayed with respect to $v_1(t)$.

Solution:

The period of both sinusoids is $T=\dfrac{2\pi}{100}=62.8319\ \text{ms}$

The difference in the phase angles is

$$\theta_2-\theta_1=-102°-(-54°)=-48°$$

The delay time is $t_d=\dfrac{-48°(62.8319)}{360°}=-8.3776\ \text{ms}$

(The minus sign indicates a delay.) The voltage $v_2(t)$ is delayed by 48.3776 ms with respect to $v_1(t)$.

P10.2-3 A sinusoidal current is given as

$$i(t) = 125\cos(5000\pi t - 135°) \text{ mA}$$

Determine the period, *T*, and the time, t_1, at which the first positive peak occurs.

Answer: *T* = 0.4 ms and t_1 = 0.15 ms.

Solution: The frequency is 5000 π rad/s or 2500 Hertz so the period is

$$T = \frac{1}{2500} = 0.0004 = 0.4 \text{ ms}$$

Converting the angle from degrees to radians, we get $-135°\left(\frac{\pi}{180°}\right) = -0.75\pi \text{ radians}$. The current sinusoid is shifted from $125\cos(5000\pi t) \text{ mA}$ by $\frac{-0.75\pi}{5000\pi} = -15 \text{ ms}$. The minus sign indicates a delay. A positive peak occurs at $t_1 = 0.15 \text{ ms}$. Since 15 ms is less than the period of $i(t_1)$, the positive peak at $t_1 = 0.15 \text{ ms}$ is the first positive peak.

P10.2-4 Express the voltage shown in Figure P10.2-7 in the general form

$$v(t) = A\cos(\omega t + \theta) \text{ V}$$

where $A \geq 0$ and $-180° < \theta \leq 180°$.

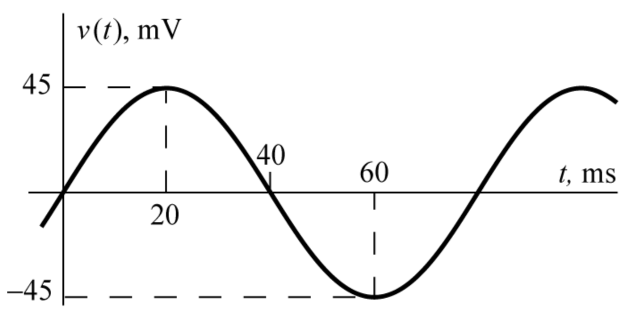


Figure P10.2-4

Solution: The amplitude is *A* = 45 mv and the period is given by $\frac{T}{2} = 60 - 20 = 40 \text{ ms}$ so the period is *T* = 80 ms. The frequency is given by $\omega = \frac{2\pi}{80 \times 10^{-3}} = 78.54 \text{ rad/s}$. Noticing that *v*(*t*) is 0 at time 0 and is increasing at time 0, we can write

$$v(t) = 45\sin(78.54t) = 45\cos(78.54t - 90°) \text{ mV}$$

P 10.2-5 Figure P 10.2-5 shows a sinusoidal voltage, $v(t)$, plotted as a function of time, t. Represent $v(t)$ by a function of the form $A\cos(\omega t+\theta)$.

Answer: $v(t) = 18\cos(393t - 27°)$

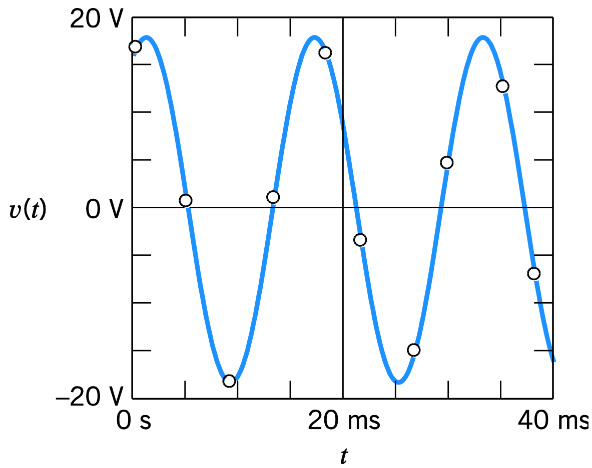
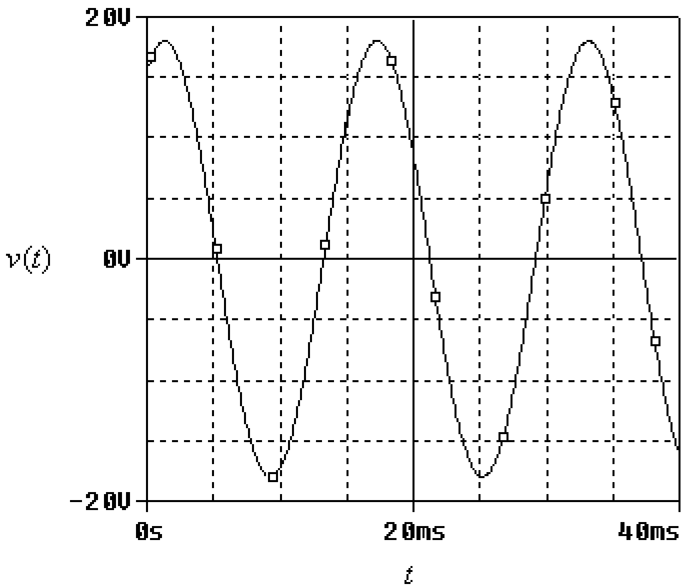


Figure P 10.2-5

Solution:



$$A = 18\text{ V}$$

$$T = 18 - 2 = 16\text{ ms}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.016} = 393\ \text{ rad/s}$$

$$\theta = -\cos^{-1}\left(\frac{16}{18}\right) = -27°$$

$$v(t) = 18\cos\left(393\,t - 27°\right)\ \text{ V}$$

P 10.2-6 Figure P 10.2-6 shows a sinusoidal voltage, $v(t)$, plotted as a function of time, t. Represent $v(t)$ by a function of the form
$A \cos(\omega t+\theta)$.

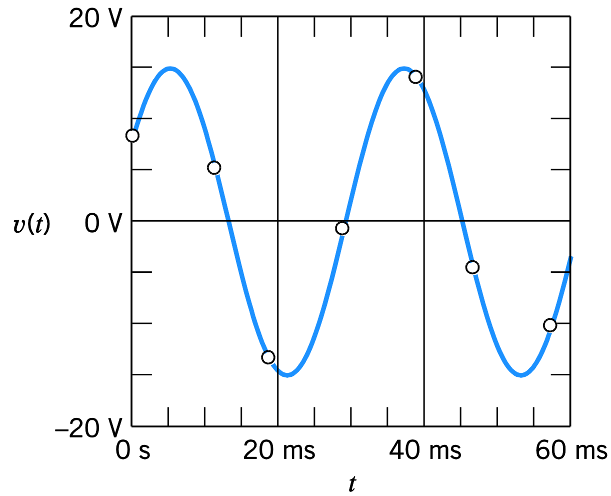
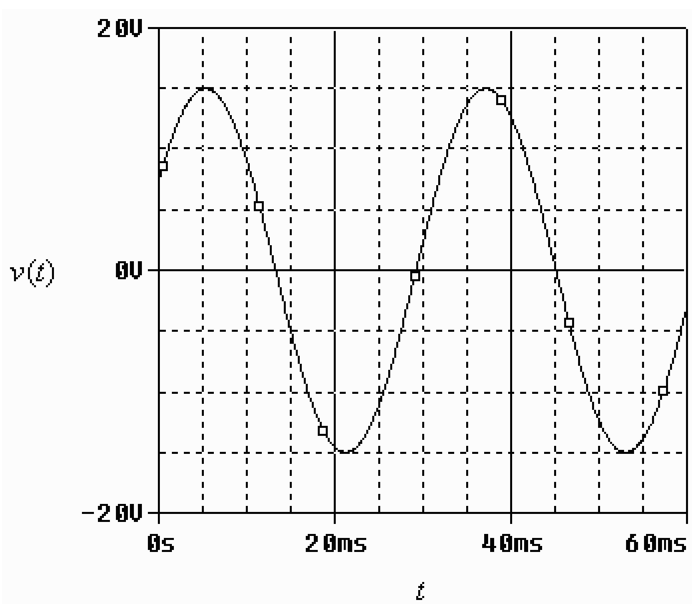


Figure P 10.2-6

Solution:



$$A = 15 \text{ V}$$

$$T = 43 - 11 = 32 \text{ ms}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.032} = 196 \text{ rad/s}$$

$$\theta = -\cos^{-1}\left(\frac{8}{15}\right) = -58°$$

$$v(t) = 15\cos(196\,t - 58°) \text{ V}$$

Section 10.3 Phasors and Sinusoids

P10.3-1 Express the current

$$i(t) = 2\cos(6t + 120°) + 2\sin(6t - 60°) \text{ mA}$$

In the general form

$$i(t) = A\cos(\omega t + \theta) \text{ mA}$$

where $A \geq 0$ and $-180° < \theta \leq 180°$.

Solution:

$$\begin{aligned} i(t) &= 2\cos(6t + 120°) + 2\sin(6t - 60°) \text{ mA} \\ &= 2\cos(6t + 120°) + 2\cos(6t - 60° - 90°) \text{ mA} \end{aligned}$$

Representing the sinusoids using phasors gives:

$$\begin{aligned} \mathbf{I}(\omega) &= 2\angle 120° + 4\angle -150° = (-1 + j1.732) + (-3.464 - j2) \\ &= -4.464 - j0.268 = 4.472\angle 183.4° = 4.472\angle -176.6° \text{ mA} \end{aligned}$$

The corresponding sinusoid is:

$$i(t) = 4.472\cos(6t - 176.6°) \text{ mA}$$

P10.3-2 Express the voltage

$$v(t) = 5\sqrt{2}\cos(8t) + 2\sin(8t + 45°) \text{ V}$$

In the general form

$$v(t) = A\cos(\omega t + \theta) \text{ V}$$

where $A \geq 0$ and $-180° < \theta \leq 180°$.

Solution:

$$\begin{aligned} v(t) &= 5\sqrt{2}\cos(8t) + 2\sin(8t + 45°) \\ &= 5\sqrt{2}\cos(8t) + 2\cos(8t + 45° - 90°) = 5\sqrt{2}\cos(8t) + 2\cos(8t - 45°) \text{ V} \end{aligned}$$

Representing the sinusoids using phasors gives:

$$\begin{aligned} \mathbf{V}(\omega) &= 7.0711 + 10\angle -45° = 7.0711 + (7.0711 - j7.0711) \\ &= 14.1422 - j7.0711 = 15.811\angle -26.6° \text{ V} \end{aligned}$$

The corresponding sinusoid is:

$$v(t) = 15.811\cos(8t - 26.6°) \text{ V}$$

P 10.3-3 Determine the polar form of the quantity

$$\frac{(25\angle 36.9°)(80\angle -53.1°)}{(4+j8)+(6-j8)}$$

Answer: $200\angle -16.2°$

Solution:

$$\frac{(25\angle 36.9°)(80\angle -53.1°)}{(4+j8)+(6-j8)} = \frac{25\cdot 80\angle(36.9°-53.1°)}{(4+6)+j(8-8)} = \frac{2000\angle -16.2°}{10} = 200\angle -16.2°$$

P 10.3-4 Determine the polar and rectangular form of the expression

$$5\angle +81.87°\left(4-j3+\frac{3\sqrt{2}\angle -45°}{7-j1}\right)$$

Answer: $88.162\angle 30.127° = 76.2520 + j44.2506$

Solution:

$$\begin{aligned}8\angle 42°\left(8-j3+\frac{40\angle -45°}{7-j12}\right) &= 8\angle 42°\left(8-j3+\frac{40\angle -45°}{13.892\angle -59.744°}\right)\\ &= 8\angle 42°(8-j3+2.8793\angle 14.744°)\\ &= 8\angle 42°(8-j3+2.7845+j0.7328)\\ &= 8\angle 42°(10.7845-j2.2672)\\ &= 8\angle 42°(11.02\angle -11.873°) = 88.162\angle 30.127° = 76.2520+j44.2506\end{aligned}$$

P 10.3-5 Determine the polar and rectangular form of the expression

$$\frac{(60\angle 120°)(-16+j12+20\angle 15°)}{5\angle -75°}$$

Solution:

$$\frac{(60\angle 120°)(-16+j12+20\angle 15°)}{5\angle -75°}=\frac{(60\angle 120°)(-16+j12+19.3185+j5.1764)}{5\angle -75°}$$

$$=\frac{(60\angle 120°)(3.3185+j17.1764)}{5\angle -75°}$$

$$=\frac{(60\angle 120°)(17.494\angle 79.065°)}{5\angle -75°}$$

$$=\frac{1049.6\angle -160.93°}{5\angle -75°}=139.95\angle 109.07°=45.714+j132.28$$

P10.3-6 The circuit shown in Figure 10.3-6 is at steady state. The input currents are

$$i_1(t)=10\cos(25t)\ \text{mA} \text{ and } i_3(t)=10\cos(25t+135°)\ \text{mA}$$

Determine the voltage $v_2(t)$.

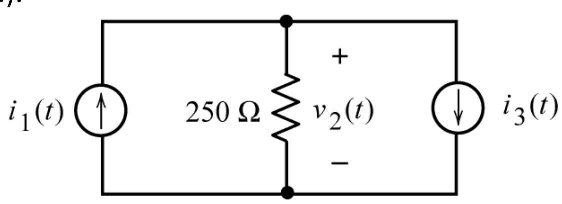
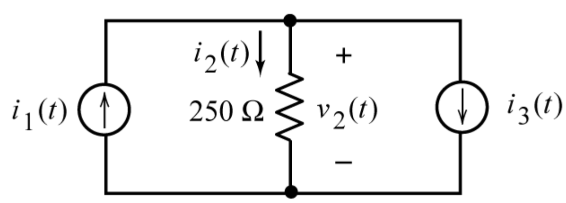


Figure 10.3-6

Solution:



Using first Ohm's law and then KCL

$$v_2(t)=250\,i_2(t)$$

and $$i_2(t)=i_1(t)-i_3(t)=10\cos(25t)-10\cos(25t+135°)\ \text{mA}$$

Using phasors

$$\mathbf{I}_2(\omega)=\mathbf{I}_1(\omega)-\mathbf{I}_3(\omega)=10-10\angle 135=10-(-7.071+j7.071)\ \text{mA}$$

$$=17.071-j7.071=18.478\angle -22.5°\ \text{mA}$$

The corresponding sinusoid is $i_2(t)=18.478\cos(25t-22.5°)\ \text{mA}$

Finally $$v_2(t)=250\,i_2(t)=4.6195\cos(25t-22.5°)\ \text{V}$$

P10.3-7 The circuit shown in Figure 10.3-7 is at steady state. The inputs to this circuit are the current source current

$$i_1(t) = 0.12\cos(100t + 45°) \text{ A}$$

and the voltage source voltage

$$v_2(t) = 24\cos(100t - 60°) \text{ V}$$

Determine the current $i_2(t)$.

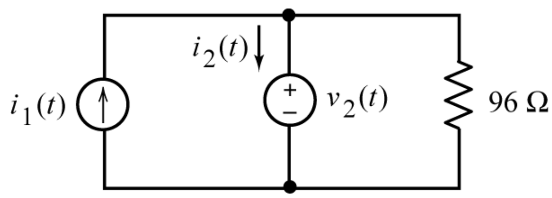


Figure P10.3-7

Solution: Using Ohm's and Kirchhoff's laws

$$i_2(t) = i_1(t) - \frac{v_2(t)}{96} = 0.12\cos(100t + 45°) - \frac{24\cos(100t - 60°)}{96}$$
$$= 0.12\cos(100t + 45°) - 0.25\cos(100t - 60°)$$

Using phasors

$$\mathbf{I}_2(\omega) = 0.12\angle 45° - 0.25\angle 60° = (0.0849 + j\,0.0849) - (0.1250 - j\,0.2165)$$
$$= -0.0401 + j\,0.3014 = 0.3040\angle 97.6° \text{ A}$$

The corresponding sinusoid is

$$i_2(t) = 0.3040\cos(100t + 97.6°) \text{ A}$$

Checked using LNAPAC 8/16/11

P 10.3-8 Given that

$i_1(t)$ = 30 cos (4t + 45°) mA

and $i_2(t)$ = – 40 cos (4t) mA

Determine $v(t)$ for the circuit shown in Figure P 10.3-8.

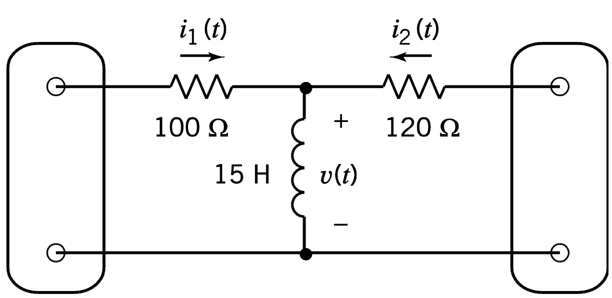


Figure P 10.3-8

Solution:

$$\mathbf{V} = j(4)(15)(\mathbf{I}_1 + \mathbf{I}_2) = j60(0.03\angle 45° - 0.04\angle 0°) = j60(0.0212 + j0.0212 - 0.04)$$
$$= -1.273 - j1.127$$
$$= 1.7\angle -138.5° \text{ V}$$

So

$$v(t) = 1.7\cos(4t - 138.5°) \text{ V}$$

(checked: LNAP 8/7/04)

P 10.3-9 For the circuit shown in Figure P 10.3-9, find (a) the impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in polar form, (b) the total combined impedance in polar form, and (c) the steady-state current $i(t)$.

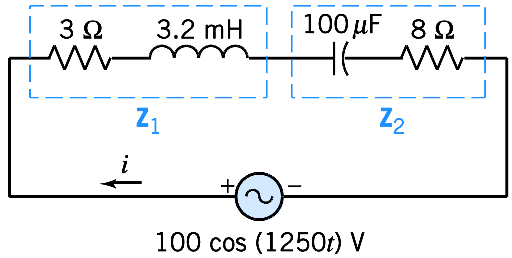


Figure P 10.3-9

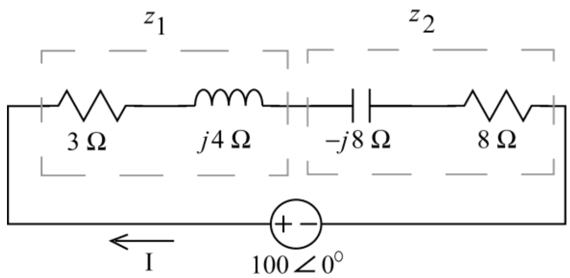
Answer:

(a) $\mathbf{Z}_1 = 5\angle 53.1°$; $\mathbf{Z}_2 = 8\sqrt{2}\angle -45°$

(b) $\mathbf{Z}_1 + \mathbf{Z}_2 = 11.7\angle -20°$

(c) $i(t) = (8.55)\cos(1250t + 20°)$ A

Solution:



(a) $\underline{\mathbf{Z}_1 = 3 + j4 = 5\angle 53.1°\ \Omega}$ and $\underline{\mathbf{Z}_2 = 8 - j8 = 8\sqrt{2}\angle -45°\ \Omega}$

(b) Total impedance $= \mathbf{Z}_1 + \mathbf{Z}_2 = 3 + j4 + 8 - j8 = 11 - j4 = \underline{11.7\angle -20.0°\ \Omega}$

(c) $$\mathbf{I} = \frac{100\angle 0°}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{100}{11.7\angle -20°} = \frac{100}{11.7}\angle 20.0° \Rightarrow \underline{i(t) = 8.55\cos(1250t + 20.0°)\ \text{A}}$$

P 10.3-10 The circuit shown in Figure P 10.3-10 is at steady state. The voltages $v_s(t)$ and $v_2(t)$ are given by

$$v_s(t) = 7.68 \cos(2t + 47°) \text{ V}$$

and

$$v_2(t) = 1.59 \cos(2t + 125°) \text{ V}$$

Find the steady-state voltage $v_1(t)$.

Answer: $v_1(t) = 7.51 \cos(2t + 35°)$ V

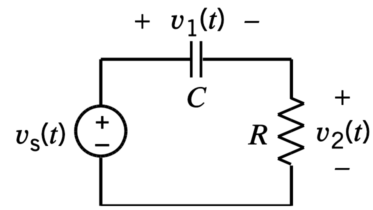


Figure P 10.3-10

Solution:

$$\begin{aligned}\mathbf{V}_1(\omega) = \mathbf{V}_s(\omega) - \mathbf{V}_2(\omega) &= 7.68\angle 47° - 1.59\angle 125° \\ &= (5.23 + j5.62) - (-0.91 + 1.30) \\ &= (5.23 + 0.91) + j(5.62 - 1.30) \\ &= 6.14 + j4.32 \\ &= 7.51\angle 35°\end{aligned}$$

$$v_1(t) = 7.51 \cos(2t + 35°) \text{ V}$$

P 10.3-11 The circuit shown in Figure P 10.3-11 is at steady state. The currents $i_1(t)$ and $i_2(t)$ are given by

$$i_1(t) = 744 \cos(2t - 118°) \text{ mA}$$

and

$$i_2(t) = 540.5 \cos(2t + 100°) \text{ mA}$$

Find the steady-state current $i(t)$.

Answer: $i(t) = 460 \cos(2t + 196°)$ mA

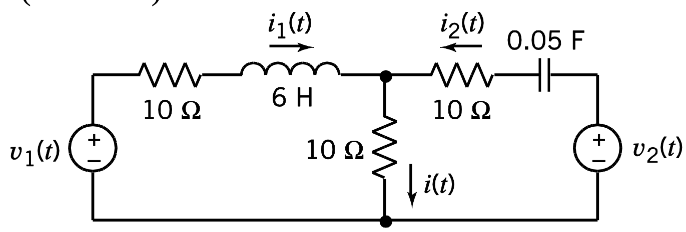


Figure P 10.3-11

Solution:

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.744\angle -118° + 0.5405\angle 100 &= (-0.349 - j\,0.657) + (-0.094 + j0.532) \\ &= (-0.349 - 0.094) + j(-0.657 + 0.532) \\ &= -0.443 - j\,0.125 \\ &= 0.460\angle 196°\end{aligned}$$

$$i(t) = 460 \cos(2t + 196°) \text{ mA}$$

P 10.3-12 Determine $i(t)$ of the *RLC* circuit shown in Figure P 10.3-12 when

$$v_s = 2\cos(4t + 30°)\ \text{V}.$$

Answer: $i(t) = 0.185\cos(4t - 26.3°)$ A

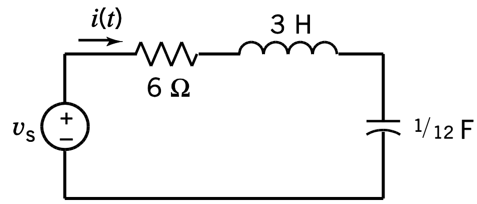
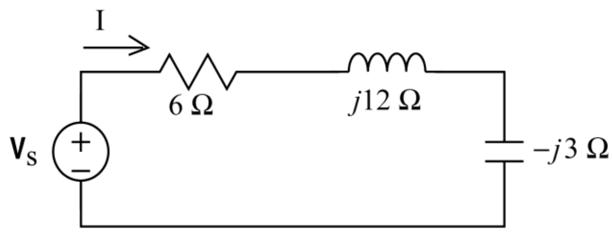


Figure P 10.3-12

Solution:



$$\mathbf{V}_s = 2\angle 30°\ \text{V}$$

and $$\mathbf{I} = \frac{2\angle 30°}{6 + j12 + 3/j} = 0.185\angle -26.3°\ \text{A}$$

$$\underline{i(t) = 0.185\cos(4t - 26.3°)\ \text{A}}$$

Section 10.4 Impedances

P10.4-1 Figure P10.4-1*a* shows a circuit represented in the time domain. Figure P10.4-1*b* shows the same circuit represented in the frequency domain, using phasors and impedances. $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, and $\mathbf{Z}_{L2}$ are the impedances corresponding to the resistor, capacitor, and two inductors in Figure P10.4-1*a*. $\mathbf{V}_s$ is the phasor corresponding to the voltage of the voltage source. Determine $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, $\mathbf{Z}_{L2}$, and $\mathbf{V}_s$.

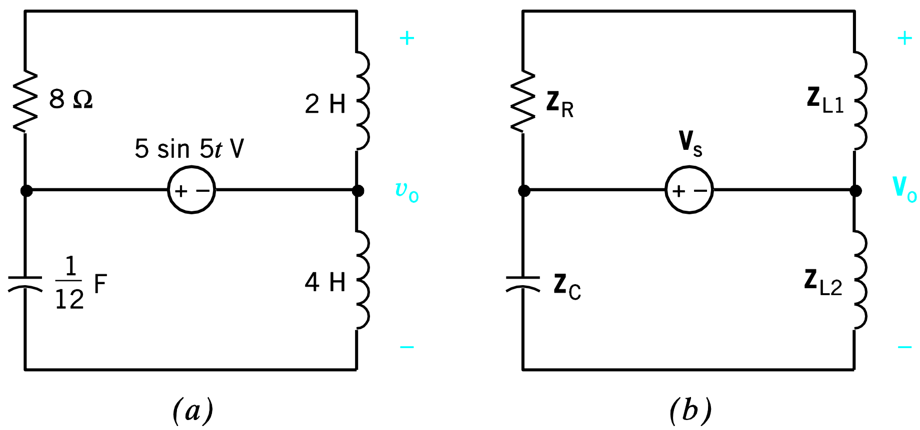


Figure P10.4-1

Hint: $5 \sin 5t = 5 \cos (5t - 90°)$

Answer: $\mathbf{Z}_R = 8\Omega, \mathbf{Z}_C = \dfrac{1}{j5\left(\dfrac{1}{12}\right)} = \dfrac{2.4}{j} = \dfrac{j2.4}{j \times j} = -j2.4\Omega, \mathbf{Z}_{L1} = j5(2) = j10\Omega,$

$\mathbf{Z}_{L2} = j5(4) = j20\ \Omega$, and $\mathbf{V}_s = 5\ \angle{-90°}$ V

Solution:

$$\mathbf{Z}_R = 8\ \Omega,\ \mathbf{Z}_C = \frac{1}{j\,5\,\frac{1}{12}} = \frac{2.4}{j} = \frac{j\,2.4}{j \times j} = -j\,2.4\ \ \Omega,\ \mathbf{Z}_{L1} = j\,5\ (2) = j\ 10\ \Omega,$$

$$\mathbf{Z}_{L2} = j\ 5\ (4) = j\ 20\ \Omega \text{ and } \mathbf{V}_S = 5\ \angle{-90°}\text{ V.}$$

P10.4-2 Figure P10.4-2*a* shows a circuit represented in the time domain. Figure P10.4-2*b* shows the same circuit represented in the frequency domain, using phasors and impedances. $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, and $\mathbf{Z}_{L2}$ are the impedances corresponding to the resistor, capacitor, and two inductors in Figure P10.4-2*a*. $\mathbf{I}_s$ is the phasor corresponding to the current of the current source. Determine $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, $\mathbf{Z}_{L2}$, and $\mathbf{I}_s$.

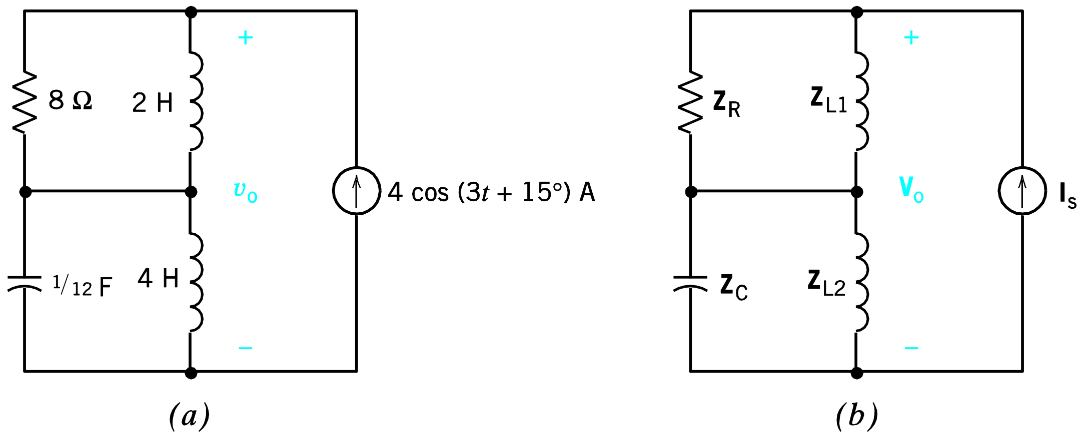


Figure P10.4-2

Answer: $\mathbf{Z}_R = 8\Omega, \mathbf{Z}_C = \dfrac{1}{j3\left(\dfrac{1}{12}\right)} = \dfrac{4}{j} = \dfrac{j4}{j \times j} = -j4\Omega, \mathbf{Z}_{L1} = j3(2) = j6\Omega,$

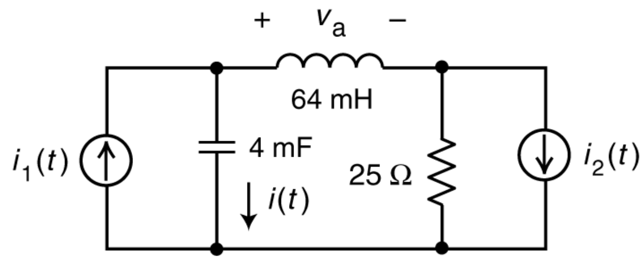
$\mathbf{Z}_{L2} = j3(4) = j12\ \Omega$, and $\mathbf{I}_s = 4\ \angle 15°$ A

Solution:

$$\mathbf{Z}_R = 8\ \Omega,\ \mathbf{Z}_C = \frac{1}{j3\frac{1}{12}} = \frac{4}{j} = \frac{j4}{j \times j} = -j4\ \Omega,\ \mathbf{Z}_{L1} = j\,3\,(2) = j\,6\ \Omega,$$

$$\mathbf{Z}_{L2} = j\,3\,(4) = j\,12\ \Omega \text{ and } \mathbf{I}_S = 4\ \angle 15°\text{ A.}$$

P10.4-3 Represent the circuit shown in Figure P10.4-3 in the frequency domain using impedances and phasors.

