HW #9 Solutions

Problems: P13.12, P13.13, P13.23, P13.51 and P13.57

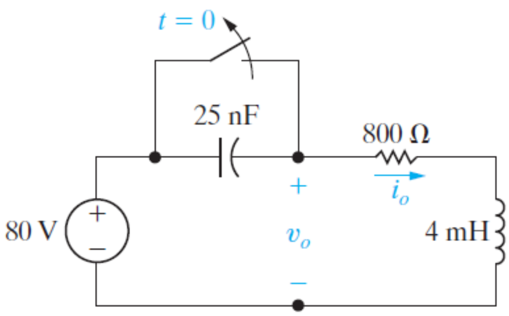
Problem 13.12

13.12 PSPICE MULTISIM The switch in the circuit in Fig. P13.12 has been closed for a long time. At $t = 0$, the switch is opened.

a) Find i_o for $t \geq 0$.

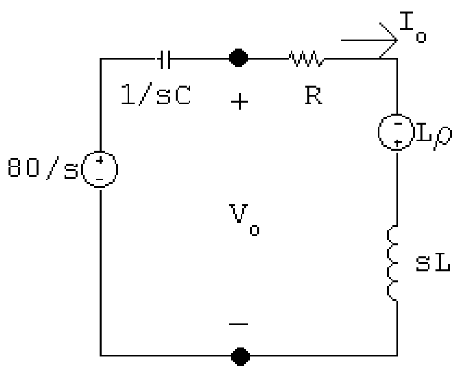
b) Find v_o for $t \geq 0$.

Figure P13.12



Solution:

[a] $i_o(0^-) = \dfrac{20}{4000} = 5\,\text{mA}$



$$I_o = \frac{80/s + L\rho}{R + sL + 1/sC} = \frac{sC(80/s + L\rho)}{s^2LC + RsC + 1}$$

$$= \frac{80/L + s\rho}{s^2 + sR/L + 1/LC} = \frac{20{,}000 + s(0.1)}{s^2 + 200{,}000s + 10^{10}}$$

$$= \frac{0.1(s + 200{,}000)}{s^2 + 200{,}000s + 10^{10}} = \frac{K_1}{(s + 100{,}000)^2} + \frac{K_2}{s + 100{,}000}$$

$K_1 = 10{,}000; \qquad K_2 = 0.1$

$$i_o(t) = [10{,}000te^{-100{,}000t} + 0.1e^{-100{,}000t}]u(t)\ \text{A}$$

$$[b]\ V_o = (R + sL)I_o - L\rho = \frac{(800 + 0.004s)(0.1s + 20{,}000)}{s^2 + 200{,}000s + 10^{10}} - 4 \times 10^{-4}$$

$$= \frac{80(s + 150{,}000)}{(s + 100{,}000)^2} = \frac{K_1}{(s + 100{,}000)^2} + \frac{K_2}{s + 100{,}000}$$

$$K_1 = 4 \times 10^6 \qquad K_2 = 80$$

$$v_o(t) = [4 \times 10^6 te^{-100{,}000t} + 80e^{-100{,}000t}]u(t)\ \text{A}$$

Problem 13.13

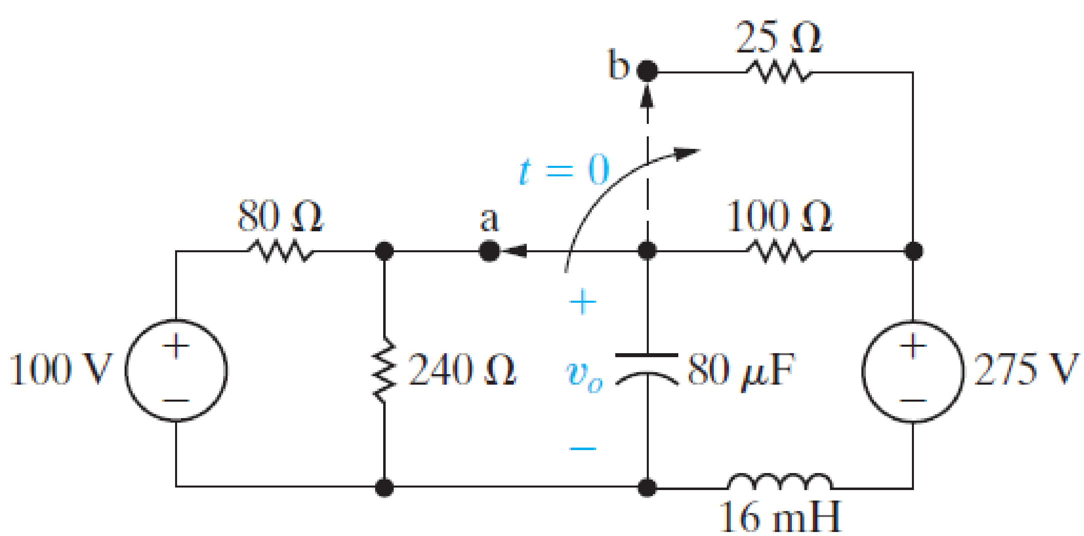
13.13 The switch in the circuit in Fig. P13.13 has been in position a for a long time. At $t = 0$, it moves instantaneously from a to b.

