SOLUTION

ECE222
Quiz 6

To get full credit, show all your work.

1) Find the partial fraction expansion of the function: $H(s) = \frac{3s+1}{s(s+1)^2}$

$$\frac{3s+1}{s(s+1)^2} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{(s+1)^2}$$

1) $\left.\frac{3s+1}{\cancel{s}(s+1)^2}\cancel{s}\right|_{s=0} = K_1$

$1 = K_1$

2) $\left.\frac{3s+1}{s(\cancel{s+1})^2}(\cancel{s+1})^2\right|_{s=-1} = K_3$

$\frac{-3+1}{-1} = 2 = K_3$

3) $\frac{3s+1}{s(\cancel{s+1})^2}(\cancel{s+1})^2 = \frac{K_1(s+1)^2}{s} + \frac{K_2(s+1)^2}{s+1} + \frac{K_3(\cancel{s+1})^2}{(\cancel{s+1})^2}$

$\Rightarrow \frac{3s+1}{s} = \frac{K_1(s+1)^2}{s} + K_2(s+1) + K_3$

$\frac{d}{ds} \Rightarrow$

$\frac{s3-(3s+1)}{s^2} = K_1\left[\frac{s\,2(s+1)-(s+1)^2}{s^2}\right] + K_2$

set $s=-1 \Rightarrow$

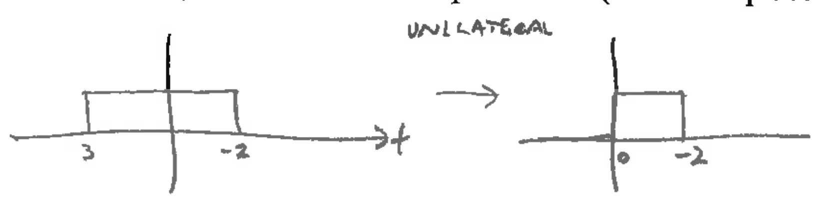
$\frac{-3-\overbrace{(-3+1)}^{-2}}{1} = K_2$

$-1 = K_2$

$$\Rightarrow \frac{3s+1}{s(s+1)^2} = \frac{1}{s} - \frac{1}{s+1} + \frac{2}{(s+1)^2}$$

2) Find the unilateral Laplace transform of the function: $f(t) = [u(t+3) - u(t-2)]$

where $u(t)$ denotes the unit step function. (See the Laplace tables over the page).



$\mathcal{L}\left[u(t+3) - u(t-2)\right] = \mathcal{L}\left\{u(t) - u(t-2)\right\}$

$= \frac{1}{s} - \frac{e^{-2s}}{s}$

An Abbreviated List of Laplace Transform Pairs

$f(t)\ (t > 0^-)$	Type	$F(s)$
$\delta(t)$	(impulse)	1
$u(t)$	(step)	$\frac{1}{s}$
t	(ramp)	$\frac{1}{s^2}$
e^{-at}	(exponential)	$\frac{1}{s+a}$
$\sin \omega t$	(sine)	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	(cosine)	$\frac{s}{s^2+\omega^2}$
te^{-at}	(damped ramp)	$\frac{1}{(s+a)^2}$
$e^{-at}\sin \omega t$	(damped sine)	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	(damped cosine)	$\frac{s+a}{(s+a)^2+\omega^2}$

An Abbreviated List of Operational Transforms

$f(t)$	$F(s)$
$Kf(t)$	$KF(s)$
$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
$e^{-at}f(t)$	$F(s+a)$
$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
$tf(t)$	$-\frac{dF(s)}{ds}$
$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$