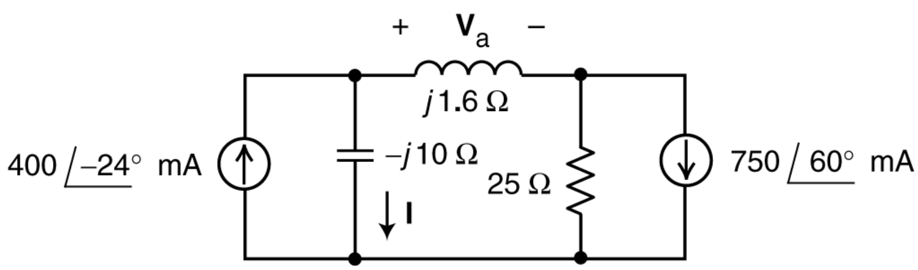


$i_1(t) = 400\cos(25t - 24°)$ mA

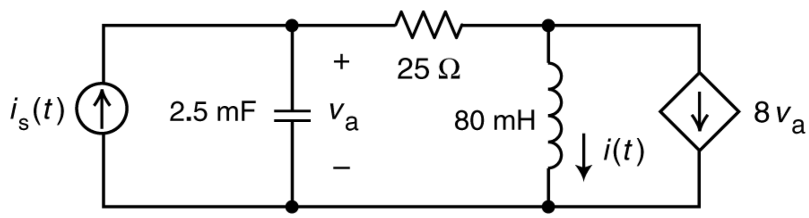
$i_2(t) = 750\cos(25t + 60°)$ mA

Figure P10.4-3

Solution:



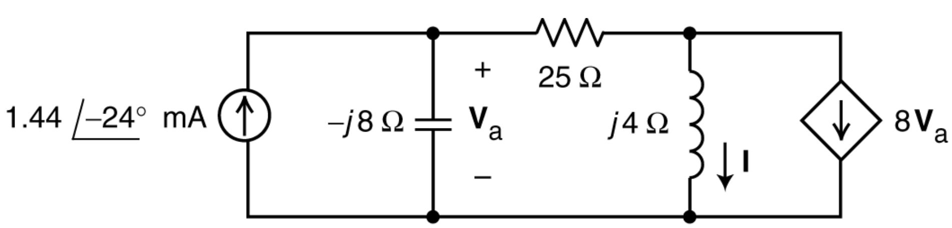
P10.4-4 Represent the circuit shown in Figure P10.4-4 in the frequency domain using impedances and phasors.



$i_s(t) = 1.44\cos(50t - 24°)$ mA

Figure P10.4-4

Solution:



P10.4-5 Determine the current, *i*(*t*), for the circuit shown in Figure P10.4-5.

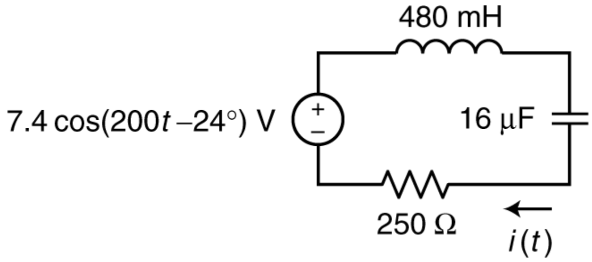
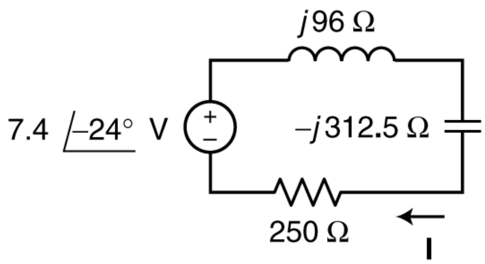


Figure P10.4-5

Solution: Represent the circuit in the frequency domain using phasors and impedances:



Using KVL: $$j96\mathbf{I} - j312.5\mathbf{I} + 250\mathbf{I} + 7.4\angle -24° = 0$$

Solving: $$\mathbf{I} = \frac{7.4\angle -24°}{j96 - j312.5 + 250} = \frac{7.4\angle -24°}{-j216.5 + 250} = \frac{7.4\angle -24°}{330.715\angle -40.9°} = 0.0224\angle 16.9 \text{ A}$$

In the time domain: $$i(t) = 22.4\cos(200t + 16.9) \text{ mA}$$

(Checked using LNAP)

P10.4-6 The input to the circuit shown in Figure P10.4-6 is the current

$$i(t) = 120\cos(4000t) \text{ mA}$$

Determine the voltage, $v(t)$, across the 40 Ω resistor.

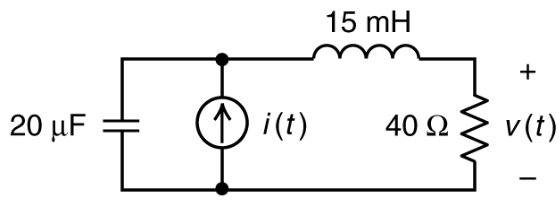
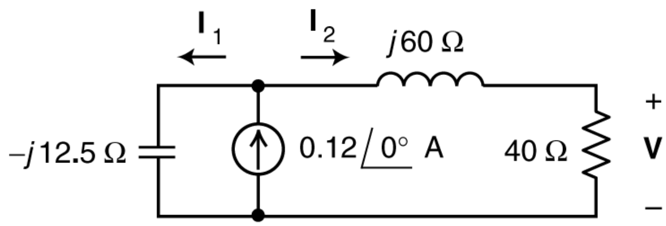


Figure P10.4-6

Solution: Represent the circuit in the frequency domain using phasors and impedances:



Using KCL $$0.120\angle 0° = \mathbf{I}_1 + \mathbf{I}_2$$

Using KVL $$j60\mathbf{I}_2 + 40\mathbf{I}_2 - (-j12.5)\mathbf{I}_1 = 0$$

$$j60\mathbf{I}_2 + 40\mathbf{I}_2 - (-j12.5)(0.12\angle 0° - \mathbf{I}_2) = 0$$

$$j60\mathbf{I}_2 + 40\mathbf{I}_2 + (-j12.5)\mathbf{I}_2 = (-j12.5)(0.12\angle 0°)$$

$$\mathbf{I}_2 = \frac{1.5\angle -90°}{j60 + 40 + (-j12.5)} = \frac{1.5\angle -90°}{40 + j47.5} = \frac{1.5\angle -90°}{62.1\angle 50°} = 0.024155\angle -140° \text{ A}$$

$$\mathbf{V} = 40\,\mathbf{I}_2 = 0.9662\angle -140° \text{ V}$$

In the time domain $$v(t) = 0.9662\cos(4000t - 140°) \text{ V}$$

(Checked using LNAP)

P 10.4-8 Each of the following pairs of element voltage and element current adheres to the passive convention. Indicate whether the element is capacitive, inductive, or resistive and find the element value.

(a) $v(t) = 15 \cos (400t + 30°)$; $i = 3 \sin (400t + 30°)$

(b) $v(t) = 8 \sin (900t + 50°)$; $i = 2 \sin (900t + 140°)$

(c) $v(t) = 20 \cos (250t + 60°)$; $i = 5 \sin (250t + 150°)$

Answer: (a) $L = 12.5$ mH

(b) $C = 277.77\ \mu$F

(c) $R = 4\ \Omega$

Solution:

(a) $v = 15\cos(400t + 30°)$ V

$i = 3\sin(400t+30°) = 3\cos(400t - 60°)$ V

v leads i by 90° $\Rightarrow$ element is an inductor

$$\left|\mathbf{Z}_L\right| = \frac{v_{\text{peak}}}{i_{\text{peak}}} = \frac{15}{3} = 5 = \omega L = 400L \Rightarrow \underline{L = 0.0125 \text{ H} = 12.5 \text{ mH}}$$

(b) i leads v by 90° $\Rightarrow$ the element is a capacitor

$$\left|\mathbf{Z}_c\right| = \frac{v_{\text{peak}}}{i_{\text{peak}}} = \frac{8}{2} = 4 = \frac{1}{\omega C} = \frac{1}{900C} \Rightarrow \underline{C = 277.77\ \mu\text{F}}$$

(c) $v = 20\cos(250t + 60°)$ V

$i = 5\sin(250t + 150°) = 5\cos(250t + 60°)$ A

Since v & i are in phase $\Rightarrow$ element is a resistor

$$\therefore R = \frac{v_{\text{peak}}}{i_{\text{peak}}} = \frac{20}{5} = \underline{4\ \Omega}$$

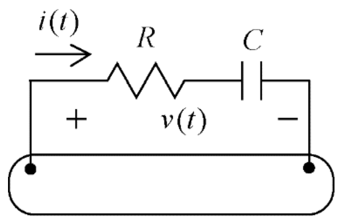


Figure P10.4-9

P10.4-9 This voltage and current for the circuit shown in Figure P10.4-9 are given by

$$v(t) = 20\cos(20t + 15°)\ \text{V} \quad \text{and} \quad i(t) = 1.49\cos(20t + 63°)\ \text{A}$$

Determine the values of the resistance, *R*, and capacitance, *C*.

Solution:

$$R - j\frac{1}{20C} = \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{20\angle 15°}{1.49\angle 63°} = \frac{20}{1.49}\angle(15° - 63°) = 13.42\angle -48° = 8.98 - j9.97\ \Omega$$

Equating real and imaginary parts gives $R = 9\ \Omega$ and $C = \dfrac{1}{20 \times 9.97} = 5\ \text{mF}$.

P10.4-10

Figure P10.4-10 shows an ac circuit represented in both the time domain and the frequency domain. Determine the values of A, B, a and b.

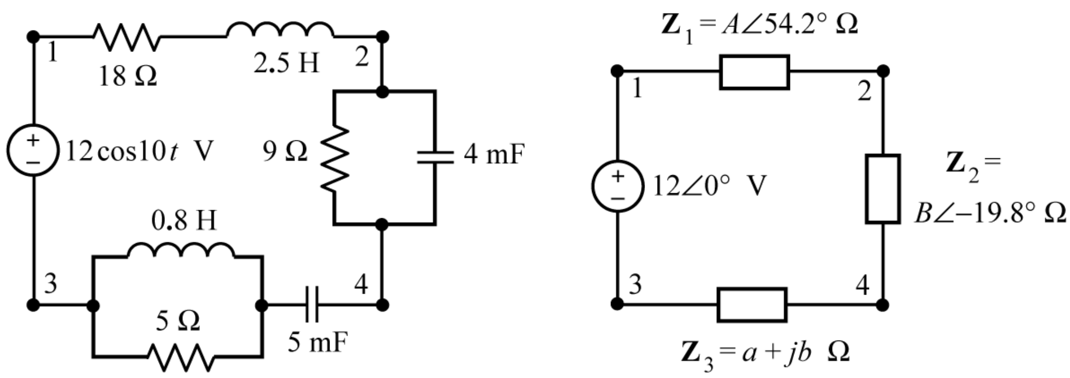


Figure P10.4-10

Solution:

The impedance between nodes a and b is given by

$$18 + j(10)(2.5) = 18 + j25 = 30.8\angle 54.2^\circ$$

To find the impedance between nodes b and c we first find the impedance of the capacitor:

$$-j\frac{1}{(10)(0.004)} = -j\frac{1}{0.04} = -j25$$

then

$$\frac{9(-j25)}{9-j25} = \frac{-j225}{26.57\angle -70.2^\circ} = \frac{225\angle -90^\circ}{26.57\angle -70.2^\circ} = 8.47\angle -19.8^\circ\ \Omega$$

The impedance between nodes c and d is given by

$$\frac{(5)(j(10)(0.88))}{5+j(10)(0.8)} - j\frac{1}{(10)(0.005)} = \frac{j40}{5+j8} - j\frac{1}{0.05} = \frac{j40}{5+j8}\left(\frac{5-j8}{5-j8}\right) - j20$$

$$= \frac{320+j200}{25+64} - j20$$

$$= 3.60 + j2.25 - j20 = 3.60 - j17.75\ \Omega$$

So

A = 30.8 V, B = 8.47 Ω, , a = 3.57 Ω and b = –17.75 Ω.

P 10.4-11 Represent the circuit shown in Figure P 10.4-11 in the frequency domain using impedances and phasors.

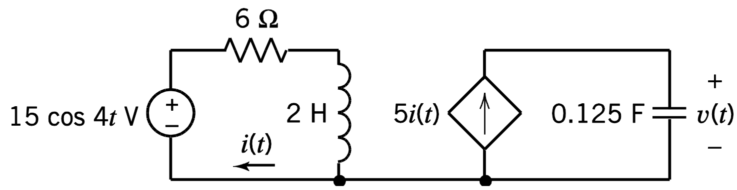
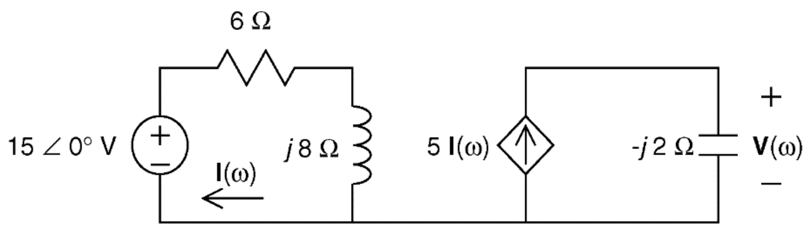


Figure P 10.4-11

Solution:



P 10.4-12 Represent the circuit shown in Figure P 10.4-12 in the frequency domain using impedances and phasors.

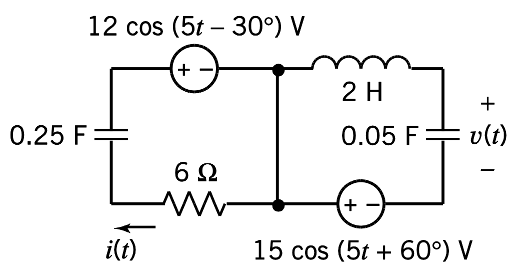
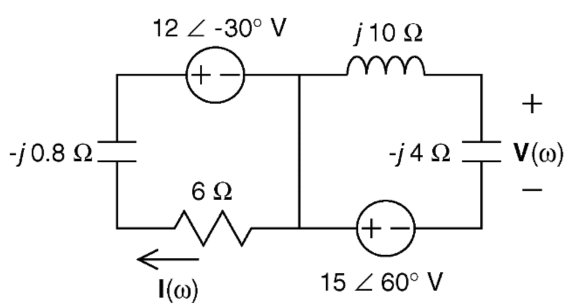


Figure P 10.4-12

Solution:



P 10.4-13 Find R and L of the circuit of Figure P 10.4-13 when $v(t) = 10 \cos(\omega t + 40°)$ V; $i(t) = 2 \cos(\omega t + 15°)$ mA, and $\omega = 2 \times 10^6$ rad/s.

Answer: $R = 4.532$ kω, $L = 1.057$ mH

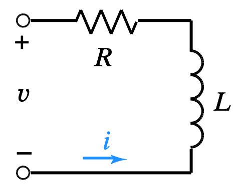


Figure P 10.4-13

Solution:

$$\mathbf{Z} = \frac{\mathbf{V}}{-\mathbf{I}} = \frac{10\ \angle 40°}{-2\times 10^{-3}\ \angle -165°} = -5000\angle 205°\Omega = 4532 + j\,2113 = R + j\omega L$$

$$\text{so } \underline{R = 4532\ \Omega} \text{ and } L = \frac{2113}{\omega} = \frac{2113}{2\times 10^6} = \underline{1.057 \text{ mH}}$$

Section 10.5 Series and Parallel Impedances

P10.5-1 Determine the steady state voltage *v*(*t*) in the circuit shown in Figure P10.5-1.

Answer: $v(t) = 32\cos(250t - 57.9°)$ V

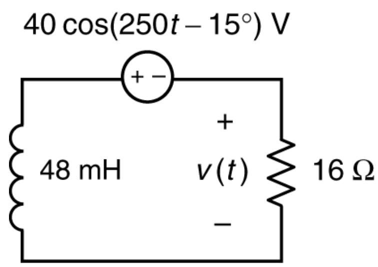
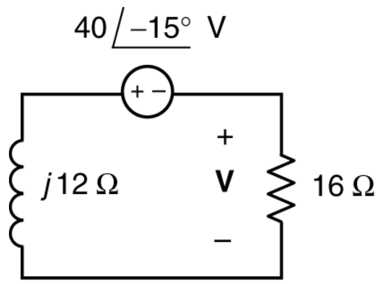


Figure P10.5-1

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using voltage division: $\mathbf{V} = -\frac{16}{16 + j12}(40\angle -15°) = \frac{16\angle 180°}{20\angle 36.9°}(40\angle -15°) = 32\angle 128.1°$ V

In the time domain $v(t) = 32\cos(250t - 57.9°)$ V

P10.5-2 Determine the voltage *v*(*t*) in the circuit shown in Figure P10.5-2.

Answer: $v(t) = 14.57\cos(800t + 111.7°)$ V

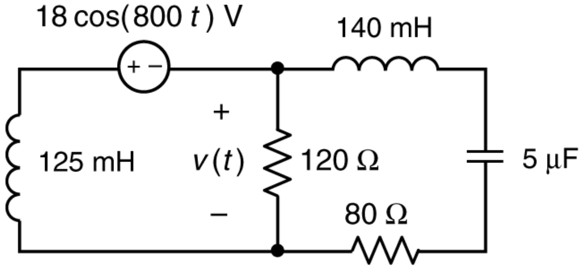
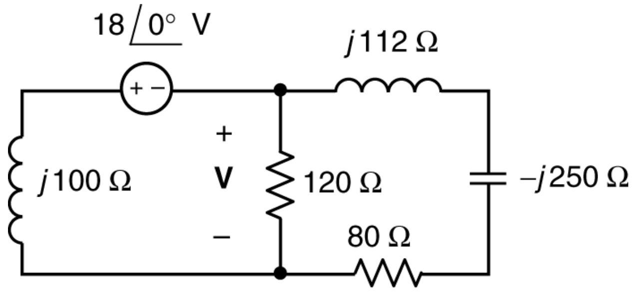


Figure P10.5-2

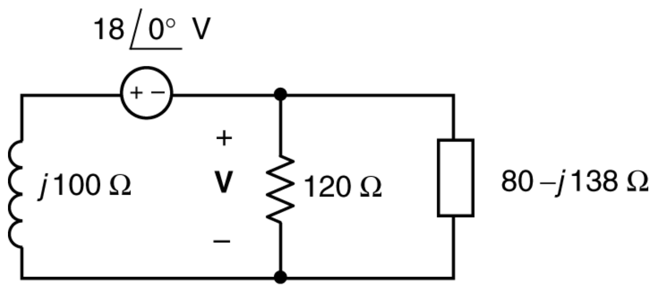
Solution:

Represent the circuit in the frequency domain:



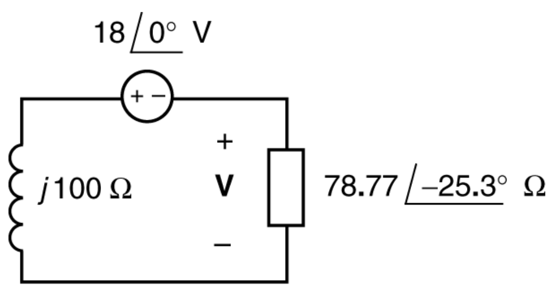
Replace the series impedances at the right of the circuit by an equivalent impedance

$$\mathbf{Z}_S = j112 + (-j250) + 80 = 80 - j138 \ \Omega$$



Replace the parallel impedances at right of the circuit by an equivalent impedance

$$\mathbf{Z}_P = \frac{(80 - j138)120}{80 - j138 + 120} = \frac{(80 - j138)120}{200 - j138} = \frac{(159.51\angle -59.9)120}{242.99\angle -34.6} = 78.77\angle 25.3° \ \Omega$$



Using voltage division

$$\mathbf{V} = -\frac{78.77\angle -25.3°}{j100 + 78.77\angle -25.3°} 18\angle 0° = -\frac{78.77\angle -25.3°}{97.325\angle 42.97°} 18\angle 0° = 14.57\angle 111.73° \text{ V}$$

In the time domain $v(t) = 14.57\cos(800t + 111.7°)$ V

(checked using LNAP)

P10.5-3 Determine the voltage $v(t)$ in the circuit shown in Figure P10.5-3.

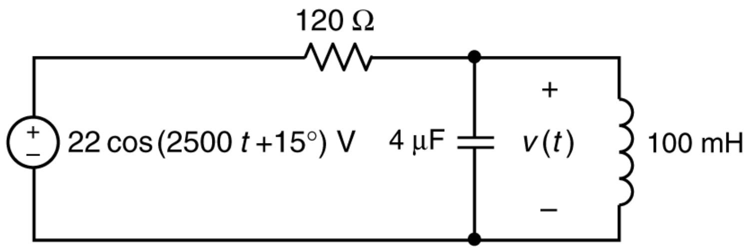
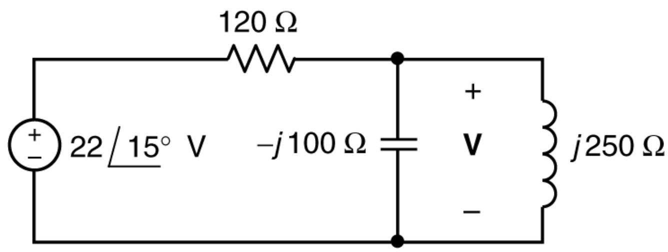


Figure P10.5-3

Answer: $v(t) = 14.1\cos(2500t - 35.2°)$ V

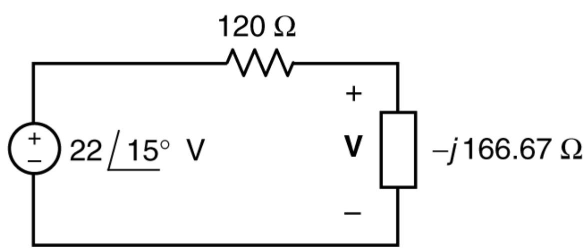
Solution:

Represent the circuit in the frequency domain:



Replace the parallel impedances at right of the circuit by an equivalent impedance:

$$\frac{(-j100)(j250)}{-j100 + j250} = -j166.67\ \Omega$$



Using voltage division

$$\mathbf{V} = -\frac{-j166.67}{200 - j166.67} 22\angle 15° = -\frac{166.67\angle -90°}{260.3\angle -39.8°} 22\angle 15° = 14.1\angle -35.2°\ \text{V}$$

In the time domain $v(t) = 14.1\cos(2500t - 35.2°)$ V

P10.5-4 The input to the circuit shown in Figure P10.5-4 is the current $i_s(t) = 48\cos(25t)$ mA. Determine the current $i_1(t)$.

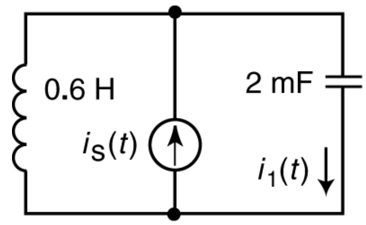
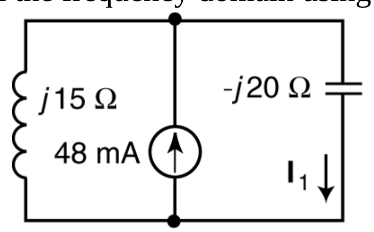


Figure P10.5-4

Answer: $i_1(t) = 144\cos(25t + 180°)$ mA

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using current division

$$\mathbf{I}_1 = \frac{j15}{j15 - j20}(48\angle 0°) = \frac{15}{-5}(48\angle 0°) = 144\angle 180° \text{ mA}$$

In the time domain $\quad i_1(t) = 144\cos(25t + 180°)$ mA

P10.5-5 The input to the circuit shown in Figure P10.5-5 is the current $i_s(t) = 48\cos(25t)$ mA. Determine the current $i_2(t)$.

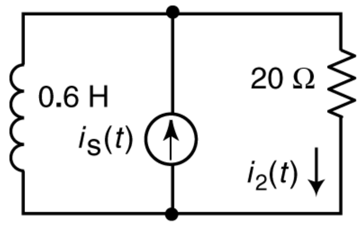
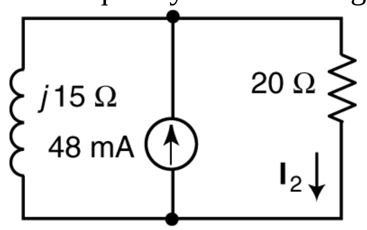


Figure P10.5-5

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using current division

$$\mathbf{I}_2(\omega) = \frac{j15}{20 + j15}(48\angle 0°) = \frac{15\angle 90°}{25\angle 36.9°}(48\angle 0°) = 28.8\angle 53.1° \text{ mA}$$

In the time domain $$i_2(t) = 28.8\cos(25t + 53.1°) \text{ mA}$$

P10.5-6 The input to the circuit shown in Figure P10.5-6 is the current $i_s(t) = 48\cos(25t)$ mA. Determine the current $i_3(t)$.

Answer: $i_3(t) = 16.85\cos(25t + 69.4°)$ mA

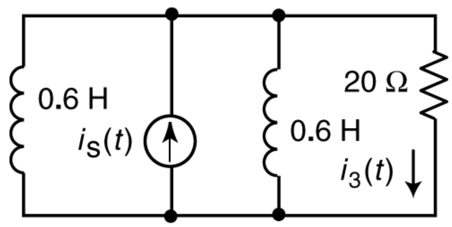
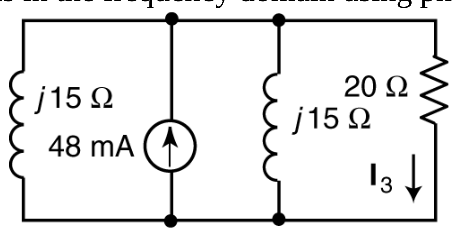


Figure P10.5-6

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using current division

$$\mathbf{I}_3 = \frac{j15 \,\|\, j15}{(j15 \,\|\, j15) + 20}(48\angle 0°) = \frac{j7.5}{j7.5 + 20}(48\angle 0°) = \frac{7.5\angle 90°}{21.36\angle 69.56°}(48\angle 0°) = 16.8539\angle 69.44° \text{ V}$$

In the time domain $\qquad i_3(t) = 16.85\cos(25t + 69.4°)$ mA

P10.5-7 Figure P10.5-7 shows a circuit represented in the frequency domain. Determine the voltage phasor $\mathbf{V}_1$.

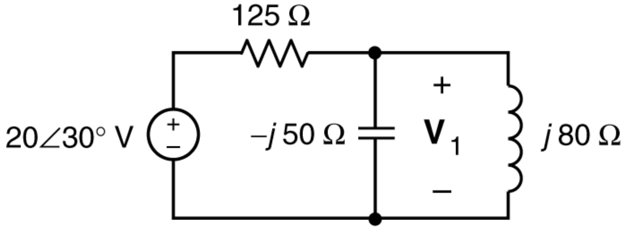


Figure P10.5-7

***Answer*:** $\mathbf{V}_1 = 14.59\angle -13.15°$ V

Solution:

$$\mathbf{V}_1 = \frac{j\,80 \,||\, -j\,50}{125+\left(j\,80 \,||\, -j\,50\right)}\left(20\angle 30°\right) = \frac{-j133.33}{125-j133.33}\left(20\angle 30°\right) = \frac{133.33\angle -90°}{182.8\angle -46.85°}\left(20\angle 30°\right)$$

$$= 14.59\angle -13.15° \text{ V}$$

P10.5-8 Figure P10.5-8 shows a circuit represented in the frequency domain. Determine the current phasor $\mathbf{I}_2$.