a) Construct the s-domain circuit for $t > 0$.

b) Find $V_o(s)$.

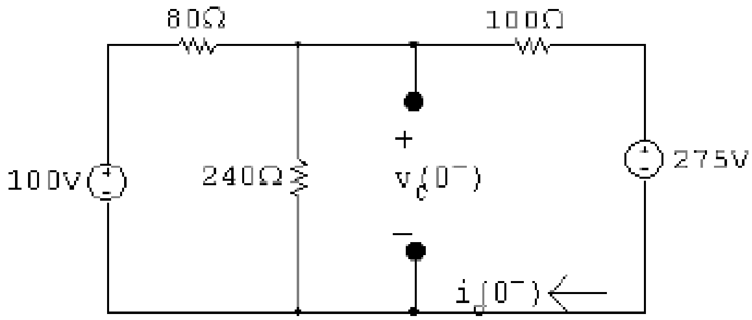
c) Find $v_o(t)$ for $t \geq 0$.

Figure P13.13



Solution:

[a] For $t < 0$:



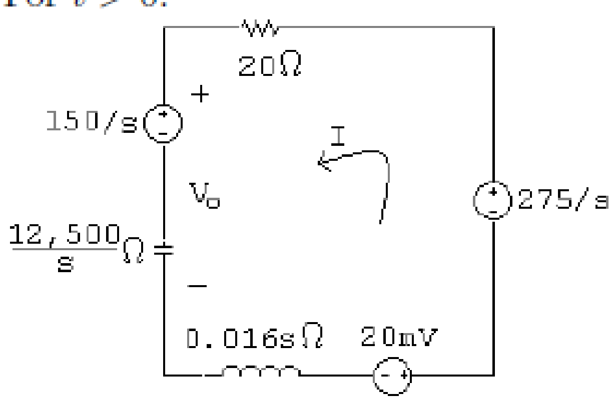
$$\frac{V_c - 100}{80} + \frac{V_c}{240} + \frac{V_c - 275}{100} = 0$$

$$V_c\left(\frac{1}{80} + \frac{1}{240} + \frac{1}{100}\right) = \frac{100}{80} + \frac{275}{100}$$

$$V_c = 150\,\text{V}$$

$$i_L(0^-) = \frac{150 - 275}{100} = -1.25\,\text{A}$$

For $t > 0$:



[b] $$V_o = \frac{12{,}500}{s}I + \frac{150}{s}$$

$$0 = -\frac{275}{s} + 20I + \frac{12{,}500}{s}I + \frac{150}{s} - 20 \times 10^{-3} + 0.016sI$$

$$I\left(20 + \frac{12{,}500}{s} + 0.016s\right) = \frac{125}{s} + 20 \times 10^{-3}$$

$$\therefore \quad I = \frac{7812.5 + 1.25s}{s^2 + 1250s + 781{,}250}$$

[c] $V_o = \frac{K_1}{s} + \frac{K_2}{s + 625 - j625} + \frac{K_2^*}{s + 625 + j625}$

$$K_1 = \frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s^2 + 1250s + 781{,}250}\bigg|_{s=0} = 275$$

$$K_2 = \frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s(s + 625 + j625)}\bigg|_{s=-625+j625} = 80.04\underline{/141.34^\circ}$$

$$v_o(t) = [2755 + 160.08e^{-625t}\cos(625t + 141.34^\circ)]u(t)\text{ V}$$

Problem 13.23:

13.23 Find v_o in the circuit shown in Fig. P13.23 if $i_g = 20u(t)$ mA. There is no energy stored in the circuit at $t = 0$.