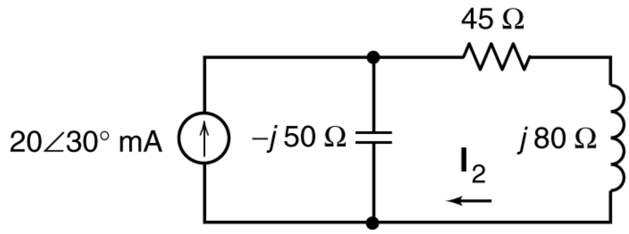


Figure P10.5-8

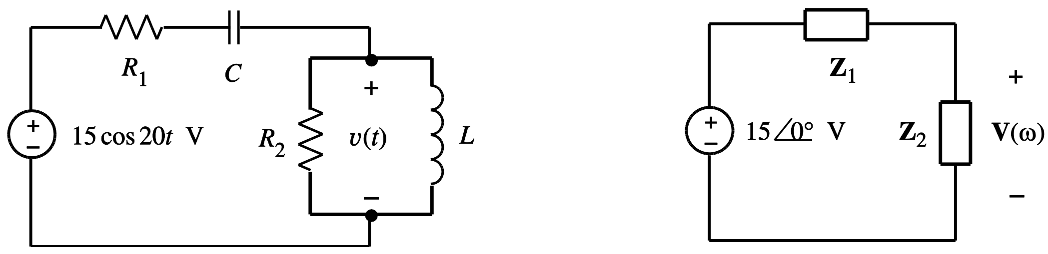
***Answer*: ** $\mathbf{I}_2 = 18.48\angle -93.7°$ mA

Solution:

$$\mathbf{I}_2 = \frac{-j50}{-j50+45+j80}\left(20\angle 30°\right) = \frac{-j50}{45+j30}\left(20\angle 30°\right)$$

$$= \frac{50\angle -90}{54.08\angle 33.7}\left(20\angle 30°\right) = 18.49\angle -93.7° \text{ mA}$$

P10.5-9 Here's an ac circuit represented both in the time domain and frequency domain:



(*a*) (*b*)

Suppose $\mathbf{Z}_1 = 15.3\angle -24.1°\ \Omega$ and $\mathbf{Z}_2 = 14.4\angle 53.1°\ \Omega$

Determine the node voltage $v(t)$ and the values of R_1, R_2, L and C.

Solution:

Consider $\mathbf{Z}_1$:

$$R_1 - j\frac{1}{20C} = 15.3\angle -24.1° = 14 - j6.25 \quad \Rightarrow \quad R_1 = 14\ \Omega \quad \text{and} \quad C = \frac{1}{20(6.25)} = 0.008\ \text{F} = 8\ \text{mF}$$

Next consider $\mathbf{Z}_2$:

$$\frac{1}{\frac{1}{R_2} + \frac{1}{j20L}} = 14.4\angle 53.1° \quad \Rightarrow \quad \frac{1}{R_2} + \frac{1}{j20L} = \frac{1}{14.4\angle 53.1°} = \frac{1}{14.4}\angle 53.1 = 0.05556 - j0.04167$$

Equating coefficients gives

$$R_2 = \frac{1}{0.05556} = 18\ \Omega \quad \text{and} \quad L = \frac{1}{20(0.04167)} = 1.2\ \text{H}$$

Next, consider the voltage divider:

$$A\angle 31.5° = \frac{14.4\angle 36.9°}{15.3\angle -24.1° + 14.4\angle 36.9°}(15\angle 0°) = \frac{(15)(14.4)\angle 36.9°}{(14 - j6.25)(11.52 + j8.64)}$$

$$= \frac{216\angle 36.9°}{25.52 + j2.39}$$

$$= \frac{216\angle 36.9°}{25.63\angle 5.4°} = 8.43\angle 31.5°\ \text{V}$$

In the time domain, $v(t) = 8.43\cos(20t + 31.5°)$ V.

P 10.5-10 Find **Z** and **Y** for the circuit of Figure P 10.5-10 operating at 10 kHz.

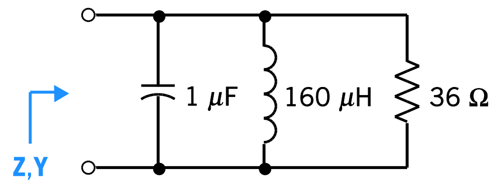


Figure P 10.5-10

Solution:

$$\omega = 2\pi f = 2\pi(10\times10^3) = 62832 \text{ rad/sec}$$

$$\mathbf{Z}_R = R = 36\,\Omega \quad\Leftrightarrow\quad \mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{36} = 0.0278 \text{ S}$$

$$\mathbf{Z}_L = j\omega L = j(62830)(160\times10^{-6}) = j10.053 \approx j10\,\Omega \quad\Leftrightarrow\quad \mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = -0.1j \text{ S}$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{(62830)(1\times10^{-6})} = -j15.915 \approx -j\,16\,\Omega \quad\Leftrightarrow\quad \mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = j0.0625 \text{ S}$$

$$\mathbf{Y}_{eq} = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = 0.0278 - j0.0375 = 0.0467\angle -53.4^\circ \text{ S}$$

$$\mathbf{Z}_{eq} = \frac{1}{\mathbf{Y}_{eq}} = 21.43\angle 53.4^\circ = \underline{12.75 + j17.22\,\Omega}$$

P 10.5-11 For the circuit of Figure P 10.5-11, find the value of *C* required so that **Z** = 590.7Ω when *f* = 1 MHz.

Answer: *C* = 0.27 nF

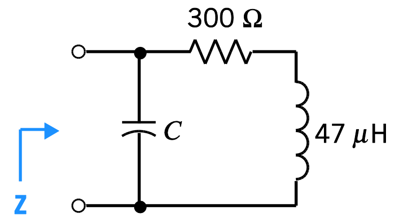


Figure P 10.5-11

Solution:

$$\mathbf{Z}_L = j\omega L = j\ (6.28\times10^6)(47\times10^{-6})\ =\ j\ 295\ \Omega$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_c \,||\, (\mathbf{Z}_R + \mathbf{Z}_L) = \frac{\left(\frac{1}{j\omega C}\right)(300+j295)}{\frac{1}{j\omega C}+300+j295}\ =\ 590.7\ \Omega$$

$$590.7\ =\ \frac{300+300\,j}{1+300\,j\,\omega C-300\,\omega C}\ \Rightarrow\ 590.7-(590.7)(295\,\omega C)+j(590.7)(300\omega C)\ =\ 300+j295$$

Equating imaginary terms $\left(\omega = 2\pi f\ = 6.28\times10^6\ \text{rad/sec}\right)$

$$(590.7)(300\omega C) = 295\ \Rightarrow\ \underline{C=0.27\ \text{nF}}$$

P 10.5-12 Determine the impedance **Z** for the circuit shown in Figure P 10.5-12.

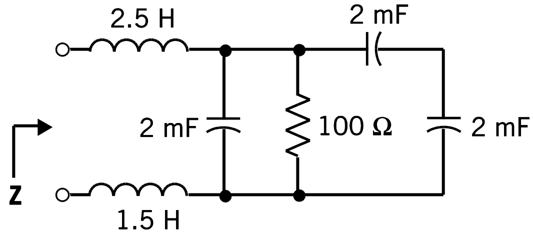
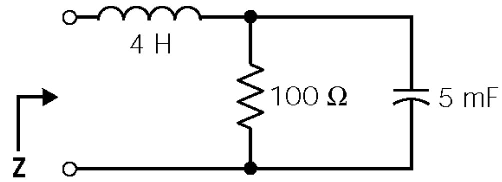


Figure P 10.5-12

Solution:

Replace series and parallel capacitors by an equivalent capacitor and series inductors by an equivalent inductor:



Then

$$\mathbf{Z} = j\omega 4 + \frac{100\dfrac{1}{j\omega\left(5\times10^{-3}\right)}}{100+\dfrac{1}{j\omega\left(5\times10^{-3}\right)}} = j\omega 4 + \frac{100\left(-j\dfrac{200}{\omega}\right)}{100+\left(-j\dfrac{200}{\omega}\right)} = j\omega 4 + \frac{-j\dfrac{200}{\omega}}{1-j\dfrac{2}{\omega}}\times\frac{1+j\dfrac{2}{\omega}}{1+j\dfrac{2}{\omega}}$$

$$\mathbf{Z} = j\omega 4 + 100\frac{\dfrac{4}{\omega^2}-j\dfrac{2}{\omega}}{1+\dfrac{4}{\omega^2}} = j\omega 4 + 100\frac{4-j2\omega}{4+\omega^2} = \frac{400}{4+\omega^2}+j\left(4\,\omega-\frac{200\,\omega}{4+\omega^2}\right)$$

P 10.5-13 The big toy from the hit movie *Big* is a child's musical fantasy come true—a sidewalk-sized piano. Like a hopscotch grid, this once-hot Christmas toy invites anyone who passes to jump on, move about, and make music. The developer of the "toy" piano used a tone synthesizer and stereo speakers as shown in Figure P 10.5-13 (Gardner, 1988). Determine the current $i(t)$ for a tone at 796 Hz when $C = 10\ \mu$F.

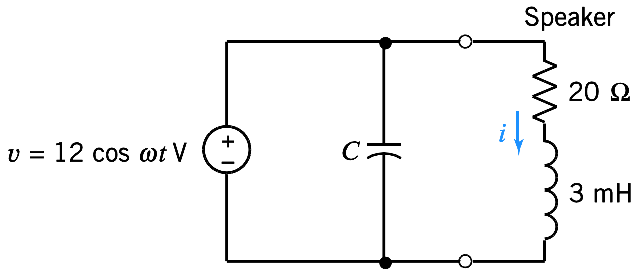
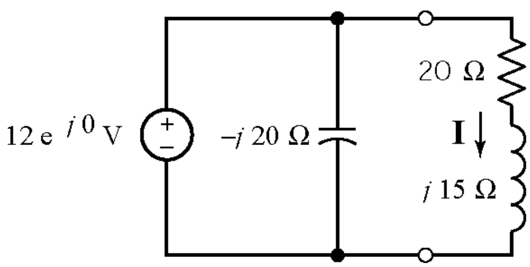


Figure P 10.5-13

Solution:



$$j(2\pi \cdot 796)(3 \cdot 10^{-3}) = j15\ \Omega$$

$$\mathbf{I} = \frac{12}{20 + j15} = 0.48\ \angle -37°\ \text{A}$$

$$i(t) = 0.48 \cos(2\pi \cdot 796\, t - 37°)\ \text{A}$$

P 10.5-14 Determine $i(t)$, $v(t)$, and L for the circuit shown in Figure P 10.5-14.

Answer: $i(t) = 1.34 \cos(2t - 87°)$ A, $v(t) = 7.29 \cos(2t - 24°)$ V, and $L = 4$ H

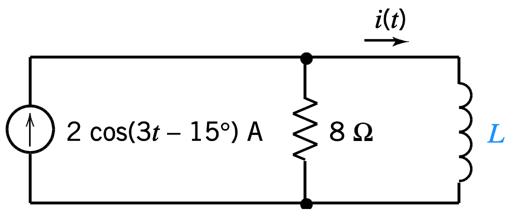


Figure P 10.5-14

Solution:

$$\mathbf{Z}_1 = R = 8\ \Omega,\quad \mathbf{Z}_2 = j3L,\quad \mathbf{I} = B\angle -51.87° \text{ and } \mathbf{I}_s = 2\angle -15° \text{ A}$$

$$\frac{\mathbf{I}}{\mathbf{I}_s} = \frac{B\angle -51.87°}{2\angle -15°} = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{8}{8 + j3L} = \frac{8\angle 0°}{\sqrt{8^2 + (3L)^2}\ \angle \tan^{-1}\left(\frac{3L}{8}\right)}$$

Equate the magnitudes and the angles.

$$\text{angles: } +36.87 = +\tan^{-1}\left(\frac{3L}{8}\right) \Rightarrow \underline{L = 2 \text{ H}}$$

$$\text{magnitudes: } \frac{8}{\sqrt{64 + 9L^2}} = \frac{B}{2} \Rightarrow \underline{B = 1.6}$$

P 10.5-15 Spinal cord injuries result in paralysis of the lower body and can cause loss of bladder control. Numerous electrical devices have been proposed to replace the normal nerve pathway stimulus for bladder control. Figure P 10.5-15 shows the model of a bladder control system where $v_s = 20 \cos \omega t$ V and $\omega = 100$ rad/s. Find the steady-state voltage across the 10-Ω load resistor.

Answer: $v(t) = 10\sqrt{2} \cos (100t + 45°)$ V

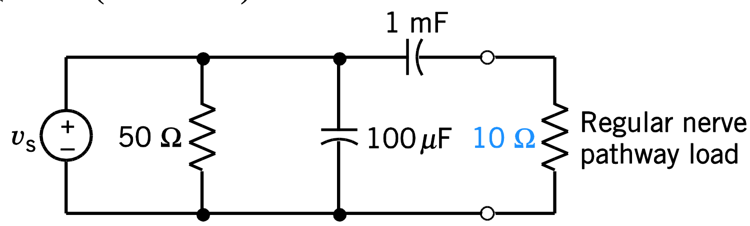
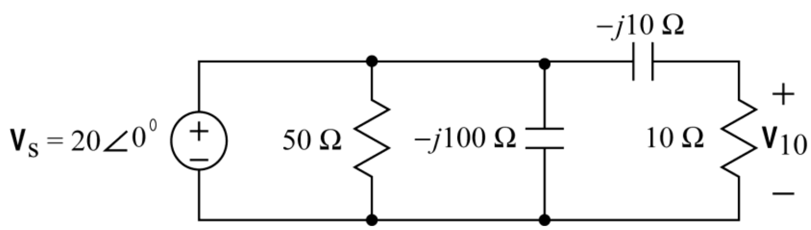


Figure P 10.5-15

Solution:



$$\mathbf{V}_{10} = \mathbf{V}_s \left(\frac{10}{10 - j10} \right)$$

$$= 20\angle 0° \left(\frac{10}{10\sqrt{2}\angle -45°} \right)$$

$$= 10\sqrt{2}\angle 45°$$

$$v_{10}(t) = 10\sqrt{2} \cos(100t + 45°) \text{ V}$$

P 10.5-16 There are 500 to 1000 deaths each year in the United States from electric shock. If a person makes a good contact with his hands, the circuit can be represented by Figure P 10.5-16, where $v_s = 160 \cos \omega t$ V and $\omega = 2\pi f$. Find the steady-state current i flowing through the body when (a) $f = 60$ Hz and (b) $f = 400$ Hz.

Answer: (a) $i(t) = 0.53 \cos(120\pi t + 5.9°)$

(b) $i(t) = 0.625 \cos(800\pi t + 59.9°)$ A

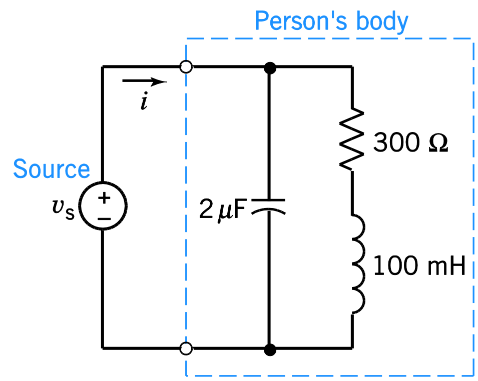
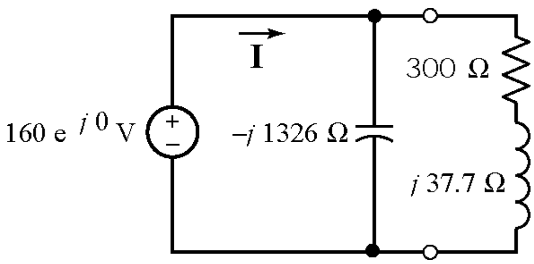


Figure P 10.5-16

Solution:

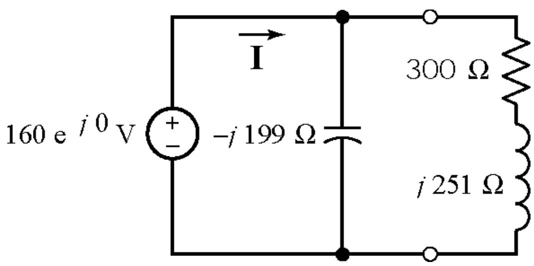
(a)



$$\mathbf{I} = \frac{160\angle 0°}{\dfrac{(-j1326)(300+j37.7)}{-j1326+300+j37.7}} = \frac{160\angle 0°}{303\angle -5.9°} = 0.53\angle 5.9° \text{ A}$$

$$\underline{i(t) = 0.53\cos(120\pi t + 5.9°) \text{ A}}$$

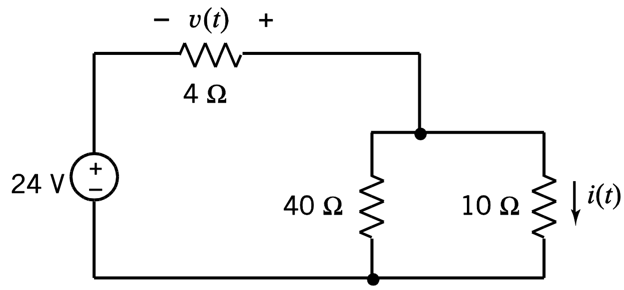
(b)



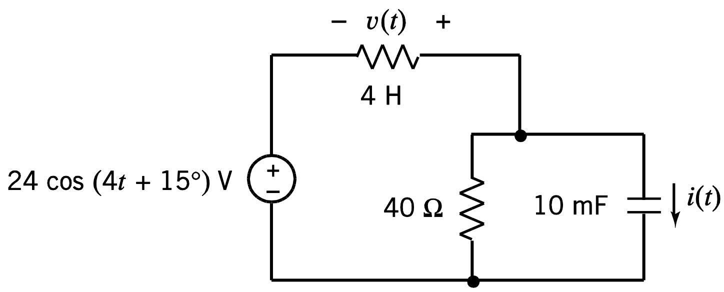
$$\mathbf{I} = \frac{160\angle 0°}{\dfrac{(-j199)(300+j251)}{-j199+300+j251}} = \frac{160\angle 0°}{256\angle -59.9°} = 0.625\angle 59.9° \text{ A}$$

$$\underline{i(t) = 0.625 \cos(800\pi t + 59.9°) \text{ A}}$$

P 10.5-17 Determine the steady-state voltage, $v(t)$, and current, $i(t)$, for each of the circuits shown in Figure P 10.5-17.



(a)



(b)

Figure P 10.5-17

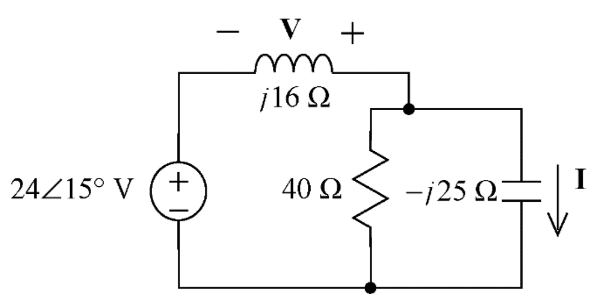
Solution:

(a)

$$v(t) = -\frac{4}{4+(40 \parallel 10)} \times 24 = -8 \text{ V}$$

$$i(t) = \frac{40}{40+10} \times \frac{24}{4+(40 \parallel 10)} = \frac{8}{5} = 1.6 \text{ A}$$

(b) Represent the circuit in the frequency domain using impedances and phasors.



$$\mathbf{V} = -\frac{j16}{j16 + \left(40 \parallel -j25\right)} \times 24\angle 15° = \frac{\left(16\angle -90°\right)\left(24\angle 15°\right)}{j16 + \dfrac{40\left(-j25\right)}{40 - j25}} = 33.66\angle -65° \text{ V}$$

$$\mathbf{I} = \frac{40}{40 - j25} \times \frac{24\angle 15°}{j16 + \dfrac{40\left(-j25\right)}{40 - j25}} = 1.78\angle 57° \text{ A}$$

so

$$v\left(t\right) = 33.66\cos\left(4t - 65°\right) \text{ V}$$

and

$$i\left(t\right) = 1.78\cos\left(4t + 57°\right) \text{ A}$$

(checked: LNAP 8/1/04)

P 10.5-18 Determine the steady-state current, $i(t)$, for the circuit shown in Figure P 10.5-18.

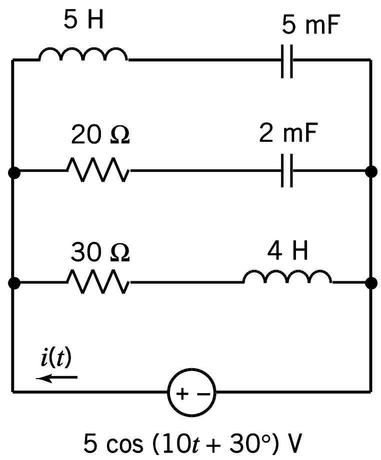


Figure P 10.5-18

Solution:

$$\mathbf{I} = \frac{5\angle 30°}{30 + j40} + \frac{5\angle 30°}{20 - j50} + \frac{5\angle 30°}{j50 - j20} = 0.100\angle -23.1° + 0.0923\angle 98.2° + 0.1667\angle -60°$$

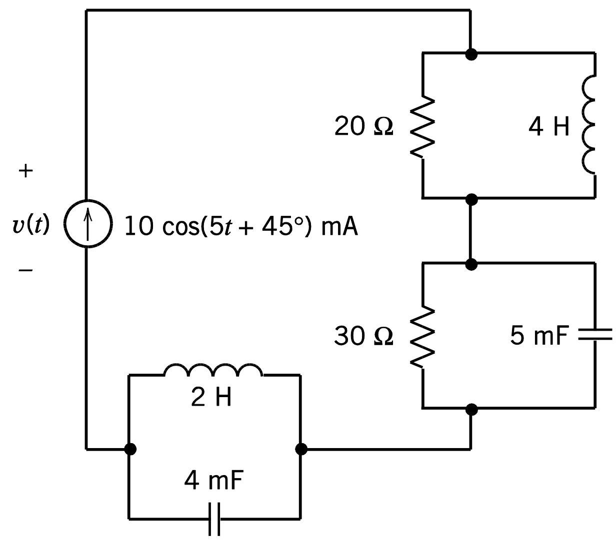
$$= 0.186\angle -29.5° \text{ A}$$

so

$$i(t) = 0.186\cos(10t - 29.5°) \text{ A}$$

(checked: LNAP 8/1/04)

P10.5-19 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:

$$\mathbf{V} = 0.01\angle 45°\left[\left(20 \| j20\right) + \left(30 \| \left(-j40\right)\right) + \left(j10 \| \left(-j50\right)\right)\right]$$

$$= 0.01\angle 45°\left[\frac{20\left(j20\right)}{20+j20} + \frac{30\left(-j40\right)}{30-j40} + \frac{j10\left(-j50\right)}{j10-j50}\right]$$

$$= 0.01\angle 45°\left[14.14\angle 45° + 24\angle -36.9° + 12.5\angle 90°\right]$$

$$= 0.01\angle 45°\left[10 + j10 + 19.2 - j14.4 + j12.5\right]$$

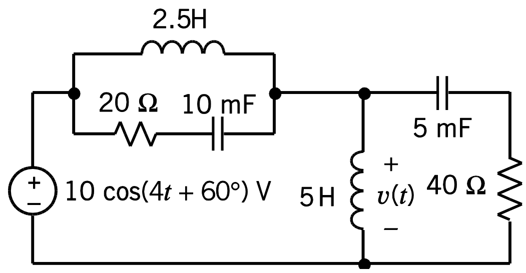
$$= 0.303\angle 60.5° \text{ V}$$

so

$$v\left(t\right) = 0.303\cos\left(5t + 60.5°\right) \text{ V}$$

(checked: LNAP 8/1/04)

P10.5-20 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:
Let

$$\mathbf{Z}_1 = \left(20 - j\frac{1}{4(0.01)}\right) \parallel j10 = \frac{(20 - j25)\, j10}{20 - j25 + j10} = \frac{250 - j200}{20 - j15} = 12.81\angle 75.5°\ \Omega$$

and

$$\mathbf{Z}_2 = j20 \parallel \left(-j\frac{1}{4(0.005)} + 40\right) = \frac{j20(40 - j50)}{j20 + 40 - j50} = \frac{1000 + j800}{40 - j30} = 25.61\angle 75.5°\ \Omega$$

Then

$$\mathbf{V} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 10\angle 60° = \frac{25.61\angle 75.5°}{12.81\angle 75.5° + 25.6\angle 75.5°} \times 10\angle 60° = 6.67\angle 60°\ \text{V}$$

so

$$v(t) = 6.67\cos(4t + 60°)\ \text{V}$$

(checked: LNAP 8/1/04)

P 10.5-21 The input to the circuit shown in Figure P 10.5-21 is the current source current

$$i_s(t) = 25 \cos(10t + 15°) \text{ mA}$$

The output is the current $i_1(t)$. Determine the steady-state response, $i_1(t)$.

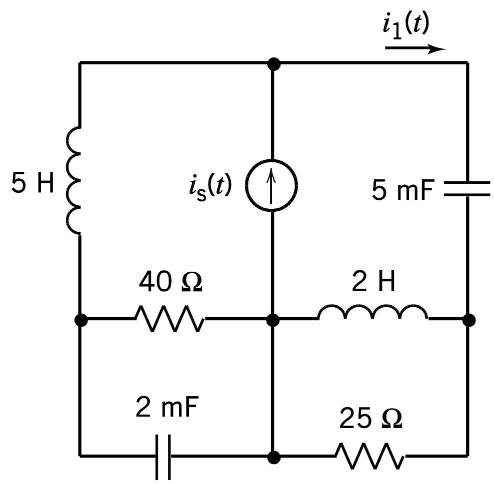


Figure P 10.5-21

Solution:
Represent the circuit in the frequency domain using impedances and phasors. Let

$$\mathbf{Z}_1 = j50 + \left(40 \| \frac{1}{j10\times 2\times 10^{-3}}\right) = j50 + \frac{40(-j50)}{40 - j50} = 39.0\angle 51.3° \ \Omega$$

and

$$\mathbf{Z}_2 = -j\frac{1}{10(5\times 10^{-3})} + j20 \| 25 = -j20 + \frac{j20(25)}{25 + j20} = 12.5\angle -38.7 \ \Omega$$

$\mathbf{Z}_1$ and $\mathbf{Z}_2$ are connected in parallel. Current division gives

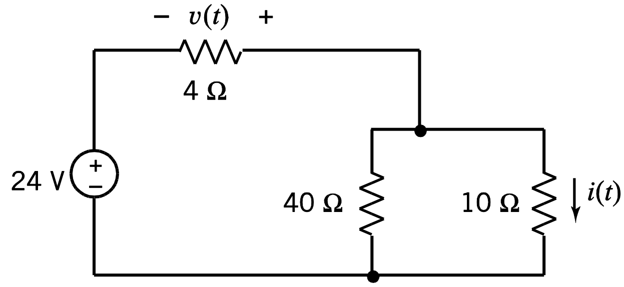
$$\mathbf{I}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 0.025\angle 15° = 0.024\angle 32.7° \text{ A}$$

so

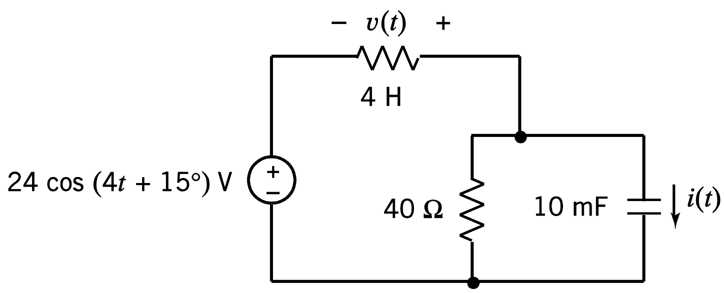
$$i_1(t) = 0.024\cos(10t + 32.7°) \text{ A}$$

(checked: LNAP 8/1/04)

P10.5-22 Determine the steady state voltage, $v(t)$, and current $i(t)$ for each of these circuits:



(a)



(b)

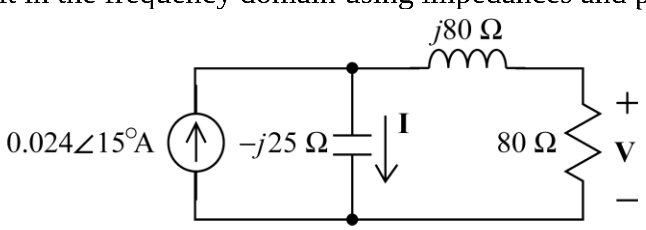
Solution:

(a)

$$i(t) = \frac{80+80}{40+(80+80)} 0.024 = 19.2 \text{ mA}$$

$$v(t) = \frac{80}{80+80} \times \left(40 \parallel (80+80)\right) 0.024 = \frac{1}{2}(32)(0.024) = 0.384 \text{ V}$$

(b) Represent the circuit in the frequency domain using impedances and phasors.



$$\mathbf{I} = \frac{80+j80}{-j25+(80+j80)} \times 0.024\angle 15° = 0.028\angle 25.5° \text{ A}$$

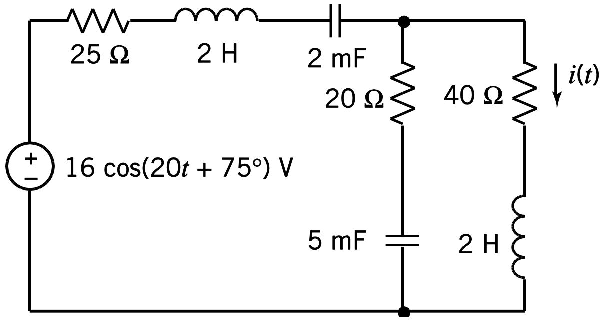
$$\mathbf{V} = \frac{80}{80+j80} \times \left[-j25 \parallel (80+j80)\right] \times 0.024\angle 15° = 0.494\angle -109.5° \text{ V}$$

So $$i(t) = 28\cos(10t + 25.5°) \text{ mA}$$

and $$v(t) = 0.494\cos(10t - 109.5°) \text{ V}$$

(checked: LNAP 8/1/04)

P10.5-23 Determine the steady state current $i(t)$ for this circuit:



Solution:

Represent the circuit in the frequency domain using impedances and phasors. Let

$$\mathbf{Z}_1 = 25 + j(20)2 + \frac{1}{j(20)(0.002)} = 25 + j15 = 29.2\angle 31°\ \Omega$$

$$\mathbf{Z}_2 = 20 + \frac{1}{j(20)(0.005)} = 20 - j10 = 22.36\angle -26.6°\ \Omega$$

$$\mathbf{Z}_3 = 40 + j(20)2 = 40 + j40 = 56.57\angle 45°\ \Omega$$

and let

$$\mathbf{Z}_p = \mathbf{Z}_2 \| \mathbf{Z}_3 = 18.86\angle -8° = 18.67 - j2.67\ \Omega$$

Then

$$\mathbf{I} = \frac{16\angle 75°}{\mathbf{Z}_1 + \mathbf{Z}_p} \times \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + \mathbf{Z}_3} = 0.118\angle 6.1°\ \text{A}$$

so

$$i(t) = 0.118\cos(20t + 6.1°)\ \text{A}$$

(checked: LNAP 8/2/04)

P 10.5-24 When the switch in the circuit shown in Figure P 10.5-24 is open and the circuit is at steady state, the capacitor voltage is

$$v(t) = 14.14 \cos(100t - 45°) \text{ V}$$

When the switch is closed and the circuit is at steady state, the capacitor voltage is

$$v(t) = 17.89 \cos(100t - 26.6°) \text{ V}$$

Determine the values of the resistances R_1 and R_2.

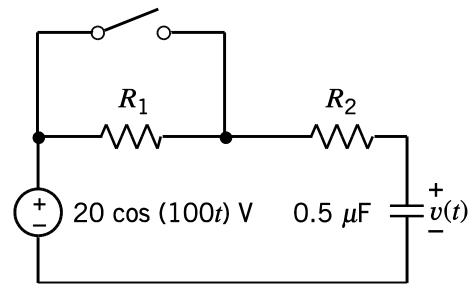


Figure P 10.5-24

Solution:

Represent the circuit in the frequency domain using phasors and impedances. The impedance capacitor is $\dfrac{1}{j(100)(0.5\times10^{-6})} = -j20{,}000$. When the switch is closed

$$17.89\angle -26.6° = \mathbf{V} = \frac{-j20{,}000}{R_2 - j20{,}000} \times 20\angle 0°$$

Equating angels gives

$$-26.6° = -90° - \tan^{-1}\left(\frac{-20{,}000}{R_2}\right) \quad \Rightarrow \quad R_2 = \frac{-20{,}000}{\tan(-63.4)} = 10015\ \Omega$$

When the switch is open

$$14.14\angle -45° = \mathbf{V} = \frac{-j20{,}000}{R_1 + R_2 - j20{,}000} \times 20\angle 0°$$

Equating angles gives

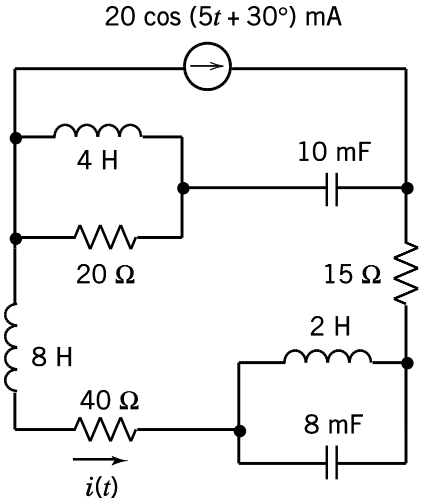
$$-45° = -90° - \tan^{-1}\left(\frac{-20{,}000}{R_1 + R_2}\right) \quad \Rightarrow \quad R_1 + R_2 = \frac{-20{,}000}{\tan(-45°)} = 20{,}000$$

So

$$R_1 = 20{,}000 - 10015 = 9985\ \Omega$$

(checked: LNAP 8/2/04)

P10.5-25 Determine the steady state current $i(t)$ for this circuit:



Solution:

Represent the circuit in the frequency domain using phasors and impedances. Let

$$\mathbf{Z}_1 = \left(j20 \parallel 20 \right) + \frac{1}{j0.05} = 10 - j10 = 14.14\angle -45° \ \Omega$$

$$\mathbf{Z}_2 = j40 + 40 + \left(j10 \parallel \frac{1}{j0.04} \right) + 15 = 55 + j56.67 = 79\angle 46.3° \ \Omega$$

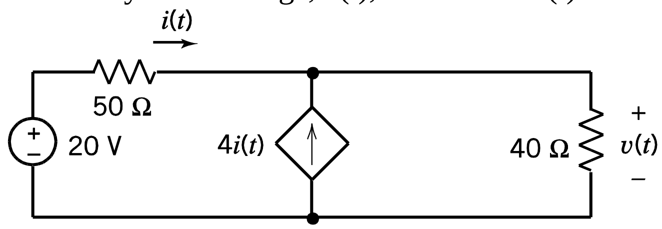
$$\mathbf{I} = -\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 20\angle 30° = 3.535\angle 129.3° \text{ mA}$$

so

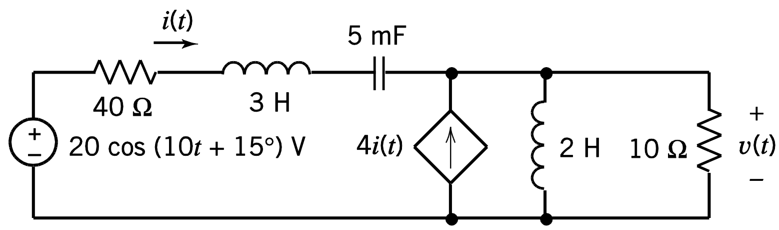
$$i(t) = 3.535 \cos\left(5t + 129.3°\right) \text{ mA}$$

(checked: LNAP 8/2/04)

P10.5-26 Determine the steady state voltage, $v(t)$, and current $i(t)$ for each of these circuits:



(a)



(b)

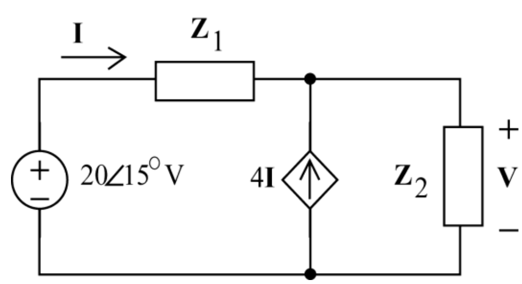
Solution:

(*a*) Using KCL and then KVL gives

$$20 = 50\,i(t) + 40\big(5\,i(t)\big) \quad \Rightarrow \quad i(t) = \frac{20}{250} = 80 \text{ mA}$$

Then
$$v(t) = 40\big(5i(t)\big) = 200(0.08) = 16 \text{ V}$$

(*b*) Represent the circuit in the frequency domain using phasors and impedances.



Where
$$\mathbf{Z}_1 = 40 + j(10)3 + \frac{1}{j(10)(0.005)} = 40 + j10 = 41.23\angle 26.6° \ \Omega$$

And
$$\mathbf{Z}_2 = j(10)2 \parallel 10 = 8 + j4 = 8.944\angle 26.6° \ \Omega$$

Using KCL and then KVL gives

$$20\angle 15° = \mathbf{Z}_1\mathbf{I} + 5\mathbf{Z}_2\mathbf{I} \qquad \Rightarrow \qquad \mathbf{I} = 0.234\angle -5.6° \text{ A}$$

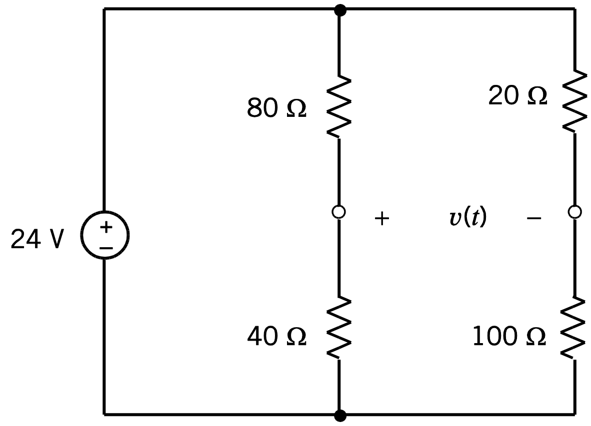
Then
$$\mathbf{V} = \mathbf{Z}_2(5\mathbf{I}) = 10.47\angle 21° \text{ A}$$

so
$$i(t) = 0.234\cos(10t - 5.6°) \text{ A}$$

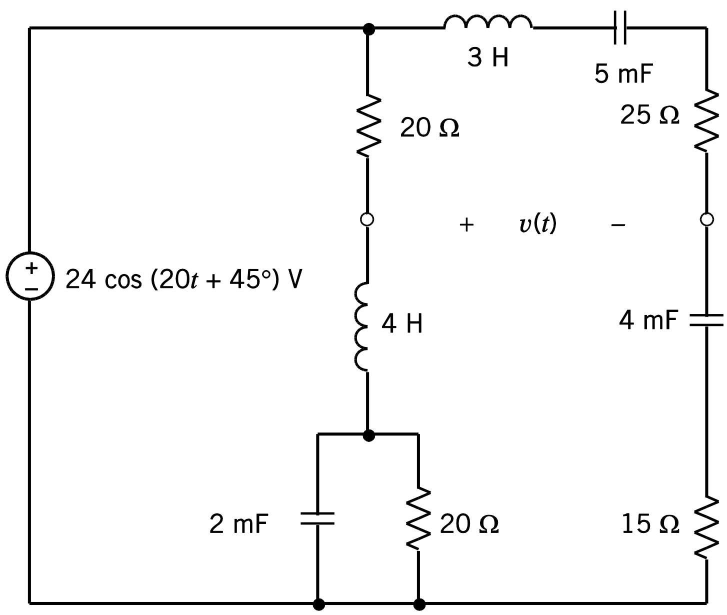
and
$$v(t) = 10.47\cos(10t + 21°) \text{ V}$$

(checked: 8/3/04)

P10.5-27 Determine the steady state voltage, $v(t)$, for each of these circuits:



(a)



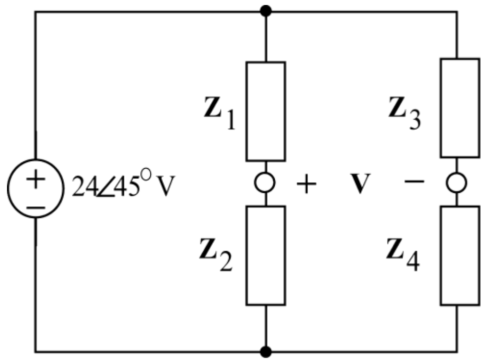
(b)

Solution:

(a) Using voltage division twice

$$v(t) = \frac{40}{40+80} \times 24 - \frac{100}{20+100} \times 24 = -12 \text{ V}$$

(b) Represent the circuit in the frequency domain using phasors and impedances.



Where

$$\mathbf{Z}_1 = 20\ \Omega$$

$$\mathbf{Z}_2 = j(20)4 + \left(\frac{1}{j(20)(0.002)} \parallel 20\right) = 12.2 + j70.2 = 71.30\angle 80.2°\ \Omega$$

$$\mathbf{Z}_3 = j(20)3 + \frac{1}{j(20)(0.005)} + 25 = 25 + j50 = 55.90\angle 63.4°\ \Omega$$

$$\mathbf{Z}_4 = \frac{1}{j(20)(0.004)} + 15 = 15 - j12.5 = 19.53\angle -39.8°\ \Omega$$

Using voltage division twice

$$\mathbf{V} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 24\angle 45° - \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} \times 24\angle 45° = 24.8\angle 80°\ \text{V}$$

so

$$v(t) = 24.8\cos(20t + 80°)\ \text{V}$$

(Checked using LNAP 10/5/04)

P 10.5-28 The input to the circuit shown in Figure P 10.5-28 is the voltage of the voltage source

$$v_s(t) = 5\cos(2t + 45°)\ \text{V}$$

The output is the inductor voltage, $v(t)$. Determine the steady-state output voltage.

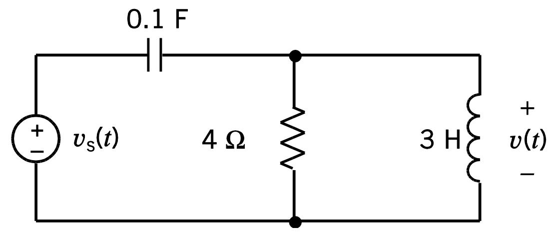
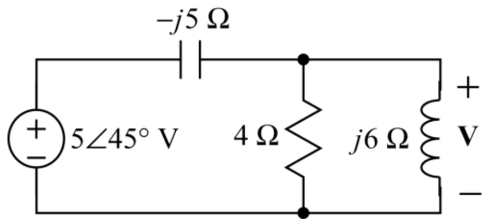


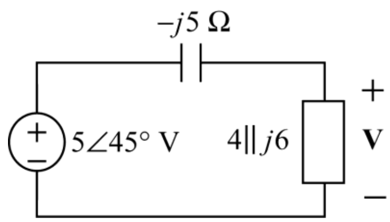
Figure P 10.5-28

Solution:

Represent the circuit in the frequency domain using phasors and impedances.



$$4 \parallel j6 = \frac{4(j6)}{4 + j6} = \frac{24\angle 90°}{7.2\angle 56°} = 3.33\angle 34° = 2.76 + j1.86\ \Omega$$



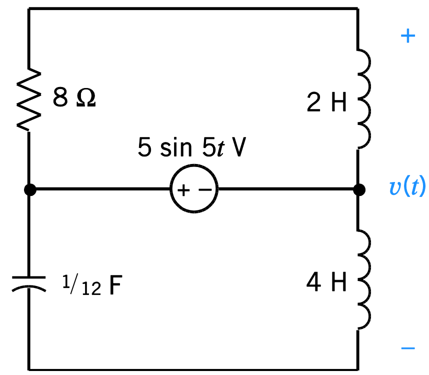
Using voltage division

$$\mathbf{V} = \frac{3.33\angle 34°}{-j5 + 2.76 + j1.86} \times 5\angle 45° = \frac{3.33\angle 34°}{2.76 - j3.14} \times 5\angle 45° = \frac{3.33\angle 34°}{4.18\angle -48°} \times 5\angle 45° = 3.98\angle 127°\ \text{V}$$

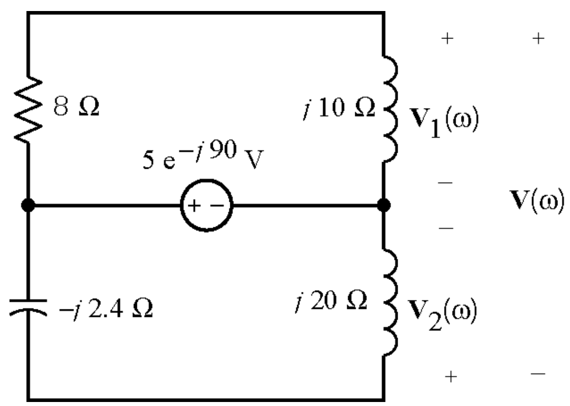
The corresponding voltage in the time domain is

$$v(t) = 3.98\cos(2t + 127°)\ \text{V}$$

P10.5-29 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:



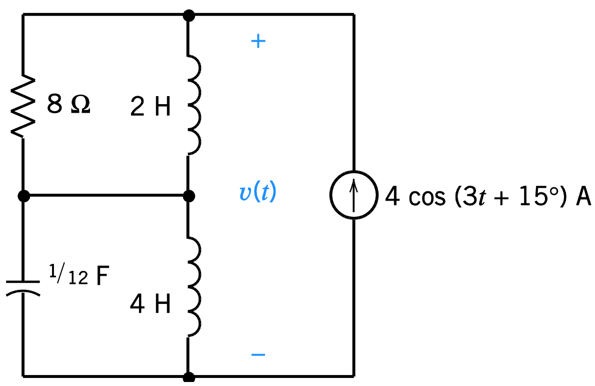
$$\mathbf{V}_1(\omega) = \frac{j10}{8+j10} 5e^{-j90} = 3.9e^{-j51} \text{ V}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20-j2.4} 5e^{-j90} = 5.68e^{-j90} \text{ V}$$

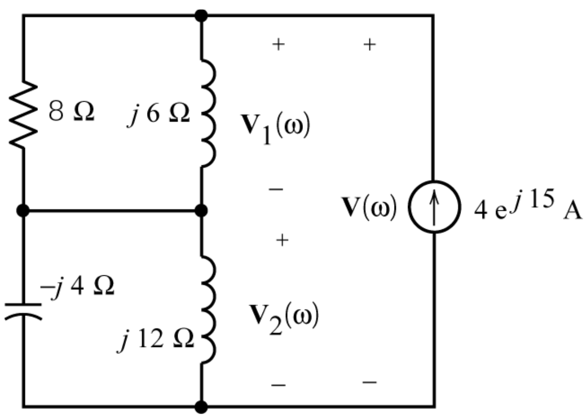
$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9e^{-j51} - 5.68e^{-j90}$$
$$= 3.58e^{j47} \text{ V}$$

Answer: $v(t) = 3.58\cos(5t + 47.2°)$ V

P10.5-30 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:



$$\mathbf{V}_1(\omega)=\frac{8(j6)}{8+j6}4e^{j15}=19.2e^{j68}\text{ V}$$

$$\mathbf{V}_2(\omega)=\frac{j12(-j4)}{j12-j4}4e^{j15}=24e^{-j75}\text{ V}$$

$$\mathbf{V}(\omega)=\mathbf{V}_1(\omega)+\mathbf{V}_2(\omega)=14.4e^{-j22}\text{ V}$$

Answer: $v(t)=14.4\cos(3t-22°)$ V

P10.5-31

The input to the circuit in Figure P10.5-31 is the voltage source voltage, $v_s(t)$. The output is the voltage $v_o(t)$. When the input is $v_s(t) = 8\cos(40t)$ V, the output is $v_o(t) = 2.5\cos(40t + 14°)$ V. Determine the values of the resistances R_1 and R_2.

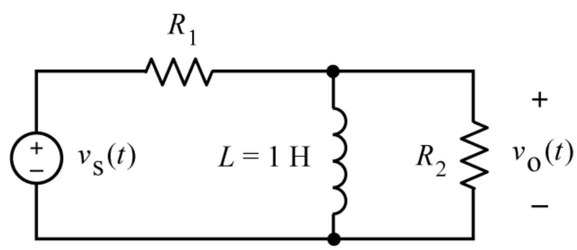
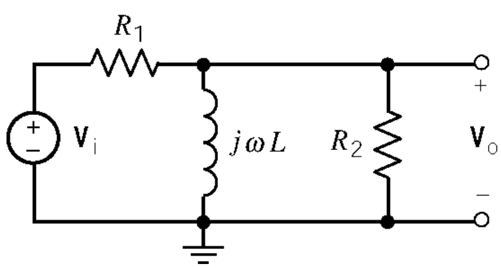


Figure P10.5-31

Solution:



Using voltage division in the frequency domain:

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

Representing the given input and output in the frequency domain:

$$\frac{2.5\angle 14°}{8\angle 0°} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}} e^{j\left(90 - \tan^{-1}\omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 14° so

$\dfrac{L}{R_p} = \dfrac{L(R_1 + R_2)}{R_1 R_2} = \dfrac{\tan(90° - 14°)}{40} = 0.1$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{8}$ so

$\dfrac{40\dfrac{L}{R_1}}{\sqrt{1+16}} = \dfrac{2.5}{8} \quad \Rightarrow \quad \dfrac{L}{R_1} = 0.0322$. One set of values that satisfies these two equations is $L = 1$ H, $R_1 = 31\,\Omega$, $R_2 = 14.76\,\Omega$.

Section 10.6 Mesh and Node Equations

P10.6-1 The input to the circuit shown in Figure P10.6-1 is the voltage

$$v_s(t) = 48\cos(2500t + 45°) \text{ V}$$

Write and solve node equations to determine the steady state output voltage $v_o(t)$.

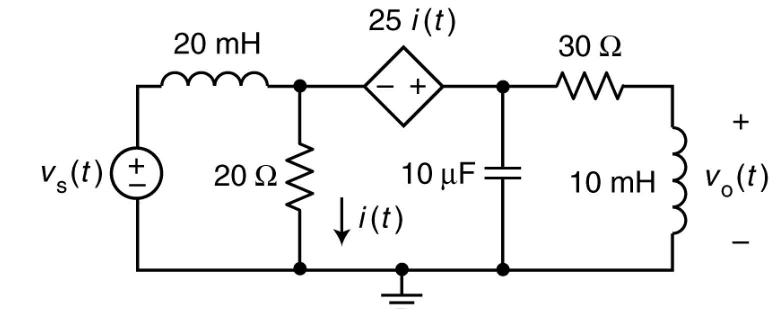
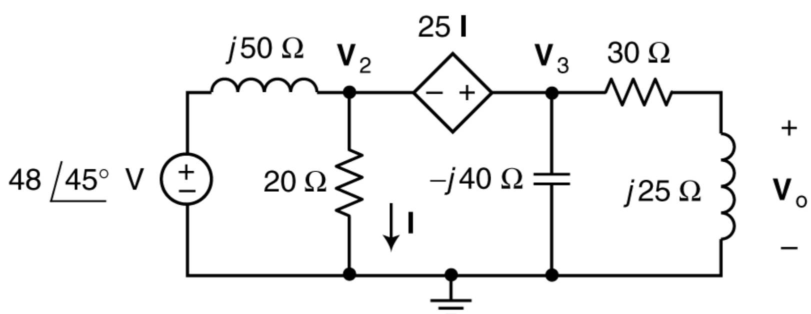


Figure P10.6-1

Solution: Represent the circuit in the frequency domain as



The node voltages are $48\angle 45° = \mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$ and $\mathbf{V}_o$. Express the dependent source voltage in terms of the node voltages:

$$\mathbf{V}_3 - \mathbf{V}_2 = 25\mathbf{I} = 25\left(\frac{\mathbf{V}_2}{20}\right) \quad \Rightarrow \quad \mathbf{V}_3 = 2.25\,\mathbf{V}_2$$

Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{48\angle 45° - \mathbf{V}_2}{j50} = \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \frac{\mathbf{V}_2}{j50} + \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \left(\frac{1}{j50} + \frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{-j40}\right)\mathbf{V}_3 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45^\circ}{j50} = \left(\frac{1}{j50}+\frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30}+\frac{1}{-j40}\right)2.25\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45^\circ}{j50} = \left(\frac{1}{j50}+\frac{1}{20}+\frac{2.25}{30}+\frac{2.25}{-j40}\right)\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

Apply KCL at the right node of the 30 Ω resistor to get

$$\frac{\mathbf{V}_3-\mathbf{V}_o}{30} = \frac{\mathbf{V}_o}{j25} \quad \Rightarrow \quad 0 = \left(-\frac{1}{30}\right)2.25\,\mathbf{V}_2 + \left(\frac{1}{30}+\frac{1}{j25}\right)\mathbf{V}_o$$

In matrix form

$$\begin{bmatrix} \frac{1}{j50}+\frac{1}{20}+\frac{2.25}{30}+\frac{2.25}{-j40} & -\frac{1}{30} \\ -\frac{2.25}{30} & \frac{1}{30}+\frac{1}{j25} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} \frac{48\angle 45^\circ}{j50} \\ 0 \end{bmatrix}$$

Solving, perhaps using MATLAB,

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} 10.18\angle -44.6^\circ \\ 14.67\angle 5.6^\circ \end{bmatrix} \text{ V}$$

P10.6-2 Figure P10.6-2 shows an ac circuit represented in the frequency domain. Determine the values of the phasor node voltages, $\mathbf{V}_b$ and $\mathbf{V}_c$.

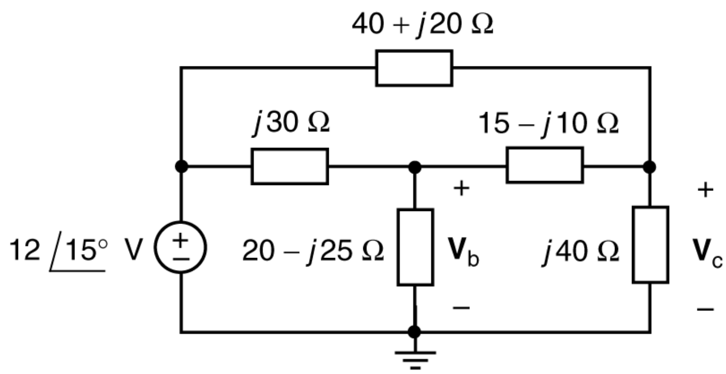


Figure P10.6-2

Solution:

Writing Node equations:

$$\frac{12\angle 45° - \mathbf{V}_b}{j30} = \frac{\mathbf{V}_b}{20 - j25} + \frac{\mathbf{V}_b - \mathbf{V}_c}{15 - j30}$$

$$\frac{\mathbf{V}_b - \mathbf{V}_c}{15 - j30} + \frac{12\angle 45° - \mathbf{V}_c}{40 + j20} = \frac{\mathbf{V}_c}{j40}$$

Rearranging:

$$\frac{12\angle 45°}{j30} = \left(\frac{1}{j30} + \frac{1}{20 - j25} + \frac{1}{15 - j30}\right)\mathbf{V}_b - \left(\frac{1}{15 - j30}\right)\mathbf{V}_c$$

$$\frac{12\angle 45°}{40 + j20} = -\left(\frac{1}{15 - j30}\right)\mathbf{V}_b + \left(\frac{1}{15 - j30} + \frac{1}{40 + j20} + \frac{1}{j40}\right)\mathbf{V}_c$$

In matrix from:

$$\begin{bmatrix} \frac{1}{j30} + \frac{1}{20 - j25} + \frac{1}{15 - j30} & -\frac{1}{15 - j30} \\ -\frac{1}{15 - j30} & \frac{1}{15 - j30} + \frac{1}{40 + j20} + \frac{1}{j40} \end{bmatrix} \begin{bmatrix} \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} \frac{12\angle 45°}{j30} \\ \frac{12\angle 45°}{40 + j20} \end{bmatrix}$$

Solving using MATLAB:

$$\mathbf{V}_b = 7.69\angle -19.8° \text{ and } \mathbf{V}_c = 10.18\angle 7.7° \text{ V}$$

Checked using LNAPAC

P10.6-3 Figure P10.6-3 shows an ac circuit represented in the frequency domain. Determine the value of the phasor node voltage **V**.

Answer: $\mathbf{V} = 71.0346\angle -39.627°\ \text{V}$

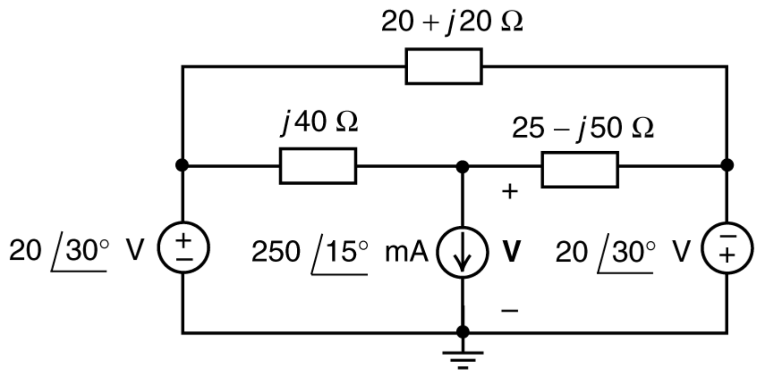


Figure P10.6-3

Solution:

Writing a node equation: $$\frac{20\angle 30° - \mathbf{V}}{j\,40} + \frac{-20\angle 30° - \mathbf{V}}{25 - j\,50} = 0.25\angle 15°$$

Rearranging $$\left(\frac{1}{j\,40} + \frac{1}{25 - j\,50}\right)\mathbf{V} = \frac{20\angle 30°}{j\,40} + \frac{-20\angle 30°}{25 - j\,50} - 0.25\angle 15°$$

Solving $$\left(0.012042\angle -48.366°\right)\mathbf{V} = 0.85537\angle 87.993°$$

Finally $$\mathbf{V} = \frac{0.85537\angle 87.993°}{0.012042\angle -48.366°} = 71.0346\angle -39.627°\ \text{V}$$

P10.6-4 Figure P10.6-4 shows an ac circuit represented in the frequency domain. Determine the values of the phasor mesh currents.