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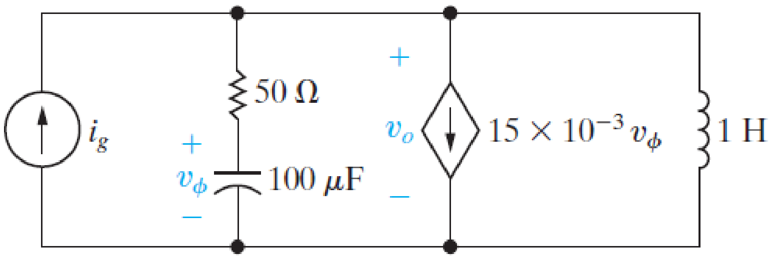
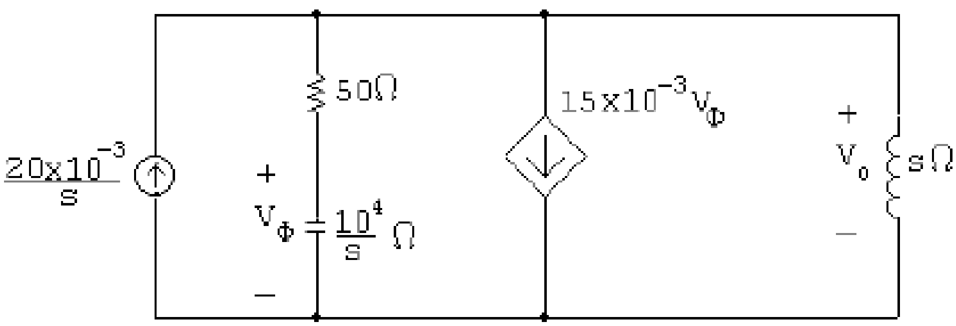
Figure P13.23



Solution:



$$\frac{20 \times 10^{-3}}{s} = \frac{V_o}{50 + 10^4/s} + 15 \times 10^{-3} V_\phi + \frac{V_o}{s}$$

$$V_\phi = \frac{10^4/s}{50 + 10^4/s} V_o = \frac{10^4 V_o}{50s + 10^4}$$

$$\therefore \quad \frac{20 \times 10^{-3}}{s} = \frac{V_o s}{50s + 10^4} + \frac{150 V_o}{50s + 10^4} + \frac{V_o}{s}$$

$$\therefore \quad V_o = \frac{s + 200}{s^2 + 200s + 10^4} = \frac{K_1}{(s+100)^2} + \frac{K_2}{s + 100}$$

$$K_1 = 100; \qquad K_2 = 1$$

$$V_o = \frac{100}{(s+100)^2} + \frac{1}{s+100}$$

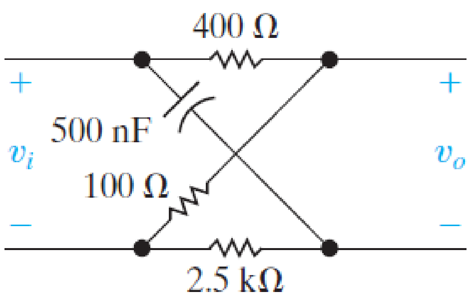
$$v_o(t) = [100te^{-100t} + e^{-100t}]u(t) \text{ V}$$

Problem 13.51:

13.51 a) Find the numerical expression for the transfer function $H(s) = V_o/V_i$ for the circuit in Fig. P13.51.

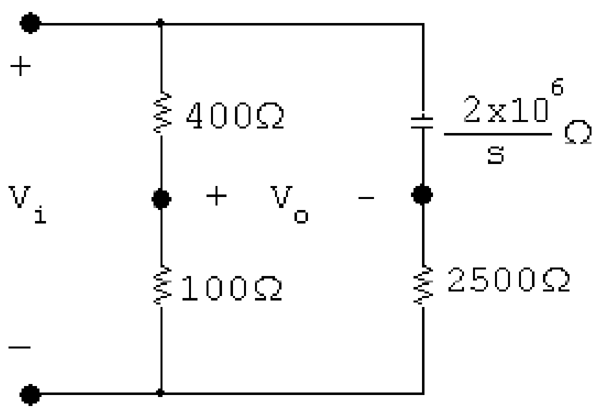
b) Give the numerical value of each pole and zero of $H(s)$.