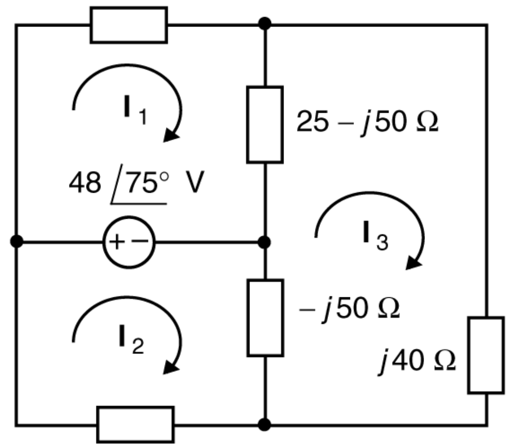


Figure P10.6-4

Solution:

Mesh 1:

$$(40+j15)\mathbf{I}_1+(25-j50)(\mathbf{I}_1-\mathbf{I}_3)-48\angle 75°=0$$
$$(65-j35)\mathbf{I}_1-(25-j50)\mathbf{I}_3=48\angle 75°$$

Mesh 2:

$$48\angle 75°+(-j50)(\mathbf{I}_2-\mathbf{I}_3)+(32+j16)\mathbf{I}_2=0$$
$$(32-j34)\mathbf{I}_2+j50\mathbf{I}_3=-48\angle 75°$$

Mesh 3:

$$j40\mathbf{I}_3-(-j50)(\mathbf{I}_2-\mathbf{I}_3)-(25-j50)(\mathbf{I}_1-\mathbf{I}_3)=0$$
$$(-25+j50)\mathbf{I}_1+j50\mathbf{I}_2+(25-j160)\mathbf{I}_3=0$$

In matrix form:

$$\begin{bmatrix} 65-j35 & 0 & -25+j50 \\ 0 & 32-j34 & +j50 \\ -25+j50 & +j50 & 25-j60 \end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\\ \mathbf{I}_3\end{bmatrix}=\begin{bmatrix}48\angle 75°\\ -48\angle 75°\\ 0\end{bmatrix}$$

Solving using MATLAB:

$$\mathbf{I}_1=0.794\angle 111°,\ \ \mathbf{I}_2=0.790\angle -61.7° \text{ and } \mathbf{I}_3=0.229\angle 176° \text{ A}$$

P 10.6-5 A commercial airliner has sensing devices to indicate to the cockpit crew that each door and baggage hatch is closed. A device called a search coil magnetometer, also known as a proximity sensor, provides a signal indicative of the proximity of metal or other conducting material to an inductive sense coil. The inductance of the sense coil changes as the metal gets closer to

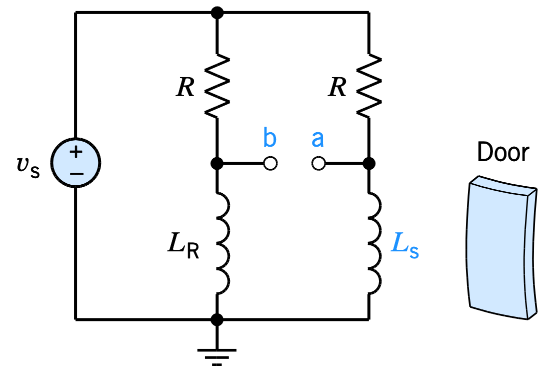


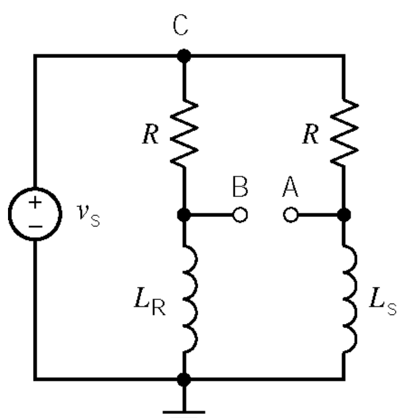
Figure P 10.6-5

the sense coil. The sense coil inductance is compared to a reference coil inductance with a circuit called a balanced inductance bridge (see Figure P 10.6-5). In the inductance bridge, a signal indicative of proximity is observed between terminals a and b by subtracting the voltage at b, v_b, from the voltage at a, v_a (Lenz, 1990).

The bridge circuit is excited by a sinusoidal voltage source $v_s = \sin(800\,\pi t)$ V. The two resistors, $R = 100\ \omega$, are of equal resistance. When the door is open (no metal is present), the sense coil inductance, L_S, is equal to the reference coil inductance, $L_R = 40$ mH. In this case, what is the magnitude of the signal $\mathbf{V}_a - \mathbf{V}_b$?

When the airliner door is completely closed, $L_S = 60$ mH. With the door closed, what is the phasor representation of the signal $\mathbf{V}_a - \mathbf{V}_b$?

Solution:



$$v_s = \sin\ (2\pi \cdot 400\,t)\ \ \text{V}$$

$$R = 100\ \Omega$$

$$L_R = 40\ \text{mH}$$

$$L_S = \begin{cases} 40\ \text{mH} & \text{door opened} \\ 60\ \text{mH} & \text{door closed} \end{cases}$$

With the door open $\left|\mathbf{V}_A - \mathbf{V}_B\right| = 0$ since the bridge circuit is balanced.

With the door closed $\mathbf{Z}_{L_R} = j(800\pi)(0.04) = j100.5\ \Omega$ and $\mathbf{Z}_{L_S} = j(800\pi)(0.06) = j150.8\ \Omega$.

The node equations are:

KCL at node B: $\dfrac{\mathbf{V}_\mathrm{B} - \mathbf{V}_\mathrm{C}}{R} + \dfrac{\mathbf{V}_\mathrm{B}}{\mathbf{Z}_{\mathrm{L_R}}} = 0 \quad \Rightarrow \mathbf{V}_\mathrm{B} = \dfrac{j100.5}{j100.5+100}\mathbf{V}_\mathrm{C}$

KCL at node A : $\dfrac{\mathbf{V}_\mathrm{A} - \mathbf{V}_\mathrm{C}}{R} + \dfrac{\mathbf{V}_\mathrm{A}}{\mathbf{Z}_{\mathrm{L_S}}} = 0$

Since $\mathbf{V}_\mathrm{C} = |\mathbf{V}_\mathrm{s}| = 1$ V $\quad \mathbf{V}_\mathrm{B} = 0.709\angle 44.86°$ V and $\mathbf{V}_\mathrm{A} = 0.833\angle 33.55$ V

Therefore

$$\mathbf{V}_\mathrm{A} - \mathbf{V}_\mathrm{B} = 0.833\angle 33.55° - 0.709\angle 44.86° = (0.694 + j.460) - (0.503 + j0.500) = 0.191 - j0.040$$
$$= 0.195\angle -11.83° \text{ V}$$

P 10.6-6 Using a tiny diamond-studded burr operating at 190,000 rpm, cardiologists can remove life-threatening plaque deposits is coronary arteries. The procedure is fast, uncomplicated, and relatively painless (McCarty, 1991). The Rotablator, an angioplasty system, consists of an advancer/catheter, a guide wire, a console, and a power source. The advancer/catheter contains a tiny turbine that drives the flexible shaft that rotates the catheter burr. The model of the operational and control circuit is shown in Figure P 10.6-6. Determine $v(t)$, the voltage that drives the tip, when $v_s = \sqrt{2}\ \cos(40t - 135°)$ V.

Answer: $v(t) = 1.414 \cos(40t + 135°)$ V

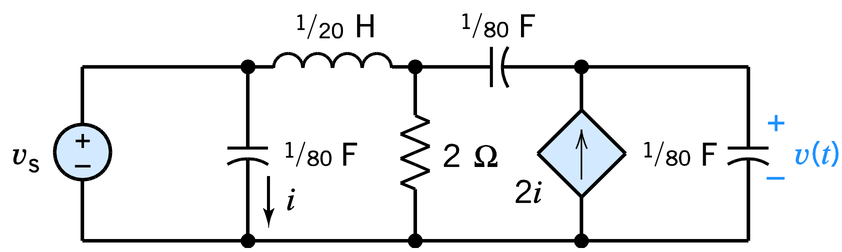
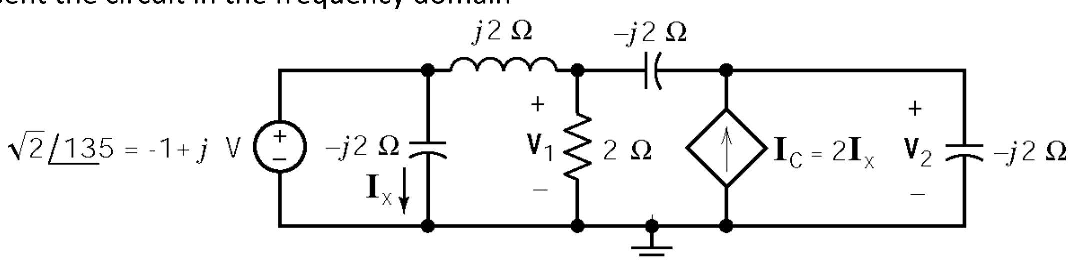


Figure P 10.6-6

Solution:

Represent the circuit in the frequency domain



The node equations are:

$$\frac{\mathbf{V}_1-(-1+j)}{j2}+\frac{\mathbf{V}_1}{2}+\frac{\mathbf{V}_1-\mathbf{V}_2}{-j2}=0$$

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{-j2}+\frac{\mathbf{V}_2}{-j2}-\mathbf{I}_C = 0$$

Also, expressing the controlling signal of the dependent source in terms of the node voltages yields

$$\mathbf{I}_x=\frac{-1+j}{-2j} \Rightarrow \mathbf{I}_C=2\mathbf{I}_x = 2\left[\frac{-1+j}{-2j}\right]=-1-j\ \text{A}$$

Solving these equations yields

$$\mathbf{V}_2=\frac{-3-j}{1+j2}=\sqrt{2}\angle -135°\ \text{V} \Rightarrow v(t)=v_2(t)=\sqrt{2}\ \cos\ (40t-135°)\ \text{V}$$

(checked: LNAP 7/19/04)

P 10.6-7 For the circuit of Figure P 10.6-7, it is known that

$v_2(t) = 0.7571 \cos(2t + 66.7°)$ V

$v_3(t) = 0.6064 \cos(2t - 69.8°)$ V

Determine $i_1(t)$.

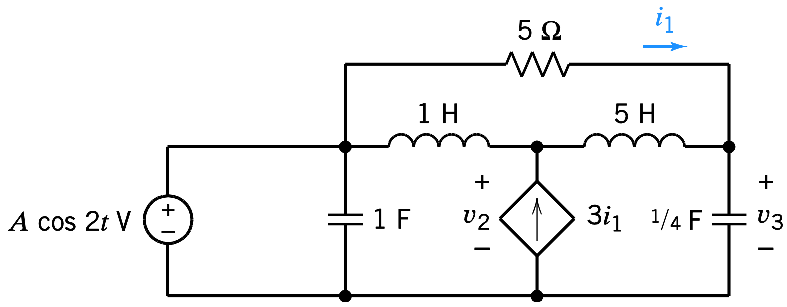
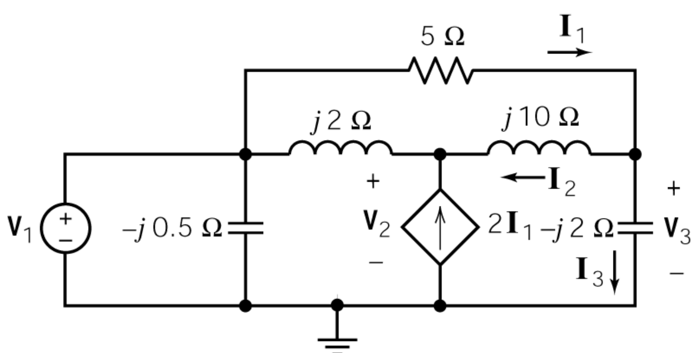


Figure P 10.6-7

Solution:



$$\mathbf{V}_2 = 0.7571\angle 66.7° \text{ V}$$
$$\mathbf{V}_3 = 0.6064\angle -69.8° \text{ V}$$

$$\left.\begin{array}{l} \mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3 \\ \mathbf{I}_2 = \dfrac{\mathbf{V}_3 - \mathbf{V}_2}{j10} \\ \mathbf{I}_3 = \dfrac{\mathbf{V}_3}{-j2} \end{array}\right\} \text{ yields } \left\{\begin{array}{l} \mathbf{I}_3 = 0.3032\angle 20.2° \text{ A} \\ \mathbf{I}_2 = 0.1267\angle -184° \text{ A} \\ \mathbf{I}_1 = 0.195\angle 36° \text{ A} \end{array}\right.$$

therefore

$$i_1(t) = 0.195\cos(2t + 36°) \text{ A}$$

(checked: MATLAB 7/18/04)

P10.6-8 The input to the circuit shown in Figure P10.6-8 is the voltage

$$v_s = 25\cos(40t + 45°)\ \text{V}$$

Determine the mesh currents i_1 and i_2 and the voltage v_o.

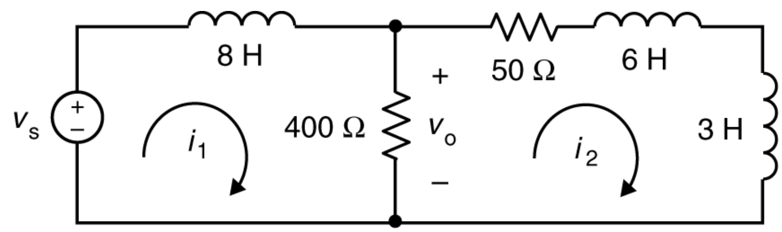
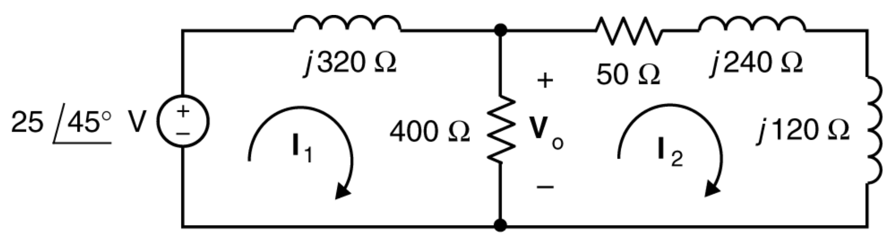


Figure P10.6-8

Solution

Represent the circuit in the frequency domain:



Apply KVL to mesh 1: $j320\,\mathbf{I}_1 + 400(\mathbf{I}_1 - \mathbf{I}_2) - 25\angle 45° = 0$

Apply KVL to mesh 2: $50\,\mathbf{I}_2 + j240\,\mathbf{I}_2 + j120\,\mathbf{I}_2 - 400(\mathbf{I}_1 - \mathbf{I}_2) = 0$

In matrix form:

$$\begin{bmatrix} 400 + j320 & -400 \\ -400 & 450 + j360 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 25\angle 45° \\ 0 \end{bmatrix}$$

Solving using MATLAB:

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 47.5\angle -24.6° \\ 33.0\angle -63.3° \end{bmatrix}\ \text{mA}$$

Using Ohm's Law $\mathbf{V}_o = 400(\mathbf{I}_1 - \mathbf{I}_2) = 12\angle 18.8°\ \text{V}$

In the time domain $i_1(t) = 47.5\cos(40t - 24.6°)\ \text{mA}$, $i_2(t) = 33\cos(40t - 63.3°)\ \text{mA}$

and $v_o(t) = 12\cos(40t + 18.8°)\ \text{V}$

P 10.6-10 The idea of using an induction coil in a lamp isn't new, but applying it in a commercially available product is. An induction coil in a bulb induces a high-frequency energy flow in mercury vapor to produce light. The lamp uses about the same amount of energy as a fluorescent bulb but lasts six times longer, with 60 times the life of a conventional incandescent bulb. The circuit model of the bulb and its associated circuit are shown in Figure P 10.6-10.

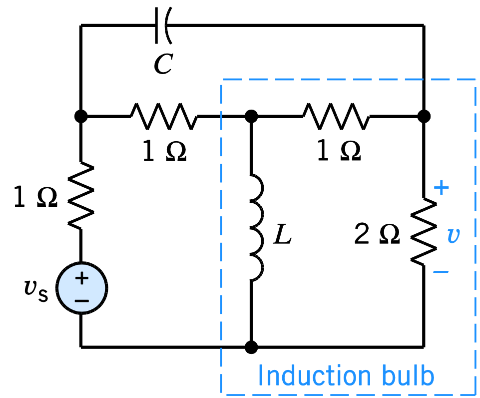


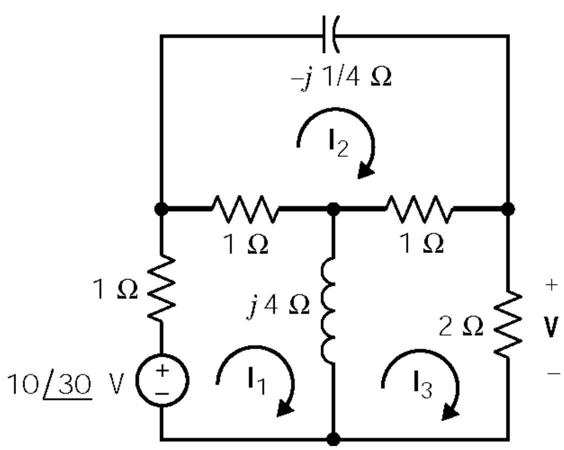
Figure P 10.6-10

Determine the voltage $v(t)$ across the 2-Ω resistor when $C = 40\ \mu F$, $L = 40\ \mu H$, $v_s = 10 \cos(\omega_0 t + 30°)$, and $\omega_0 = 10^5$ rad/s.

Answer: $v(t) = 6.45 \cos(10^5 t + 44°)$ V

Solution:

Represent the circuit in the frequency domain:



The mesh equations are:

$$\begin{bmatrix} (2+j4) & -1 & -j4 \\ -1 & (2+1/j4) & -1 \\ -j4 & -1 & (3+j4) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

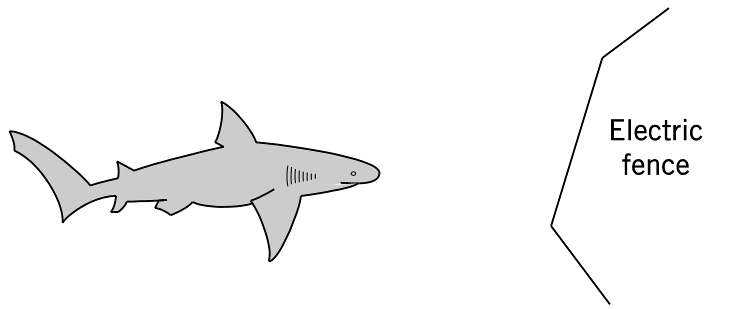
Using Cramer's rule yields

$$\mathbf{I}_3 = \frac{2+j8}{12+j\ 22.5}(10\angle 30°) = 3.225\angle 44°\ \text{A}$$

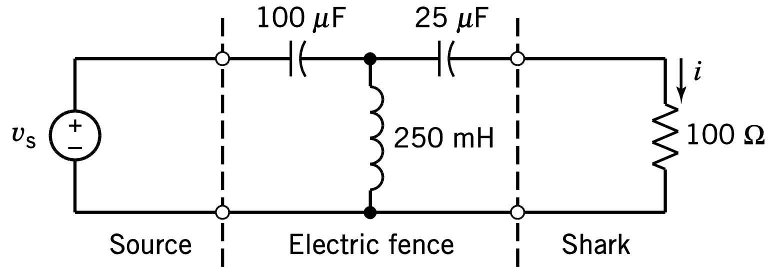
Then $\mathbf{V} = 2\ \mathbf{I}_3 = 2(3.225\angle 44°) = 6.45\angle 44°\ \text{V} \Rightarrow v(t) = 6.45\cos(10^5 t + 44°)\ \text{V}$

(checked: LNAP 7/19/04)

P 10.6-11 The development of coastal hotels in various parts of the world is a rapidly growing enterprise. The need for environmentally acceptable shark protection is manifest where these developments take place alongside shark-infested waters (Smith, 1991). One concept is to use an electrified line submerged in the water in order to deter the sharks, as shown in Figure P 10.6-11*a*. The circuit model of the electric fence is shown in Figure P 10.6-11*b*, where the shark is represented by an equivalent resistance of 100Ω. Determine the current flowing through the shark's body, $i(t)$, when $v_s = 375 \cos 400t$ V.



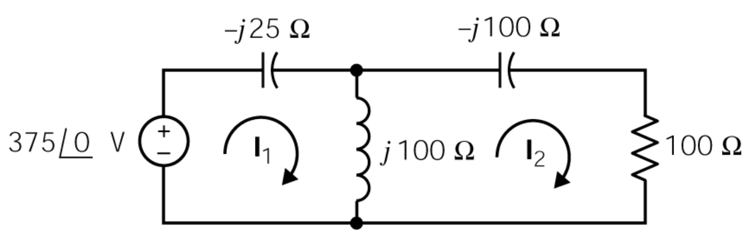
(a)



(b)

Figure P 10.6-11

Solution: Represent the circuit in the frequency domain:



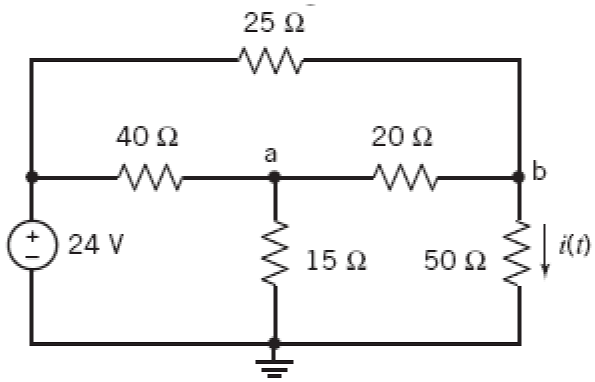
Mesh Equations:

$$j\,75\mathbf{I}_1 - j100\mathbf{I}_2 = 375$$
$$-j100\mathbf{I}_1 + (100 + j100)\mathbf{I}_2 = 0$$

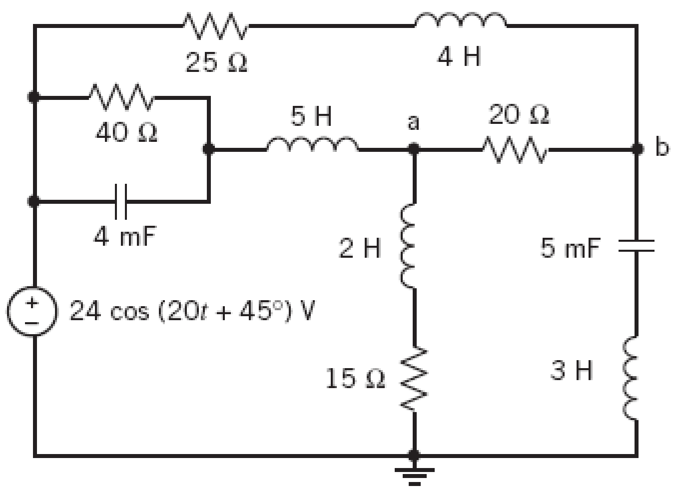
Solving for $\mathbf{I}_2$ yields $\mathbf{I}_2 = 4.5 + j1.5 = 3\angle 53.1°$ A $\Rightarrow$ $i_2(t) 3\cos(400t + 53.1°)$ A

(checked: LNAP 7/19/04)

P 10.6-12 Determine the node voltage at nodes a and b in each of these circuits:



(a)



(b)

Solution

(a)

The node equations are

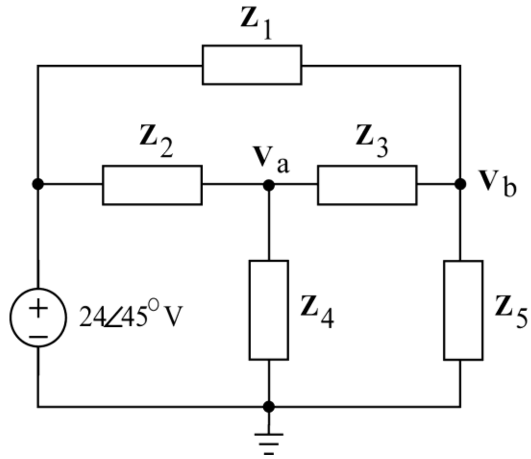
$$\frac{24-v_a}{40}=\frac{v_a-v_b}{20}+\frac{v_a}{15}$$
$$\frac{24-v_b}{25}+\frac{v_a-v_b}{20}=\frac{v_b}{50}$$

or

$$\begin{bmatrix} \frac{1}{40}+\frac{1}{20}+\frac{1}{15} & -\frac{1}{20} \\ -\frac{1}{20} & \frac{1}{25}+\frac{1}{20}+\frac{1}{50} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \end{bmatrix}=\begin{bmatrix} \frac{24}{40} \\ \frac{24}{25} \end{bmatrix}$$

Solving using MATLAB gives $v_a = 8.713$ V and $v_b = 12.69$ V

(b) Use phasors and impedances to represent the circuit in the frequency domain as



where

$$\mathbf{Z}_1 = 25 + j(20)4 = 25 + j80 = 83.82\angle 72.7°\ \Omega$$

$$\mathbf{Z}_2 = \left(40 \parallel \frac{1}{j(20)(0.004)}\right) + j(20)5 = 3.56 + j88.6 = 88.68\angle 87.7°\ \Omega$$

$$\mathbf{Z}_3 = 20\ \Omega$$

$$\mathbf{Z}_4 = 15 + j(20)2 = 15 + j40 = 42.72\angle 69.4°$$

$$\mathbf{Z}_5 = j(20)3 + \frac{1}{j(20)(0.005)} = j50 = 50\angle 90°\ \Omega$$

The node equations are

$$\frac{24\angle 45° - \mathbf{V}_a}{\mathbf{Z}_2} = \frac{\mathbf{V}_a}{\mathbf{Z}_4} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3}$$

$$\frac{24\angle 45° - \mathbf{V}_b}{\mathbf{Z}_1} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3} = \frac{\mathbf{V}_b}{\mathbf{Z}_5}$$

$$\begin{bmatrix} \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4} & -\frac{1}{\mathbf{Z}_3} \\ -\frac{1}{\mathbf{Z}_3} & \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{24\angle 45°}{\mathbf{Z}_2} \\ \frac{24\angle 45°}{\mathbf{Z}_1} \end{bmatrix}$$

Solving using MATLAB gives

$$\mathbf{V}_a = 7.89\angle 44.0°$$

$$\mathbf{V}_b = 8.45\angle 45.1°$$

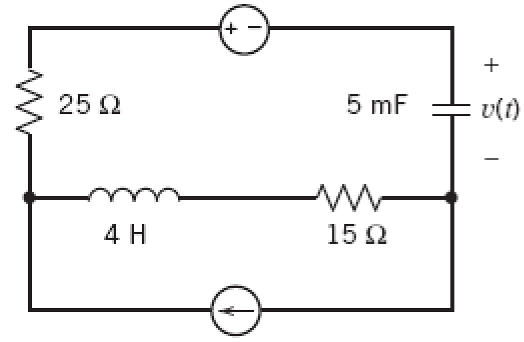
So

$$v_a(t) = 7.89\cos(20t + 44°)\ \text{V}$$

$$v_a(t) = 8.45\cos(20t + 45.1)\ \text{V}$$

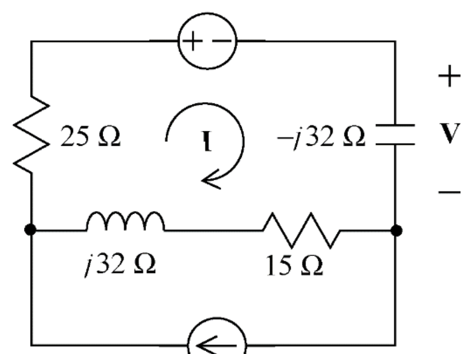
(checked: LNAP 8/3/04)

P 10.6-13 Determine the voltage *v*(*t*):



Solution:

Represent the circuit in the frequency domain using impedances and phasors



The mesh currents are **I** and $0.05\angle-30°\ \text{A}$. Apply KVL to the top mesh to get

$$15\angle 45° + (-j25)\mathbf{I} + (15 + j32)(\mathbf{I} - 0.05\angle -30°) + 25\mathbf{I} = 0$$

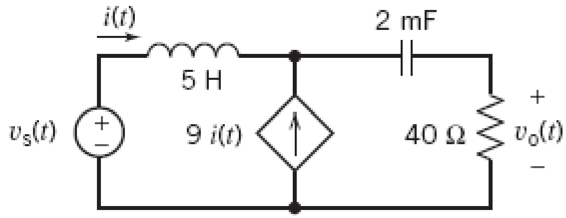
So $$\mathbf{I} = \frac{-15\angle 45° + (15 + j32)(0.05\angle -30°)}{25 - j25 + 15 + j32} = 0.3266\angle -143.6° = -0.2629 - j0.1939\ \text{A}$$

Then $$\mathbf{V} = (-j25)\mathbf{I} = 8.166\angle 126.4° = -4.8475 + j6.5715\ \text{V}$$

So $$v(t) = 8.166\cos(8t + 126.4°)\ \text{V}$$

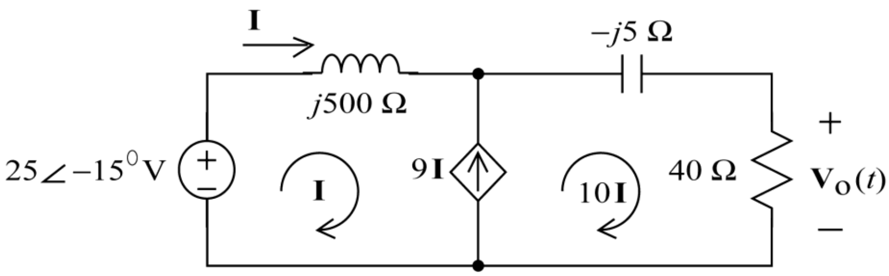
(checked: LNAP 8/3/04)

P 10.6-14 Determine the voltage $v_o(t)$ when $v_s(t)$ = 25 cos (100t-15°) V.



Solution:

Represent the circuit in the frequency domain using impedances and phasors.



The mesh currents are **I** and 10**I**. Apply KVL to the supermesh corresponding to the dependant current source to get

$$(j500)\mathbf{I}+(-j5)(10\mathbf{I})+40(10\mathbf{I})-25\angle-15°=0$$

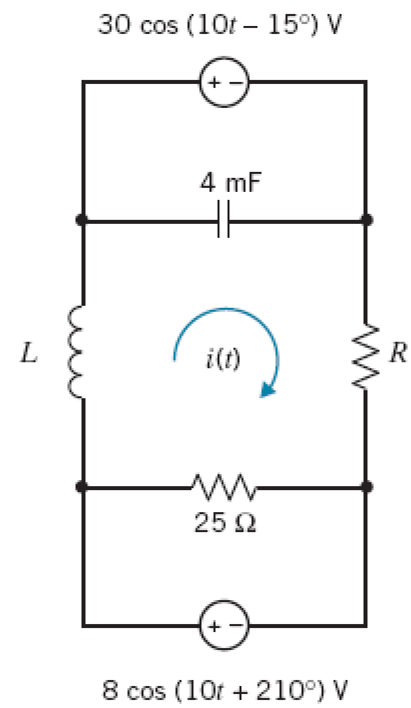
So
$$\mathbf{I}=\frac{25\angle-15°}{400+j450}=0.04152\angle-63.37°\text{ A}$$

The output voltage is
$$\mathbf{V}=40(10\mathbf{I})=16.61\angle-63.37°\text{ V}$$

So
$$v(t)=16.61\cos(100t-63.37°)\text{ V}$$

(checked: LNAP 8/3/04)

P 10.6-15 Determine the mesh current is when *i*(*t*) = 0.8394 cos (10*t*-138.5°) A. Determine the values of *L* and *R*.



Solution:

Represent the circuit in the frequency domain using phasors and impedances. Apply KVL to the center mesh to get

$$0.8394\angle 138.5° = \mathbf{I} = \frac{8\angle 210° - 30\angle -15°}{R + j10L} \quad \Rightarrow \quad R + j10L = 35 + j25 = 35 + j(10)2.5$$

So $$R = 35\ \Omega \quad \text{and} \quad L = 2.5\ \text{H}$$

(checked: LNAP 8/3/04)

P 10.6-16 The circuit shown in Figure P 10.6-16 has two inputs:

$$v_1(t) = 50 \cos (20t - 75°) \text{ V}$$

$$v_2(t) = 35 \cos (20t + 110°) \text{ V}$$

When the circuit is at steady state, the node voltage is

$$v(t) = 21.25 \cos (20t - 168.8°) \text{ V}$$

Determine the values of R and L.

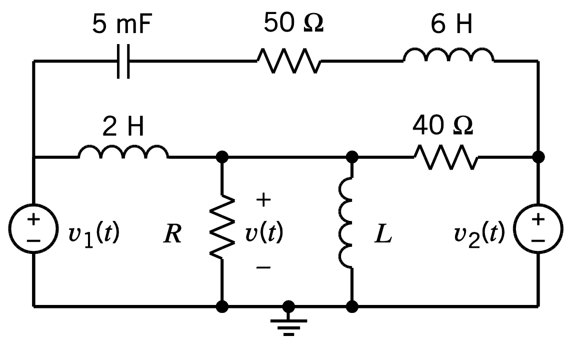


Figure P 10.6-16

Solution:

Represent the circuit in the frequency domain using phasors and impedances. Apply KCL at the top node of R and L to get

$$\frac{(50\angle -75°) - \mathbf{V}}{j40} + \frac{35\angle 100° - \mathbf{V}}{40} = \frac{\mathbf{V}}{R \parallel j\omega L}$$

$$\Rightarrow \quad \frac{50\angle -75°}{40\angle 90°} + \frac{35\angle 110°}{40} = \left(\frac{1}{j40} + \frac{1}{40} + \frac{1}{R} - j\frac{1}{20L} \right) \mathbf{V}$$

Using the given equation for $v(t)$ we get

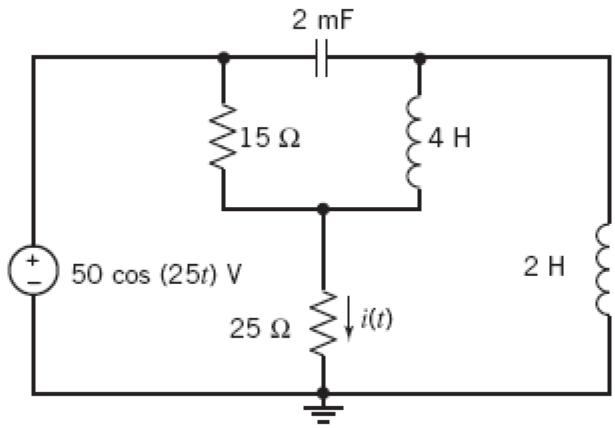
$$21.25\angle -168.8° = \mathbf{V} = \frac{1.587\angle 161.7°}{0.025(1-j) + \frac{1}{R} - j\frac{1}{20L}}$$

Then $$\frac{1}{R} - j\frac{1}{20L} = \frac{1.587\angle 161.7°}{21.25\angle -168.8°} - 0.025(1-j) = 0.04 - j0.01176$$

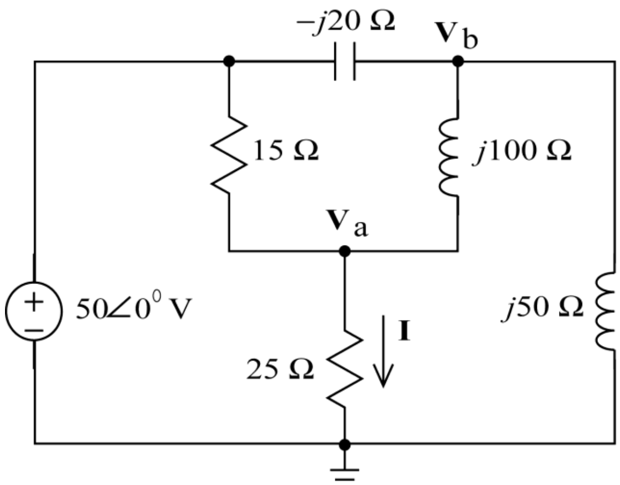
Finally $$R = \frac{1}{0.04} = 25\ \Omega \quad \text{and} \quad L = \frac{1}{20(0.01176)} = 4.25 \text{ H}$$

(checked: LNAP 8/3/04)

P 10.6-17 Determine the steady state current *i*(*t*):



Solution: Represent the circuit in the frequency domain using phasors and impedances.



The node equations are

$$\frac{50\angle 0° - \mathbf{V}_a}{15} + \frac{\mathbf{V}_b - \mathbf{V}_a}{j100} = \frac{\mathbf{V}_a}{25}$$

$$\frac{50\angle 0° - \mathbf{V}_b}{-j20} = \frac{\mathbf{V}_b - \mathbf{V}_a}{j100} + \frac{\mathbf{V}_b}{j50}$$

or

$$\begin{bmatrix} \frac{1}{15} + \frac{1}{j100} + \frac{1}{25} & -\frac{1}{j100} \\ -\frac{1}{j100} & \frac{1}{j50} + \frac{1}{j100} + \frac{1}{-j20} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{50\angle 0°}{15} \\ \frac{50\angle 0°}{-j20} \end{bmatrix}$$

$$\begin{bmatrix} 0.1067 - j0.010 & j0.010 \\ j0.010 & j0.020 \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} 3.333 \\ j2.5 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

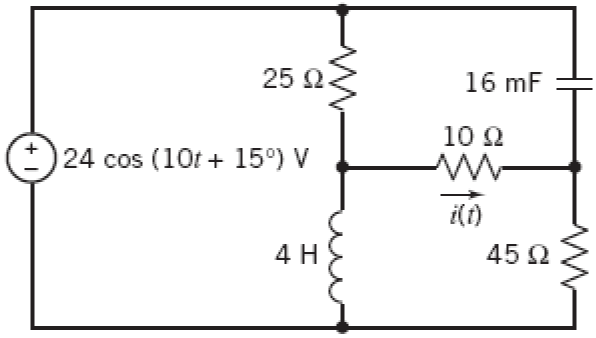
$$\mathbf{V}_a = 33.05\angle -12.6° \text{ V} \quad \text{and} \quad \mathbf{V}_b = 108.9\angle 1.9° \text{ V}$$

Then $$\mathbf{I} = \frac{\mathbf{V}_a}{25} = 1.322\angle -12.6° \text{ A}$$

So $$i(t) = 1.322 \cos(25t - 12.6°) \text{ A}$$

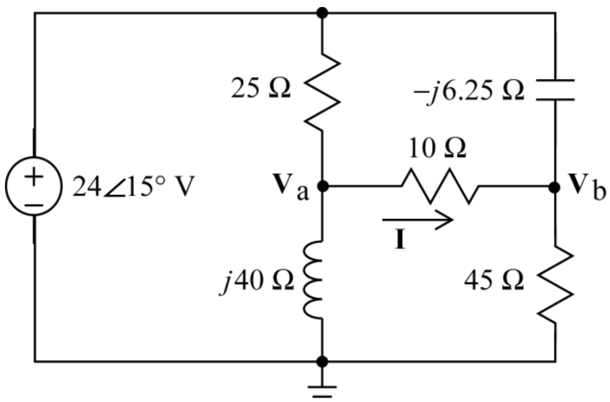
(checked: LNAP 8/3/04)

P 10.6-18 Determine the steady state current *i*(*t*):



Solution:

Represent the circuit in the frequency domain using phasors and impedances. Label the node voltages.



The node equations are

$$\frac{24\angle 15° - \mathbf{V}_a}{25} = \frac{\mathbf{V}_a}{j40} + \frac{\mathbf{V}_a - \mathbf{V}_b}{10}$$

$$\frac{24\angle 15° - \mathbf{V}_b}{-j6.25} + \frac{\mathbf{V}_a - \mathbf{V}_b}{10} = \frac{\mathbf{V}_b}{45}$$

or

$$\begin{bmatrix} \frac{1}{25} - j\frac{1}{40} + \frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & j\frac{1}{6.25} + \frac{1}{45} + \frac{1}{10} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{24\angle 15°}{25} \\ \frac{24\angle 15°}{6.25\angle -90°} \end{bmatrix}$$

$$\begin{bmatrix} 0.140 - j0.025 & -0.10 \\ -0.10 & 0.1222 + j0.160 \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} 0.960\angle 15° \\ 3.840\angle 105° \end{bmatrix}$$

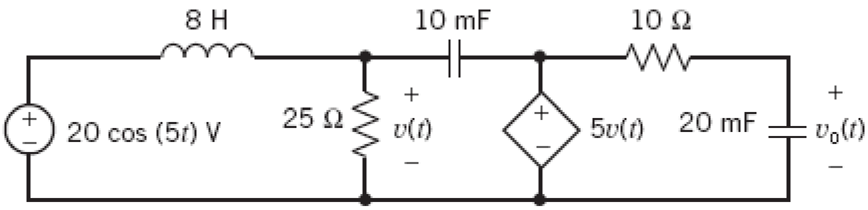
Solving gives $\mathbf{V}_a = 24.67\angle 32.6°\ \text{V}$ and $\mathbf{V}_b = 25.59\angle 25.2°\ \text{V}$

Then $$\mathbf{I} = \frac{\mathbf{V}_a - \mathbf{V}_b}{10} = 0.3347\angle 134.9°\ \text{A}$$

So $$i(t) = 0.3347\cos(10t + 134.9°)\ \text{A}$$

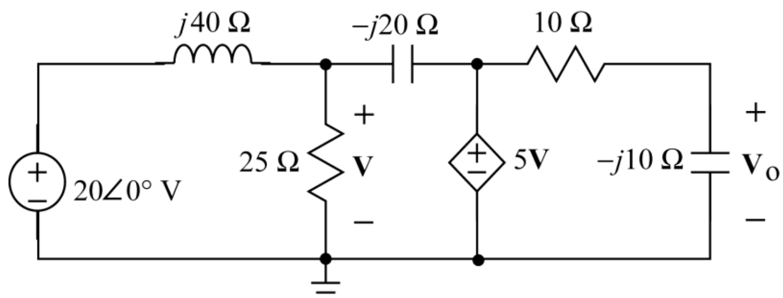
(checked: LANP 8/4/04)

P 10.6-19 Determine the steady state voltage $v_o(t)$:



Solution:

Represent the circuit in the frequency domain using phasors and impedances.



The node equations are

$$\frac{20\angle 0° - \mathbf{V}}{j40} = \frac{\mathbf{V}}{25} + \frac{\mathbf{V} - 5\mathbf{V}}{-j20}$$

$$\frac{5\mathbf{V} - \mathbf{V}_o}{10} = \frac{\mathbf{V}_o}{-j10}$$

$$\begin{bmatrix} \frac{1}{25} - j\frac{1}{5} - j\frac{1}{40} & 0 \\ -\frac{1}{2} & \frac{1}{10} + j\frac{1}{10} \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} -j0.5 \\ 0 \end{bmatrix}$$

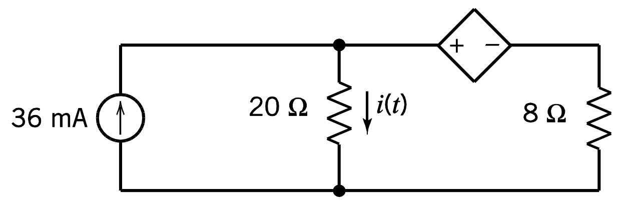
$$\begin{bmatrix} 0.04 - j0.225 & 0 \\ -0.50 & 0.10 + j0.10 \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} -j0.5 \\ 0 \end{bmatrix}$$

Solving gives $\mathbf{V} = 2.188\angle -10.1° \text{ V}$ and $\mathbf{V}_o = 7.736\angle -55.1° \text{ V}$

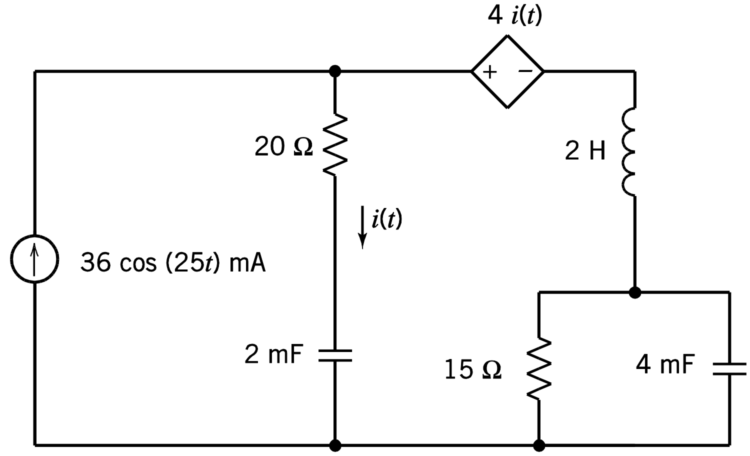
So $v_o(t) = 7.736\cos(5t - 55.1°) \text{ V}$

(checked: LNAP 8/4/04)

P 10.6-20 Determine the steady state current *i*(*t*) in each of these circuits:



(a)



(b)

Solution:

(a) Use KVL to see that the voltage across the 8 Ω resistor is $20i(t)-4i(t)=16i(t)$.

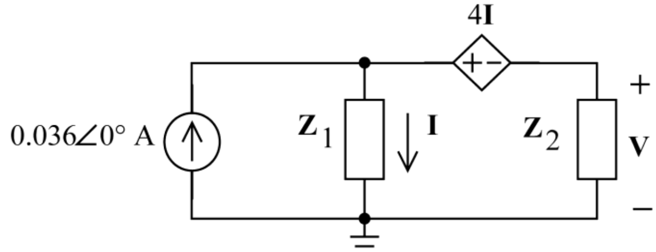
Apply KCL to the supernode corresponding to the dependent voltage source to get

$$0.036=i(t)+\frac{16i(t)}{8}=3i(t)$$

so

$$i(t)=12 \text{ mA}$$

(b) Represent the circuit in the frequency domain using phasors and impedances.



$$\mathbf{Z}_1 = 20 + \frac{1}{j(25)(0.002)} = 20 - j20\ \Omega$$

where

$$\mathbf{Z}_2 = j50 + \left(15 \parallel \frac{1}{j(25)(0.004)} \right) = 43.3\angle 83.9°\ \Omega$$

Use KVL to get $$\mathbf{V} = \mathbf{Z}_1\mathbf{I} - 4\mathbf{I} = (\mathbf{Z}_1 - 4)\mathbf{I}$$

Then apply KCL to the supernode corresponding to the dependent source to get

$$0.036\angle 0° = \mathbf{I} + \frac{(\mathbf{Z}_1 - 4)\mathbf{I}}{\mathbf{Z}_2} = \left(\frac{\mathbf{Z}_1 + \mathbf{Z}_2 - 4}{\mathbf{Z}_2} \right)\mathbf{I}$$

so $$\mathbf{I} = \frac{\mathbf{Z}_2(0.036\angle 0°)}{\mathbf{Z}_1 + \mathbf{Z}_2 - 4} = 50.4\angle 35.7°\ \text{mA}$$

so $$i(t) = 50.4\cos(25t + 35.7°)\ \text{mA}$$

(checked: LNAP 8/4/04)

10.6-21 The input to the circuit show in Figure 10.9-24 is the current

$$i_s(t) = 50\cos(200t) \text{ mA}$$

Determine the steady-state mesh current i_2.

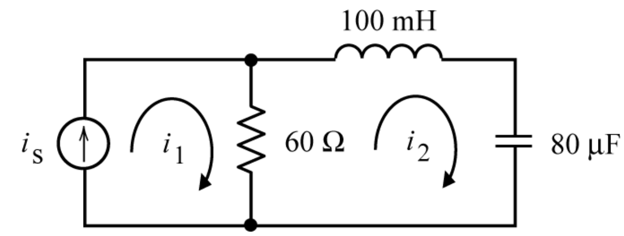
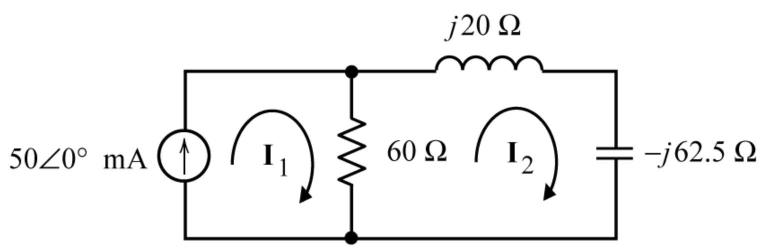


Figure P10.6-21

Solution:

Represent the circuit in the frequency domain using phasors and impedances:



Apply KVL to the right mesh to get:

$$(j20 - j62.5)\mathbf{I}_2 + 60(\mathbf{I}_2 - 0.050\angle 0°) = 0 \quad \Rightarrow \quad \mathbf{I}_2 = \frac{3\angle 0°}{60 - j42.5} = 0.0408\angle 35.3° \text{ A}$$

In the time domain

$$i_2(t) = 40.8\cos(200t + 35.3°) \text{ mA}$$

MATLAB, 11/20/09

10.6-22 The input to the circuit show in Figure 10.9-24 is the current

$$i_s(t) = 80\cos(250t) \text{ mA}$$

The steady-state mesh current in the right mesh is

$$i_s(t) = 66.56\cos(250t + 33.7°) \text{ mA}$$

Determine the value of the resistance *R*.

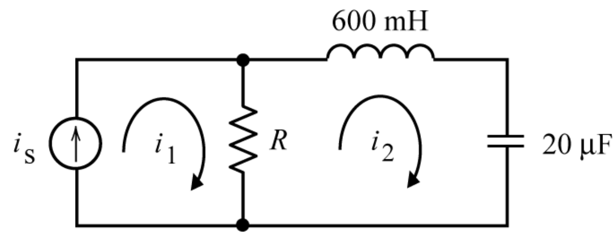


Figure P10.6-22

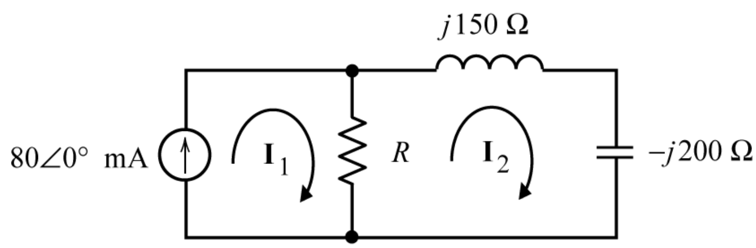
Solution:

Represent the circuit in the frequency domain using phasors and impedances. The mesh currents are

$$\mathbf{I}_1 = 0.080\angle 0° \text{ A}$$

and

$$\mathbf{I}_2 = 0.06656\angle 33.7° \text{ A}$$



Apply KVL to the right to get

$$(j150 - j200)(0.06656\angle 33.7°) + R(0.06656\angle 33.7° - 0.080\angle 0°) = 0$$

$$(-j50)(0.06656\angle 33.7°) + R(0.044376\angle 123.7°) = 0$$

$$R = \frac{(50\angle 90°)(0.06656\angle 33.7°)}{0.044376\angle 123.7°} = 74.9955 \cong 75\ \Omega$$

MATLAB, 11/20/09

P10.6-23

This circuit shown in Figure P10.6-23 is at steady state. The voltage source voltages are given by

$$v_1(t) = 12\cos(2t - 90°)\text{ V} \quad \text{and} \quad v_2(t) = 5\cos(2t + 90°)\text{ V}$$

The currents are given by

$$i_1(t) = 744\cos(2t - 118°)\text{ mA}\ ,\ \ i_2(t) = 540.5\cos(2t + 100°)\text{ mA}$$

Determine the values of R_1, R_2, L and C.

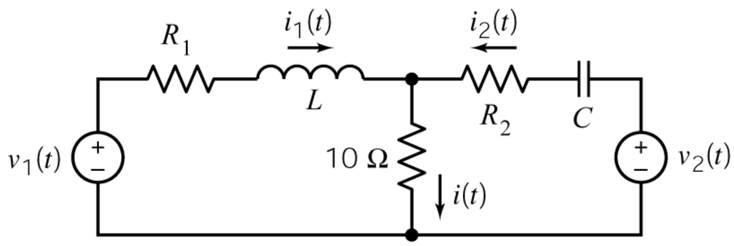
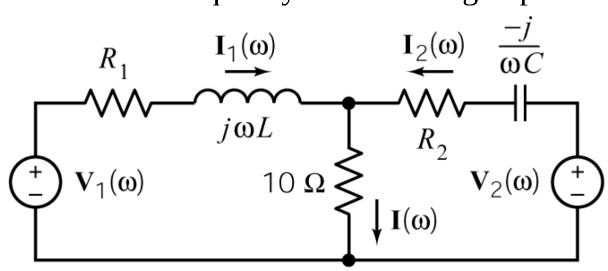


Figure P10.6-23

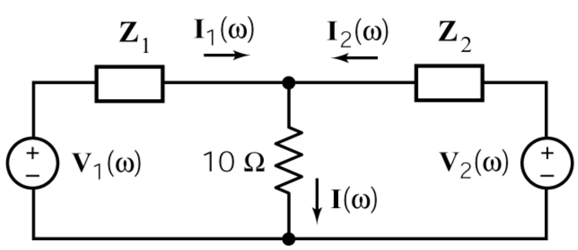
Solution: Represent the circuit in the frequency domain using impedances and phasors:



$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.744\angle -118° + 0.5405\angle 100 &= (-0.349 - j\,0.657) + (-0.094 + j0.532)\\ &= (-0.349 - 0.094) + j(-0.657 + 0.532)\\ &= -0.443 - j\,0.125\\ &= 0.460\angle -164°\end{aligned}$$

In the time domain $\qquad i(t) = 460\cos(2t - 164°)\ \text{mA}$

Replacing series impedances by equivalent impedances gives



and

$$\mathbf{Z}_1 = R_1 + j\,\omega\,L$$

$$\mathbf{Z}_2 = R_2 - j\frac{1}{\omega\,C}$$

From KVL

$$\mathbf{Z}_1\mathbf{I}_1 + 10\mathbf{I} - \mathbf{V}_1 = 0 \quad \Rightarrow \quad \mathbf{Z}_1 = \frac{\mathbf{V}_1 - 10\mathbf{I}}{\mathbf{I}_1} = \frac{12\angle -90° - 10\left(0.460\angle -164°\right)}{0.744\angle -118°}$$

$$= \frac{-j12 - 10\left(-0.443 - j\,0.125\right)}{0.744\angle -118°}$$

$$= \frac{4.43 - j10.75}{0.744\angle -118°} = \frac{11.63\angle -67.6°}{0.744\angle -118°}$$

$$= 15.63\angle 50.4°$$

$$= 10 + j12\ \Omega$$

and

$$-\mathbf{Z}_2\mathbf{I}_2 + \mathbf{V}_2 - 10\mathbf{I} = 0 \quad \Rightarrow \quad \mathbf{Z}_2 = \frac{\mathbf{V}_2 - 10\mathbf{I}}{\mathbf{I}_2} = \frac{5\angle 90° - 10\left(0.460\angle -164°\right)}{0.5405\angle 100°}$$

$$= \frac{j5 - 10\left(-0.443 - j\,0.125\right)}{0.5405\angle 100°}$$

$$= \frac{4.43 + j\,6.25}{0.5405\angle 100°} = \frac{7.66\angle 54.7°}{0.5405\angle 100°}$$

$$= 14.14\angle -55.3°$$

$$= 10 - j10\ \Omega$$

Next $$10 + j12 = R_1 + j\,\omega\,L = R_1 + j\,2\,L \quad \Rightarrow \quad R_1 = 10\ \Omega \quad \text{and} \quad L = \frac{12}{2} = 6\ \text{H}$$

and $$10 - j10 = R_2 - j\frac{1}{\omega\,C} = R_2 - j\frac{1}{2\,C} \quad \Rightarrow \quad R_2 = 10\ \Omega \quad \text{and} \quad C = \frac{1}{2\left(10\right)} = 0.05\ \text{F}$$

10.7 Thevenin and Norton Equivalent Circuits

P10.7-1 Determine the Thevenin equivalent circuit of the circuit shown in Figure P10.7-1 when (*a*) ω = 1000 rad/s, (*b*) ω = 2000 rad/s and (*c*) ω = 4000 rad/s.

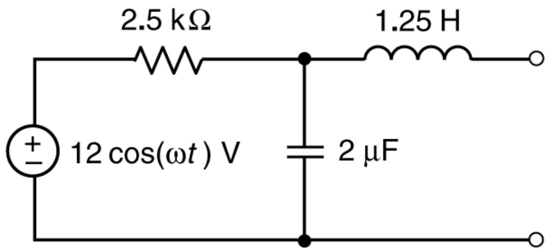
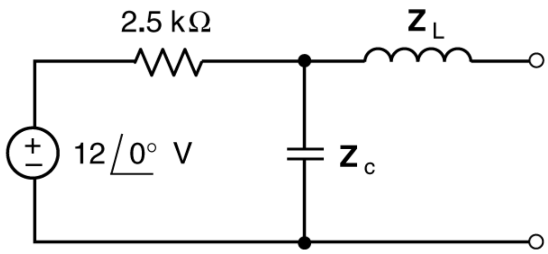
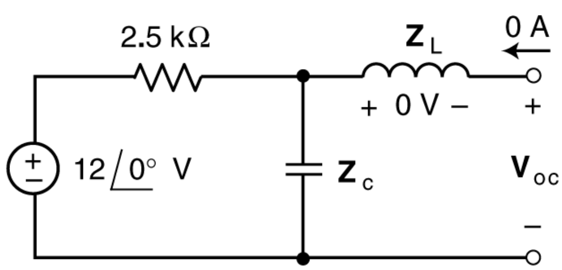


Figure P10.7-1

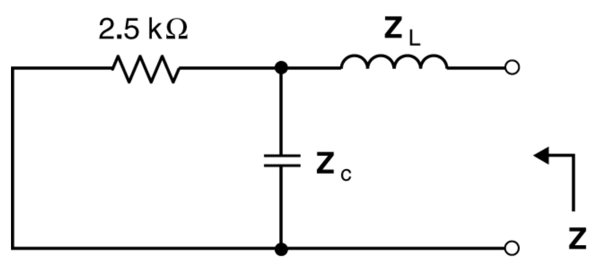
Solution: Represent the circuit in the frequency domain as



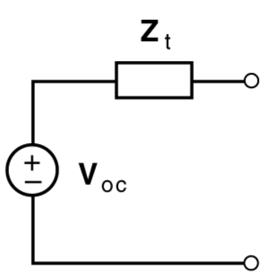
Determine the open circuit voltage and Thevenin impedance:



$$\mathbf{V}_{oc} = \frac{\mathbf{Z}_C}{2500+\mathbf{Z}_C}(12\angle 0°)$$



$$\mathbf{Z}_t = \frac{2500\,\mathbf{Z}_C}{2500+\mathbf{Z}_C}+\mathbf{Z}_L$$



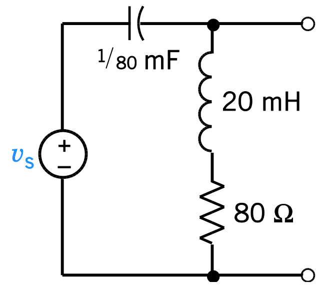
a. $\mathbf{V}_{oc} = 2.3534\angle -78.69°$ V and $\mathbf{Z}_t = 775.22\angle 82.875°\ \Omega$

b. $\mathbf{V}_{oc} = 1.194\angle -84.29°$ V and $\mathbf{Z}_t = 2252.6\angle 89.37°\ \Omega$

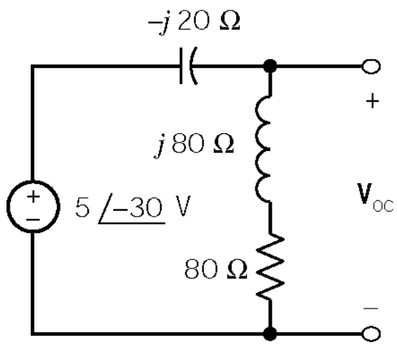
c. $\mathbf{V}_{oc} = 0.59925\angle -87.14°$ V and $\mathbf{Z}_t = 4875.3\angle 89.93°\ \Omega$

The Thevenin Equivalent Circuit changes whenever the input frequency changes.