Figure P13.51



Solution:

[a]



$$\frac{100}{500}V_i = V_o + \frac{2500V_i}{2500 + (2 \times 10^6/s)}$$

$$0.2V_i - \frac{sV_i}{s + 800} = V_o$$

$$\therefore \quad \frac{V_o}{V_i} = H(s) = \frac{-0.8(s - 200)}{(s + 800)}$$

[b] $-z_1 = 200\,\text{rad/s}$

$$-p_1 = -800\,\text{rad/s}$$

Problem 13.57:

13.57 a) Find the transfer function I_o/I_g as a function of μ for the circuit seen in Fig. P13.57.

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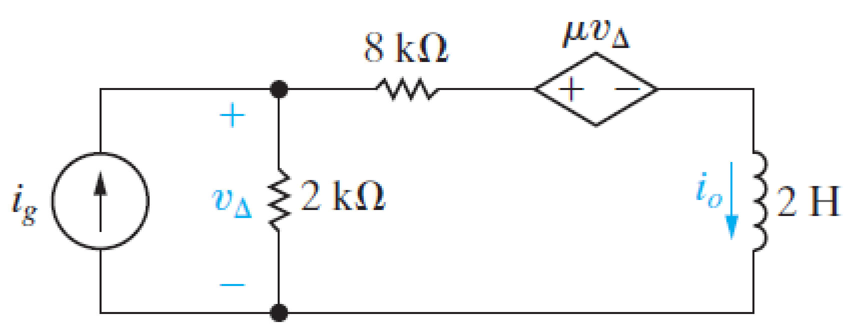
b) Find the largest value of μ that will produce a bounded output signal for a bounded input signal.

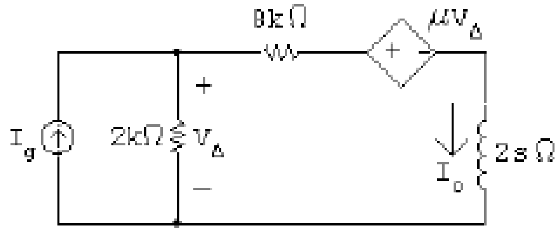
c) Find i_o for $\mu = -3, 0, 4, 5$, and 6 if $i_g = 5u(t)$ A.

Figure P13.57



Solution:

[a]



$$2000(I_o - I_g) + 8000I_o + \mu(I_g - I_o)(2000) + 2sI_o = 0$$

$$\therefore \quad I_o = \frac{1000(1-\mu)}{s + 1000(5-\mu)} I_g$$

$$\therefore \quad H(s) = \frac{1000(1-\mu)}{s + 1000(5-\mu)}$$

[b] $\mu < 5$

[c]

μ	$H(s)$	I_o
-3	$4000/(s+8000)$	$20{,}000/s(s+8000)$
0	$1000/(s+5000)$	$5000/s(s+5000)$
4	$-3000/(s+1000)$	$-15{,}000/s(s+1000)$
5	$-4000/s$	$-20{,}000/s^2$
6	$-5000/(s-1000)$	$-25{,}000/s(s-1000)$

$\mu = -3$:

$$I_o = \frac{2.5}{s} - \frac{2.5}{(s+8000)}; \qquad i_o = [2.5 - 2.5e^{-8000t}]u(t)\text{ A}$$

$\mu = 0$:

$$I_o = \frac{1}{s} - \frac{1}{s+5000}; \qquad i_o = [1 - e^{-5000t}]u(t)\text{ A}$$

$\mu = 4$:

$$I_o = \frac{-15}{s} - \frac{15}{s+1000}; \qquad i_o = [-15 + 15e^{-1000t}]u(t)\text{ A}$$

$\mu = 5$:

$$I_o = \frac{-20{,}000}{s^2}; \qquad i_o = -20{,}000t\,u(t)\text{ A}$$

$\mu = 6$:

$$I_o = \frac{25}{s} - \frac{25}{s-1000}; \qquad i_o = 25[1 - e^{1000t}]u(t)\text{ A}$$