P10.7-2 Determine the Thevenin equivalent of this circuit when $v_s(t)$ = 5 cos (4000t-30°) V.

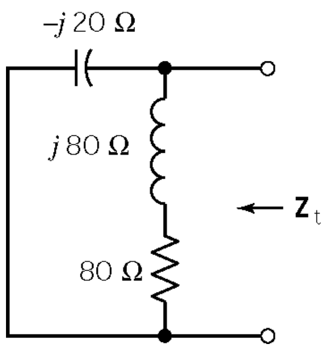


Solution:



Find $\mathbf{V}_{oc}$:

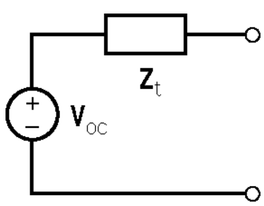
$$\mathbf{V}_{oc} = (5\angle -30°)\left(\frac{80 + j80}{80 + j80 - j20}\right)$$
$$= (5\angle -30°)\left(\frac{80\sqrt{2}\angle -45°}{100\angle 36.90°}\right)$$
$$= 4\sqrt{2}\angle -21.9° \text{ V}$$



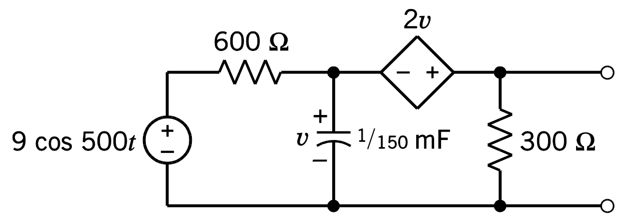
Find $\mathbf{Z}_t$:

$$\mathbf{Z}_t = \frac{(-j20)(80 + j80)}{-j20 + 80 + j80} = 23\angle -81.9° \ \Omega$$

The Thevenin equivalent is

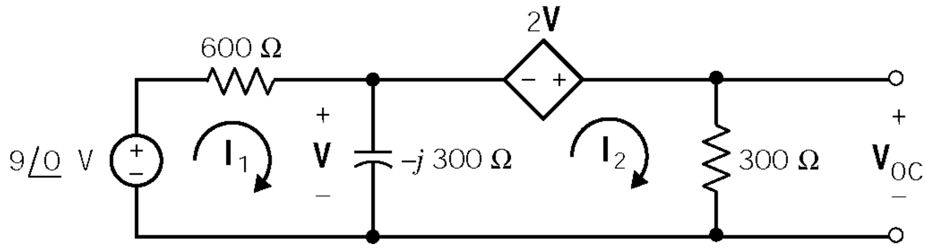


P10.7-3 Determine the Thevenin equivalent of this circuit



Solution:

First, determine $\mathbf{V}_{oc}$:



The mesh equations are

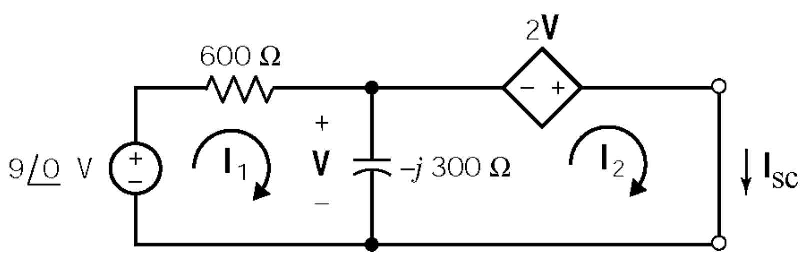
$$600\mathbf{I}_1 - j300(\mathbf{I}_1 - \mathbf{I}_2) = 9 \;\Rightarrow\; (600 - j300)\mathbf{I}_1 + j300\mathbf{I}_2 = 9\angle 0°$$

$$-2\mathbf{V} + 300\mathbf{I}_2 - j300(\mathbf{I}_1 - \mathbf{I}_2) = 0 \quad \text{and} \quad \mathbf{V} = j300(\mathbf{I}_1 - \mathbf{I}_2) \;\Rightarrow\; j3\mathbf{I}_1 + (1 - j3)\mathbf{I}_2 = 0$$

Using Cramer's rule: $\mathbf{I}_2 = 0.0124\angle -16°\text{ A}$

Then $\mathbf{V}_{oc} = 300\mathbf{I}_2 = 3.71\angle -16°\text{ V}$

Next, determine $\mathbf{I}_{sc}$:

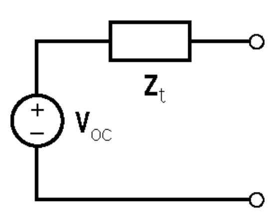


$$-2\mathbf{V} - \mathbf{V} = 0 \;\Rightarrow\; \mathbf{V} = 0 \;\Rightarrow\; \mathbf{I}_{sc} = \frac{9\angle 0°}{600} = 0.015\angle 0°\text{ A}$$

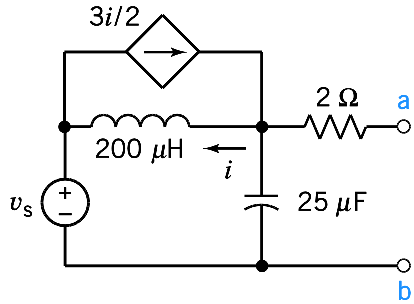
The Thevenin impedance is

$$\mathbf{Z}_T = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{3.545\angle -16°}{0.015\angle 0°} = 247\angle -16°\;\Omega$$

The Thevenin equivalent is



P10.7-4 Determine the Thevenin equivalent of this circuit when $v_s(t) = 10\cos(10{,}000t - 53.1°)$ V.



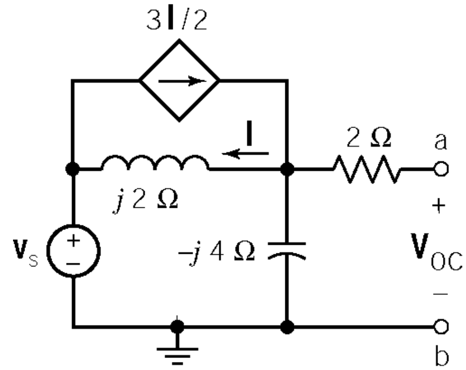
Solution:

First, determine $\mathbf{V}_{oc}$:

The node equation is:

$$\frac{\mathbf{V}_{oc}}{-j4} + \frac{\mathbf{V}_{oc} - (6+j8)}{j2} - \frac{3}{2}\left(\frac{\mathbf{V}_{oc} - (6+j8)}{j2}\right) = 0$$

$$\mathbf{V}_{oc} = 3 + j4 = 5\angle 53.1° \text{ V}$$



$$\mathbf{V}_s = 10\angle 53° = 6 + j8 \text{ V}$$

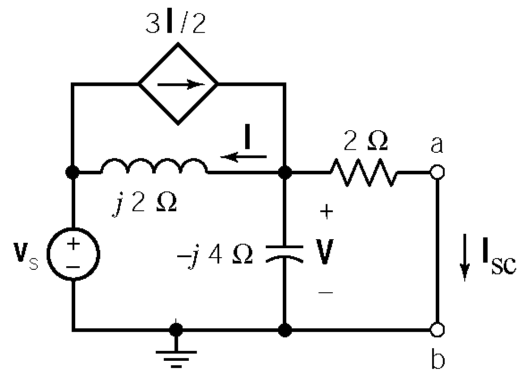
Next, determine $\mathbf{I}_{sc}$:

The node equation is:

$$\frac{\mathbf{V}}{2} + \frac{\mathbf{V}}{-j4} + \frac{\mathbf{V} - (6+j8)}{j2} - \frac{3}{2}\left[\frac{\mathbf{V} - (6+j8)}{j2}\right] = 0$$

$$\mathbf{V} = \frac{3+j4}{1-j}$$

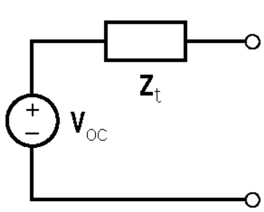
$$\mathbf{I}_{sc} = \frac{\mathbf{V}}{2} = \frac{3+j4}{2-j2}$$



$$\mathbf{V}_s = 10\angle 53° = 6 + j8 \text{ V}$$

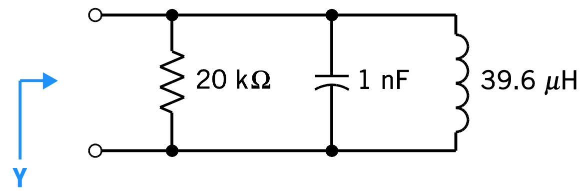
The Thevenin impedance is $\mathbf{Z}_T = \dfrac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = 3 + j4\left(\dfrac{2-j2}{3+j4}\right) = 2 - j2\ \Omega$

The Thevenin equivalent is



(checked: LNAP 7/18/04)

P10.7-5 Determine the frequency at which Y is a pure conductance



Solution:

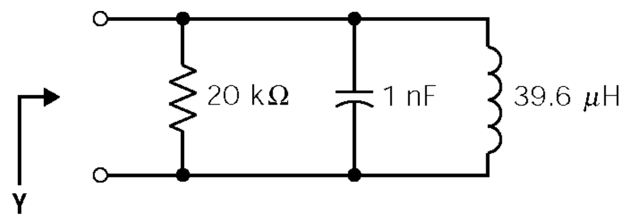
$$\mathbf{Y} = G + \mathbf{Y}_L + \mathbf{Y}_C$$

$$\mathbf{Y} = G \text{ when } \mathbf{Y}_L + \mathbf{Y}_C = 0 \text{ or } \frac{1}{j\omega L} + j\omega C = 0$$

$$\omega_O = \frac{1}{\sqrt{LC}},\ f_O = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{39.6\times 10^{-15}}}$$

$$= 0.07998\times 10^7 \text{ Hz=800 kHz}$$

(80 on the dial of the radio)



P 10.7-6 Consider the circuit of Figure P 10.7-6, where we wish to determine the current **I**. Use a series of source transformations to reduce the part of the circuit connected to the 2-Ω resistor to a Norton equivalent circuit, and then find the current in the 2-Ω resistor by current division.

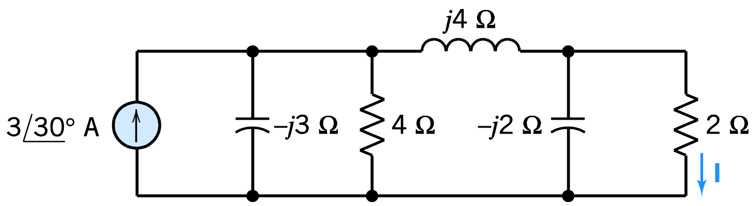
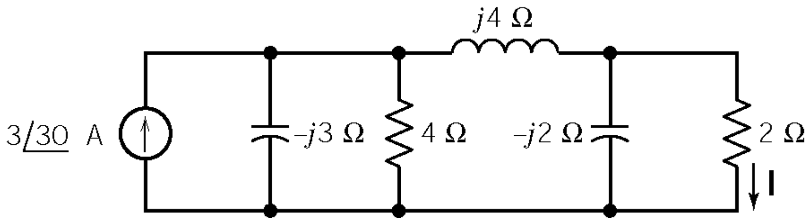
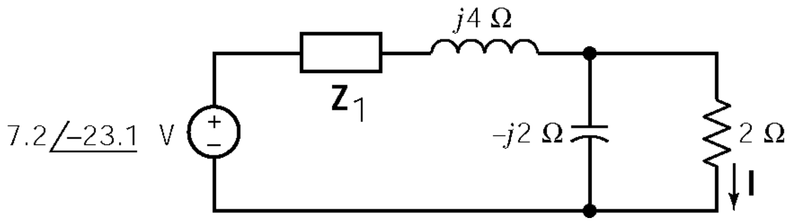


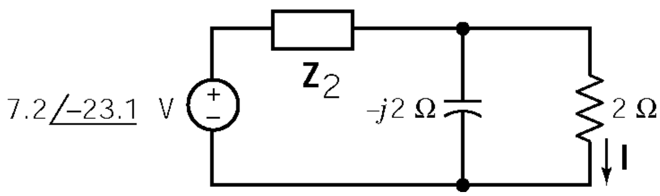
Figure P 10.7-6

Solution:

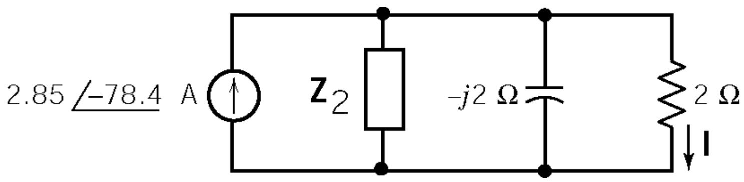




$$\mathbf{Z}_1 = \frac{(-j3)(4)}{-j3+4} = 2.4\angle -53.1°\ \Omega$$
$$= 1.44 - j1.92\ \Omega$$



$$\mathbf{Z}_2 = \mathbf{Z}_1 + j4$$
$$= 1.44 + j2.08$$
$$= 2.53\angle 55.3°\ \Omega$$



$$\mathbf{Z}_3 = 3.51\angle -37.9°\ \Omega$$
$$= 2.77 - j2.16\ \Omega$$

$$\mathbf{I} = (2.85\angle -78.4°)\left(\frac{3.51\angle -37.9°}{2.77 - j2.16 + 2}\right) = (2.85\angle -78.4°)\frac{(3.51\angle -37.9°)}{(5.24\angle -24.4°)} = 1.9\angle -92°\ \text{A}$$

(checked: LNAP 7/18/04)

P 10.7-7 For the circuit of Figure P 10.7-7, determine the current **I** using a series of source transformations.

The source has $\omega = 25 \times 10^3$ rad/s.

Answer: $i(t) = 4 \cos(25{,}000t - 44°)$ mA

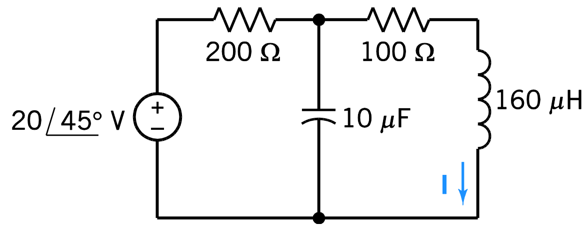
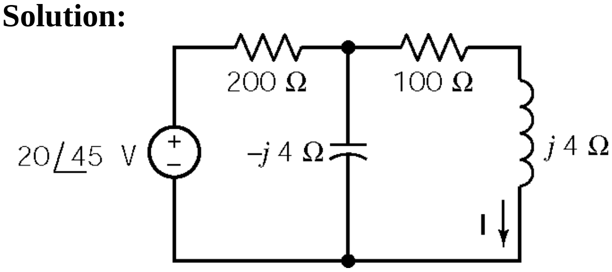
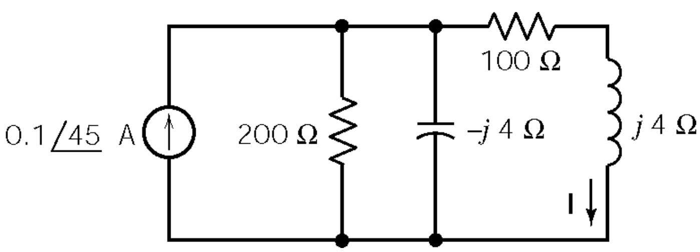
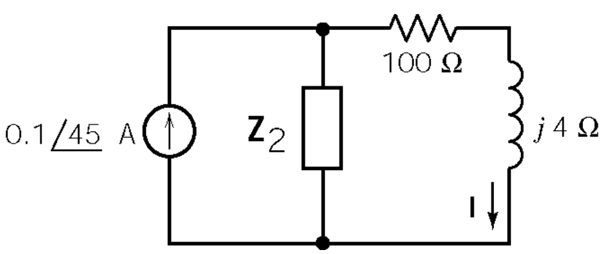


Figure P 10.7-7

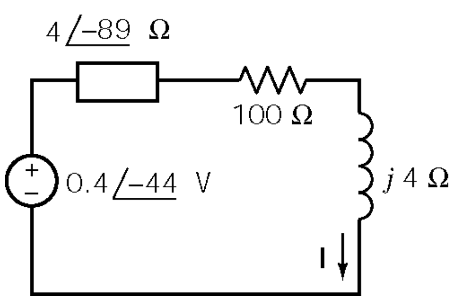
Solution:







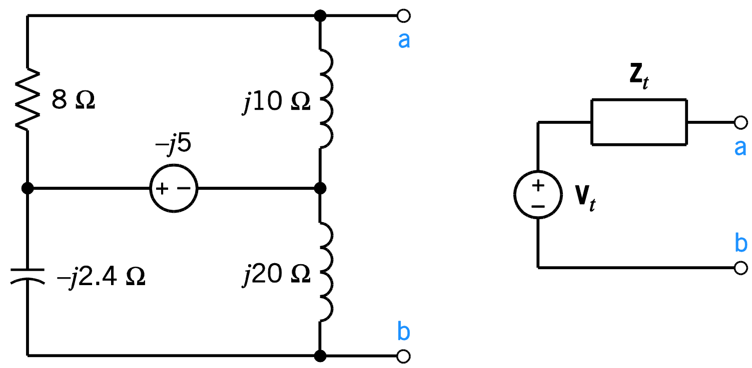
$$\mathbf{Z}_2 = \frac{(200)(-j4)}{200-j4} = 4\angle -88.8°\ \Omega$$



$$\mathbf{I} = \frac{0.4\angle -44°}{-4j+100+j4} = 4\angle -44°\ \text{ mA}$$

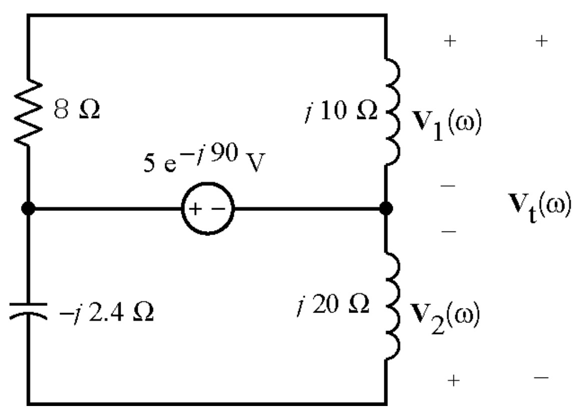
$$i(t) = 4\cos(25000t - 44°)\ \text{mA}$$

P10.7-8 Determine the Thevenin equivalent of the circuit in (*a*):



(*a*) (*b*)

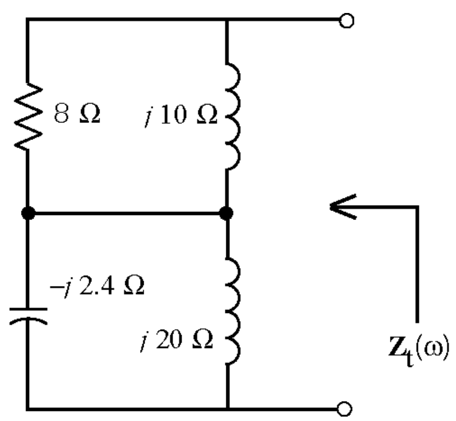
Solution:



$$\mathbf{V}_1 = \frac{j10}{8+j10} 5e^{-j90} = 3.9e^{-j51}$$

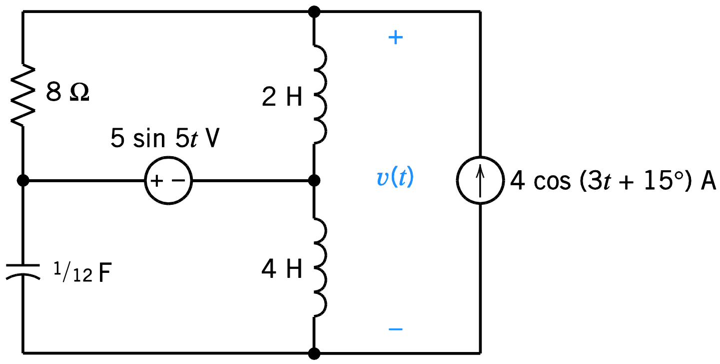
$$\mathbf{V}_2 = \frac{j20}{j20-j2.4} 5e^{-j90} = 5.68e^{-j90}$$

$$\mathbf{V}_t = \mathbf{V}_1 - \mathbf{V}_2 = 3.9e^{-j51} - 5.68e^{-j90}$$
$$= 3.58e^{j47}$$

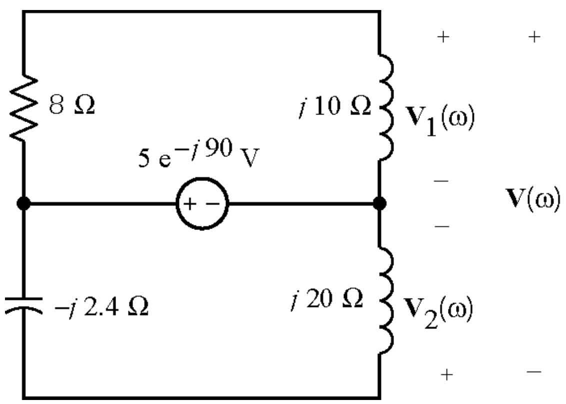


$$\mathbf{Z}_t = \frac{8(j10)}{8+j10} + \frac{-j2.4(j20)}{-j2.4+j20} = 4.9 + j\,1.2$$

P10.7-9 Determine the voltage $v(t)$ for this circuit:



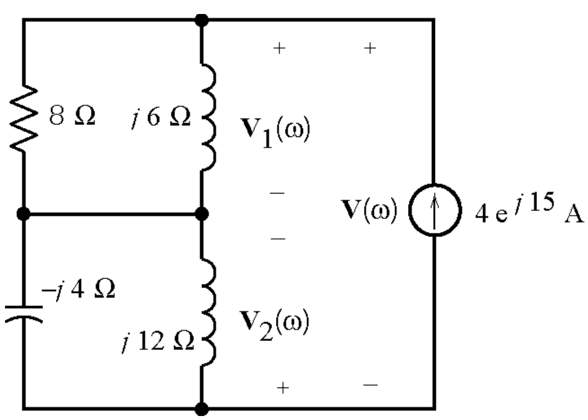
Solution:



$$\mathbf{V}_1(\omega)=\frac{j10}{8+j10}5e^{-j90}=3.9e^{-j51}$$

$$\mathbf{V}_2(\omega)=\frac{j20}{j20-j2.4}5e^{-j90}=5.68e^{-j90}$$

$$\mathbf{V}(\omega)=\mathbf{V}_1(\omega)-\mathbf{V}_2(\omega)=3.9e^{-j51}-5.68e^{-j90}$$
$$=3.58e^{j47}$$



$$\mathbf{V}_1(\omega)=\frac{8(j6)}{8+j6}4e^{j15}=19.2e^{j68}$$

$$\mathbf{V}_2(\omega)=\frac{j12(-j4)}{j12-j4}4e^{j15}=24e^{-j75}$$

$$\mathbf{V}(\omega)=\mathbf{V}_1(\omega)+\mathbf{V}_2(\omega)=14.4e^{-j22}$$

Using superposition: $v(t)$ = 3.58 cos (5t + 47°) + 14.4 cos (3t - 22°) V.

10.8 Superposition

P10.8-1 Determine the steady state current $i(t)$ in the circuit shown in Figure P10.8-1 when the voltage source voltages are

$$v_{s1}(t) = 12\cos(2500\,t)\text{ V} \quad \text{and} \quad v_{s2}(t) = 12\cos(4000\,t)\text{ V}$$

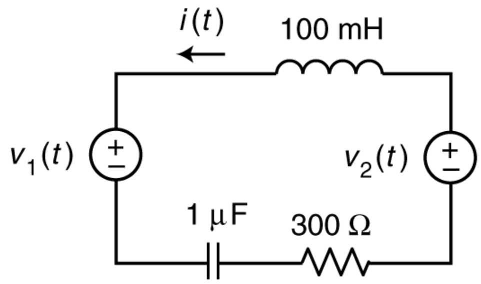
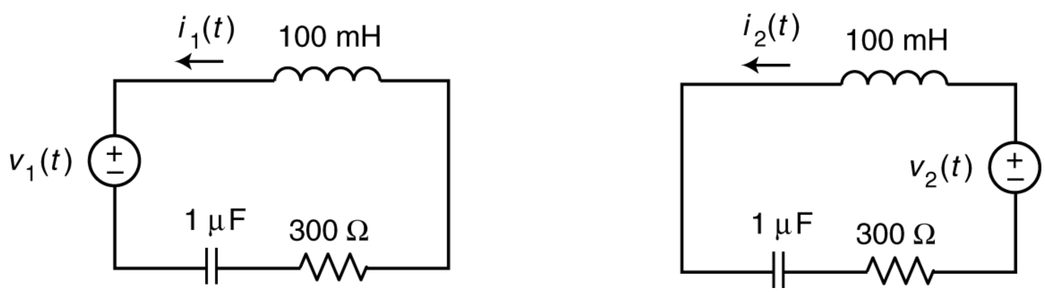


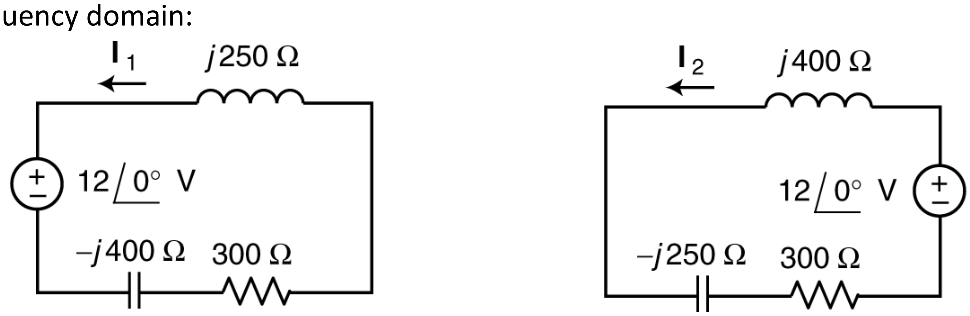
Figure 10.8-1

Solution:

Use superposition *in the time domain*. These circuits can be used to find the part of i_o caused by v_{s1} and the part of i_o caused by v_{s2}.



In the frequency domain:



$$\mathbf{I}_1 = \frac{12\angle 0^\circ}{300 + j\,250 - j400} = 0.03578\angle 26.6^\circ \text{ V}$$

$$\mathbf{I}_2 = \frac{12\angle 0^\circ}{300 - j\,250 + j400} = 0.03578\angle -26.6^\circ \text{ V}$$

In the time domain

$$i_{o1}(t) = 35.78\cos(2500t + 26.6^\circ) \text{ mA} \quad \text{and} \quad i_{o2}(t) = 35.78\cos(4000t - 26.6^\circ) \text{ mA}$$

and

$$i_o(t) = i_{o1}(t) + i_{o2}(t) = 35.78\cos(2500t + 26.6^\circ) + 35.78\cos(4000t - 26.6^\circ) \text{ mA V}$$

P10.8-2 Determine the steady state voltage *v*(*t*) in the circuit shown in Figure P10.8-2 when the current source current is (*a*) 400 rad/s and (*b*) 200 rad/s.

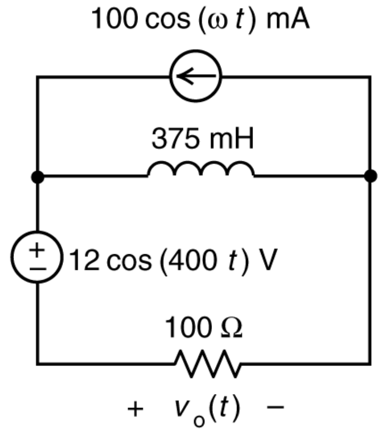
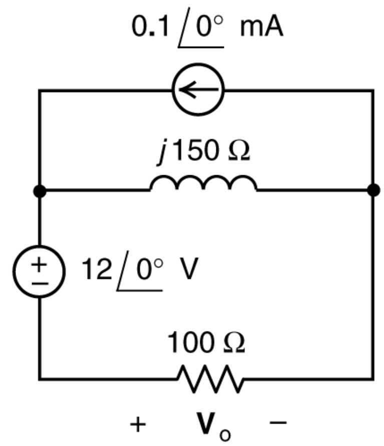


Figure 10.8-2

Solution:

(*a*) Represent the circuit in the frequency domain as



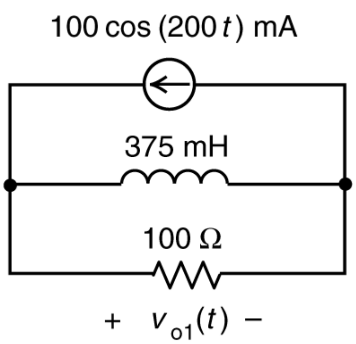
Use superposition *in the frequency domain* to write

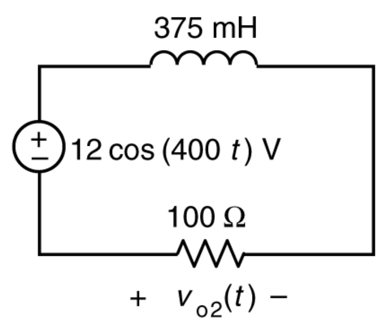
$$\mathbf{V}_o = -\frac{100}{100+j150}(12\angle 0°)+100\frac{j150}{100+j150}(0.1\angle 0°)$$

$$= \frac{-1200+j1500}{100+j150} = 10.66\angle 72.35°$$

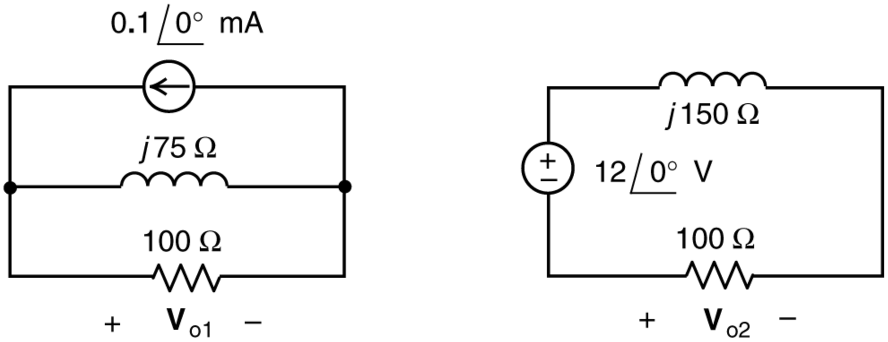
In the time domain $v_o(t) = 10.66\cos(400t+72.35)$ V

(*b*) Use superposition *in the time domain*. These circuits can be used to find the part of v_o caused by the current source and the part of v_o caused by the voltage source.





In the frequency domain:



$$\mathbf{V}_{o1} = 100\frac{j75}{100+j75}(0.1\angle 0°) = 6\angle 53.1° \text{ V}$$

$$\mathbf{V}_{o2} = -\frac{100}{100+j150}(12\angle 0°) = 6.656\angle 123.7° \text{ V}$$

In the time domain

$$v_{o1}(t) = 6\cos(200t + 53.1°) \text{ V} \text{ and } v_{o2}(t) = 6.656\cos(400t + 123.7°) \text{ V}$$

and

$$v_o(t) = v_{o1}(t) + v_{o2}(t) = 6\cos(200t + 53.1°) + 6.656\cos(400t + 123.7°) \text{ V}$$

P10.8-3 Determine the steady state current $i(t)$ in the circuit shown in Figure P10.8-3 when the voltage source voltage is

$$v_s(t) = 8 + 8\cos(400t - 135°)\ \text{V}$$

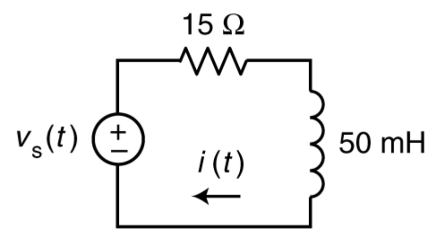
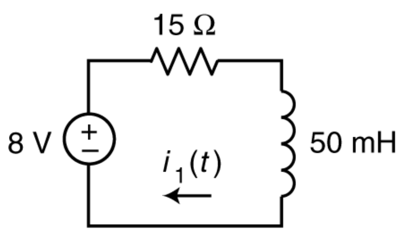
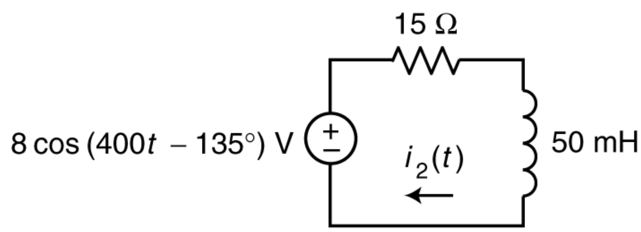


Figure 10.8-3

Solution:

Use superposition *in the time domain*:

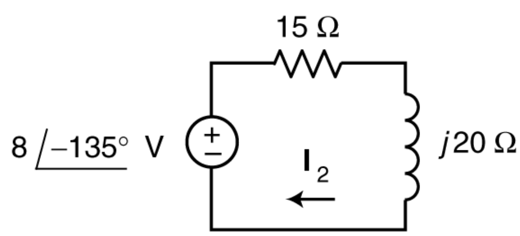




An inductor in a dc circuit acts like a short circuit so:

$$i_1(t) = \frac{8}{15} = 0.533\ \text{A}$$

Represent the right circuit the frequency domain:



$$\mathbf{I}_2 = \frac{8\angle -135°}{15 + j20} = 0.32\angle -188°\ \text{A}$$

In the time domain $\quad i_2(t) = 0.32\cos(400t - 188°)\ \text{A}$

and

$$i(t) = i_1(t) + i_2(t) = 0.533 + 0.32\cos(400t - 188°)\ \text{A}$$

P10.8-4 Determine the steady state current $i(t)$ in the circuit shown in Figure P10.8-4 when the voltage source voltages are

$$v_{s1}(t) = 10\cos(800t + 30°)\text{ V} \quad \text{and} \quad v_{s2}(t) = 15\sin(200t - 30°)\text{ V}$$

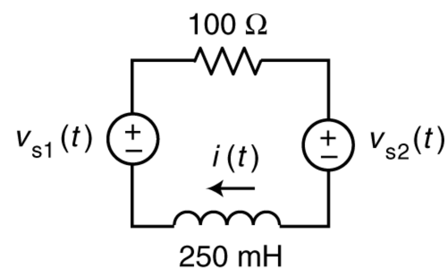
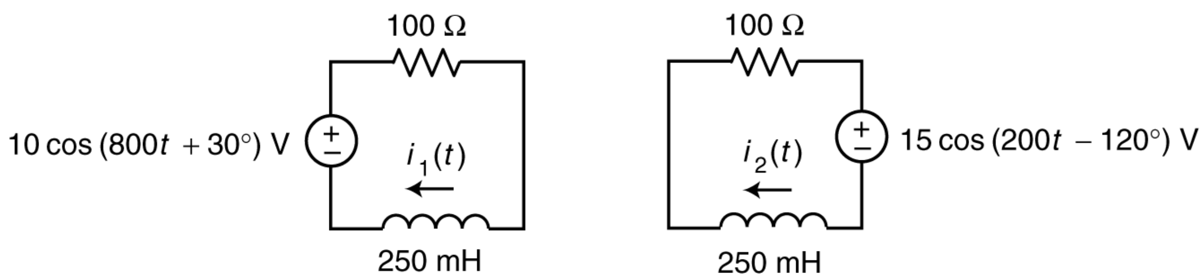


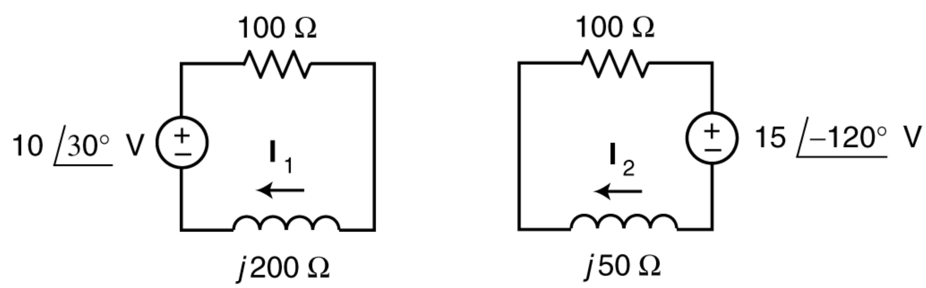
Figure 10.8-4

Solution:

Use superposition *in the time domain*:



Represent these circuits the frequency domain:



$$\mathbf{I}_1 = \frac{10\angle 30°}{100 + j200} = 0.0447\angle -33.4°\text{ A} \quad \text{and} \quad \mathbf{I}_2 = -\frac{15\angle -120°}{100 + j50} = 0.1342\angle 33.4°\text{ A}$$

In the time domain

$$i(t) = i_1(t) + i_2(t) = 44.7\cos(800t - 33.4°) + 134.2\cos(200t + 33.4°)\ \text{mA}$$

P 10.8-5 The input to the circuit shown in Figure P 10.8-5 is the current source current

$i_s(t)$ = 36 cos (25t) + 48 cos (50t + 45°) mA

Determine the steady-state current, $i(t)$.

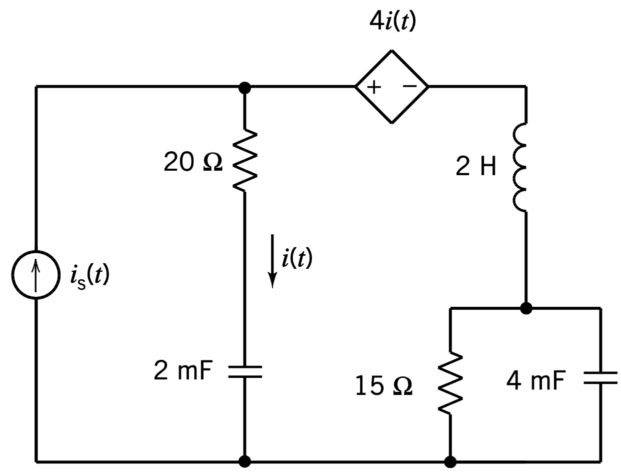


Figure P 10.8-5

Solution:

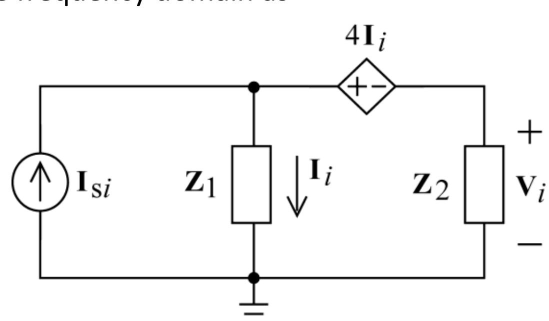
Use superposition in the time domain. Let

$$i_{s1}(t)=36\cos(25t)\text{ mA} \quad\text{and}\quad i_{s2}(t)=48\cos(50t+45°)\text{ mA}$$

We will find the response to each of these inputs separately. Let $i_i(t)$ denote the response to $i_{si}(t)$ for i = 1,2. The sum of the two responses will be $i(t)$, i.e.

$$i(t)=i_1(t)+i_2(t)$$

Represent the circuit in the frequency domain as



Use KVL to get

$$\mathbf{V}_i=\mathbf{Z}_i\mathbf{I}_i-4\mathbf{I}_i$$

Apply KCL to the supernode corresponding to the dependent voltage source.

$$\mathbf{I}_{si}=\mathbf{I}_i+\frac{\mathbf{V}_i}{\mathbf{Z}_2}=\frac{\mathbf{Z}_1+\mathbf{Z}_2-4}{\mathbf{Z}_2}\mathbf{I}_i$$

or

$$\mathbf{I}_i=\frac{\mathbf{Z}_2\mathbf{I}_{si}}{\mathbf{Z}_1+\mathbf{Z}_2-4}$$

Consider the case i = 1 : $i_{s1}(t)$ = 26cos(25t) mA.

Here ω = 25 rad/s and

$$\mathbf{I}_{si} = 36\angle 0° \text{ mA}$$

$$\mathbf{Z}_1 = 20 + \frac{1}{j(25)(0.002)} = 20 - j20\ \Omega$$

$$\mathbf{Z}_2 = j50 + \left(15 \parallel \frac{1}{j(25)(0.004)}\right) = 43.3\angle 83.9°\ \Omega$$

and

$$\mathbf{I}_1 = 50.4\angle 35.7° \text{ mA}$$

so

$$i(t) = 50.4\cos(25t + 35.7°) \text{ mA}$$

Next consider i = 2 : i_{s2} = 48cos(50t + 45°) mA.

Here ω = 50 rad/s and

$$\mathbf{I}_{s2} = 48\angle 45° \text{ mA}$$

$$\mathbf{Z}_1 = 20 + \frac{1}{j(50)(0.002)} = 20 - j10\ \Omega$$

$$\mathbf{Z}_2 = j100 + \left(15 \parallel \frac{1}{j(50)(0.004)}\right) = 95.5\angle 89.1°\ \Omega$$

(Notice that $\mathbf{Z}_1$ and $\mathbf{Z}_2$ change when ω changes.)

$$\mathbf{I}_2 = 52.5\angle 55.7° \text{ mA}$$

so

$$i_2(t) = 52.5\cos(50t + 55.7°) \text{ mA}$$

Finally, using superposition <u>in the time</u> domain gives

$$i(t) = 50.4\cos(25t + 35.7°) + 52.5\cos(50t + 55.7°) \text{ mA}$$

(checked: LNAP 8/7/04)

P 10.8-6 The inputs to the circuit shown in Figure P 10.8-6 are

$$v_{s1}(t) = 30\cos(20t + 70°)\text{ V}$$

and $$v_{s2}(t) = 18\cos(10t - 15°)\text{ V}$$

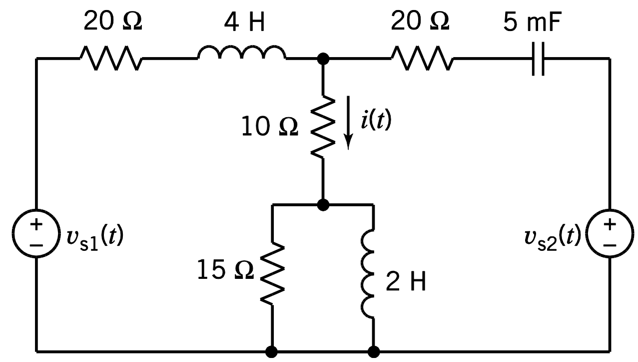
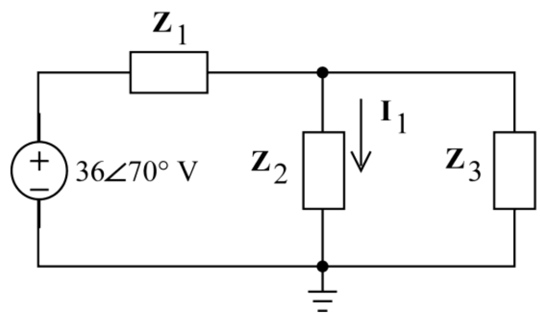


Figure P 10.8-6

Solution:

Use superposition in the time domain. Let $i_1(t)$ be the part of $i(t)$ due to $v_{s1}(t)$ and $i_2(t)$ be the part of $i(t)$ due to $v_{s2}(t)$. To determine $i_1(t)$, set $v_{s2}(t) = 0$. Represent the resulting circuit in the frequency domain to get



where

$$\mathbf{Z}_1 = 20 + j80 = 82.46\angle 76°\ \Omega$$

$$\mathbf{Z}_2 = 10 + \left(j40 \parallel 15\right) = 23.15 + j4.93 = 23.67\angle 12°\ \Omega$$

$$\mathbf{Z}_3 = 20 + \frac{1}{j(20)(0.005)} = 20 - j10 = 22.36\angle -26.6°\ \Omega$$

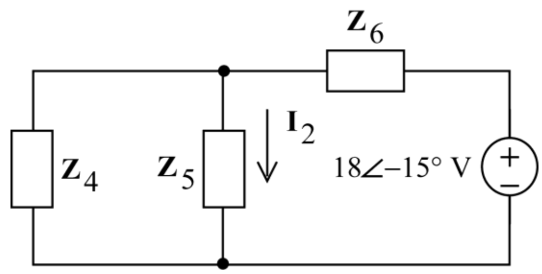
Next, using Ohm's law and current division gives

$$\mathbf{I}_1 = \frac{30\angle 70°}{\mathbf{Z}_1 + \left(\mathbf{Z}_2 \parallel \mathbf{Z}_3\right)} \times \frac{\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{\mathbf{Z}_3\left(30\angle 70°\right)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3} = 0.182\angle -17.6°\text{ A}$$

so

$$i(t) = 0.182\cos\left(20t - 17.6°\right)\text{ A}$$

To determine $i_2(t)$, set $v_{s1}(t) = 0$. Represent the resulting circuit in the frequency domain to get



where

$$\mathbf{Z}_4 = 20 + j40 = 44.72\angle 63.4°\ \Omega$$

$$\mathbf{Z}_5 = 10 + \left(j20 \parallel 15\right) = 19.6 + j7.2 = 20.88\angle 20.2°\ \Omega$$

$$\mathbf{Z}_6 = 20 + \frac{1}{j(10)(0.005)} = 20 - j20 = 28.28\angle -45°\ \Omega$$

Next, using Ohm's law and current division gives

$$\mathbf{I}_2 = \frac{18\angle -15°}{\mathbf{Z}_6 + \left(\mathbf{Z}_4 \parallel \mathbf{Z}_5\right)} \times \frac{\mathbf{Z}_4}{\mathbf{Z}_4 + \mathbf{Z}_5} = \frac{\mathbf{Z}_1\left(18\angle -15°\right)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3} = 0.377\angle 18°\ \text{A}$$

so

$$i_2(t) = 0.377\cos\left(10t + 18°\right)\ \text{A}$$

Using superposition,

$$i(t) = i_1(t) + i_2(t) = 0.182\cos\left(20t - 17.6°\right) + 0.377\cos\left(10t + 18°\right)\ \text{A}$$

(checked: LNAP 8/8/04)

P 10.8-7 The input to the circuit shown in Figure P 10.8-7 is the voltage source voltage

$$v_s(t) = 5 + 30\cos(100t)\text{ V}$$

Determine the steady-state current, $i(t)$.

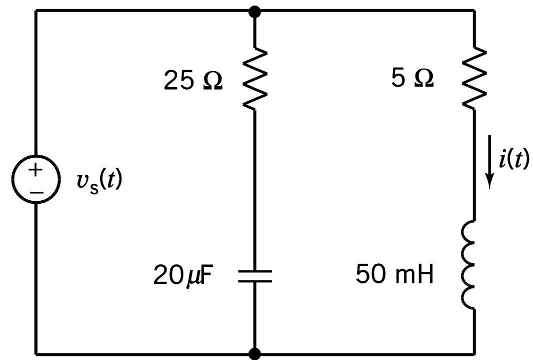


Figure P 10.8-7

Solution:

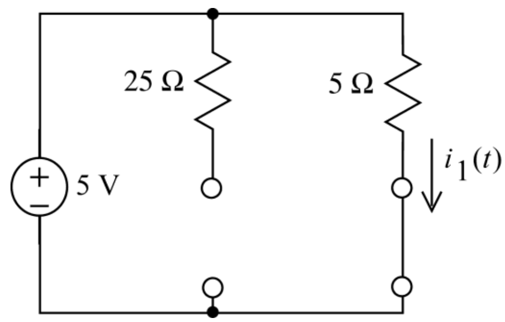
Use superposition in the time domain. Let $v_{s1}(t)$ = 5 V and $v_{s2}(t) = 30\cos(100t)$ V.

Find the steady state response to $v_{s1}(t)$.

When the input is constant and the circuit is at steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

So

$$i_1(t) = \frac{5}{5} = 1\text{ A}$$



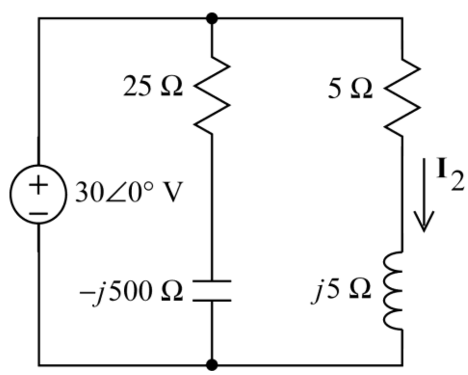
Find the steady state response to $v_{s2}(t)$.

Represent the circuit in the frequency domain using impedances and phasors.

$$\mathbf{I}_2 = \frac{30\angle 0^\circ}{5 + j5} = 4.243\angle -45\text{ A}$$

So

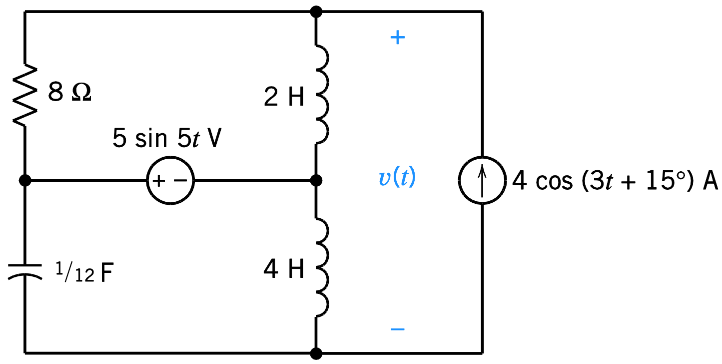
$$i_2(t) = 4.243\cos(100t - 45^\circ)\text{ A}$$



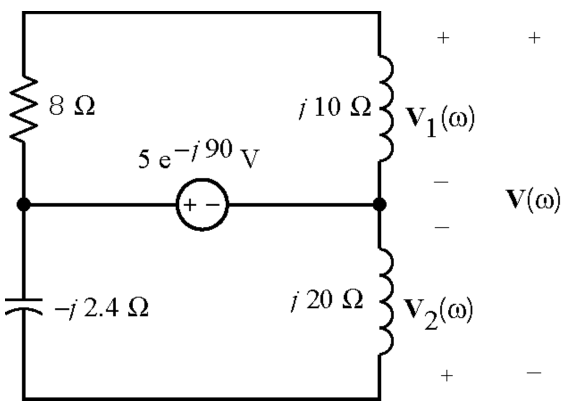
Using superposition

$$i(t) = i_1(t) + i_2(t) = 1 + 4.243\cos(100t - 45^\circ)$$

P10.8-8 Determine the voltage $v(t)$ for the circuit



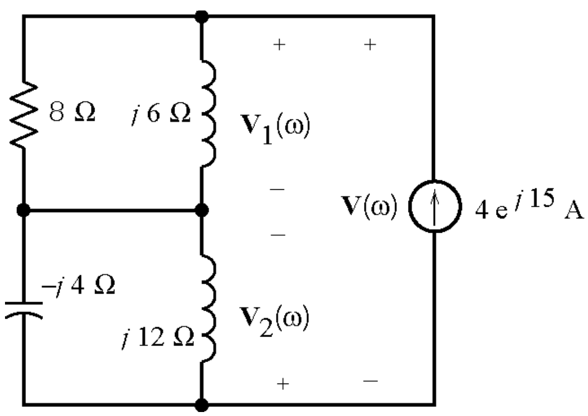
Solution:



$$\mathbf{V}_1(\omega) = \frac{j10}{8 + j10} 5e^{-j90} = 3.9e^{-j51}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20 - j2.4} 5e^{-j90} = 5.68e^{-j90}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9e^{-j51} - 5.68e^{-j90}$$
$$= 3.58e^{j47}$$



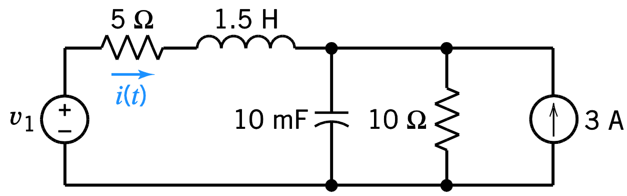
$$\mathbf{V}_1(\omega) = \frac{8(j6)}{8 + j6} 4e^{j15} = 19.2e^{j68}$$

$$\mathbf{V}_2(\omega) = \frac{j12(-j4)}{j12 - j4} 4e^{j15} = 24e^{-j75}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) + \mathbf{V}_2(\omega) = 14.4e^{-j22}$$

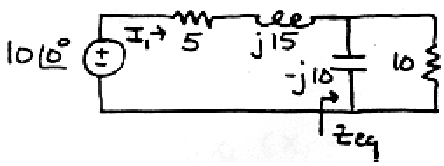
Using superposition: $v(t)$ = 3.58 cos (5t + 47°) + 14.4 cos (3t - 22°) V.

P10.8-9 Determine the current *i*(*t*) for this circuit when $v_1(t)$=10cos(10*t*) V



Solution:

Use superposition. First, find the response to the voltage source acting alone:



$$\mathbf{Z}_{\text{eq}} = \frac{-j10 \cdot 10}{10 - j10} = 5(1-j)\ \Omega$$

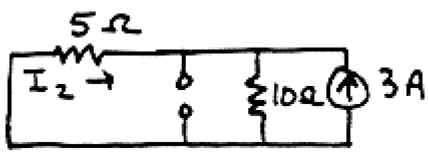
Replacing the parallel elements by the equivalent impedance. The write a mesh equation :

$$-10 + 5\,\mathbf{I}_1 + j15\,\mathbf{I}_1 + 5(1-j)\,\mathbf{I}_1 = 0 \quad \Rightarrow \quad \mathbf{I}_1 = \frac{10}{10+j10} = 0.707\angle -45°\ \text{A}$$

Therefore:

$$i_1(t) = 0.707\cos(10t - 45^\circ)\ \text{A}$$

Next, find the response to the dc current source acting alone:



Current division: $I_2 = -\frac{10}{15} \times 3 = -2\ \text{A}$

Using superposition:

$$i(t) = 0.707\cos(10t - 45°) - 2\ \ \text{A}$$

Section 10-9: Phasor Diagrams

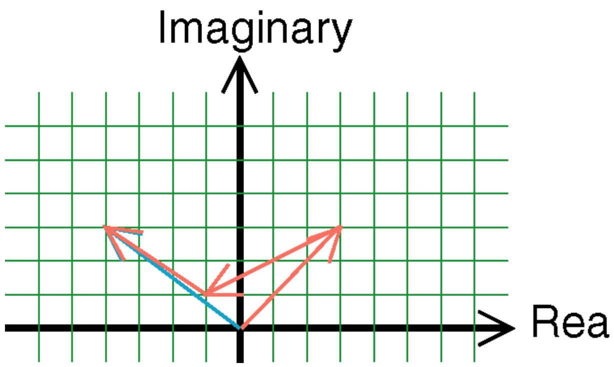
P 10.9-1 Using a phasor diagram, determine **V** when

$\mathbf{V} = \mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_3^*$ and $\mathbf{V}_1 = 3 + j3$, $\mathbf{V}_2 = 4 + j2$, and $\mathbf{V}_3 = -3 - j2$. (Units are volts.)

Answer: $\mathbf{V} = 5 \angle 143.1° \text{ V}$

.

Solution:



$$\mathbf{V} = \mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_3^* = (3 + j3) - (4 + j2) + (-3 - j2)^* = -4 + j3$$

P 10.9-2 Consider the series *RLC* circuit of Figure P 10.9-2 when

$$R = 10\ \Omega,\ L = 1\text{ mH},\ C = 100\ \mu\text{F, and } \omega = 10^3\text{ rad/s.}$$

Find **I** and plot the phasor diagram

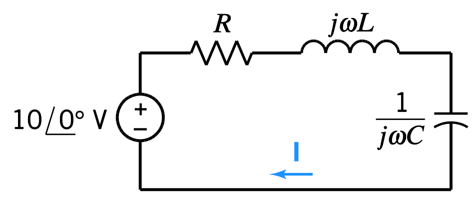
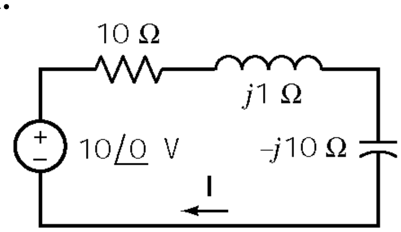


Figure P 10.9-2

Solution:



$$\mathbf{I} = \frac{10\angle 0°}{10 + j1 - j10} = 0.74\angle 42°\ \text{A}$$

$$\mathbf{V}_R = R\,\mathbf{I} = 7.4\angle 42°\ \text{V}$$

$$\mathbf{V}_L = \mathbf{Z}_L\mathbf{I} = (1\angle 90°)(0.74\angle 42°) = 0.74\angle 132°\ \text{V}$$

$$\mathbf{V}_C = \mathbf{Z}_C\mathbf{I} = (10\angle -90°)(0.74\angle 42°) = 7.4\angle -48°\ \text{V}$$

$$\mathbf{V}_S = 10\angle 0°\ \text{V}$$