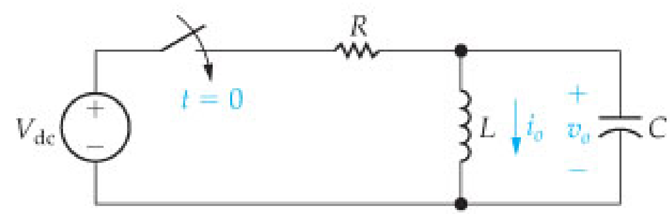
Problem 12.30

The switch in the circuit in the figure has been open for a long time. At $t = 0$, the switch closes.



Part A

Derive the integrodifferential equation that governs the behavior of the voltage v_o for $t \geq 0$.

ANSWER:

- $v_0 + \frac{L}{R}\int_0^t v_0 d\tau + RC\frac{dv_0}{dt} = V_{\mathrm{dc}}$
- $v_0 + \frac{R}{C}\int_0^t v_0 d\tau + RL\frac{dv_0}{dt} = V_{\mathrm{dc}}$
- $v_0 - \frac{R}{L}\int_0^t v_0 d\tau - RC\frac{dv_0}{dt} = V_{\mathrm{dc}}$
- $v_0 + \frac{R}{L}\int_0^t v_0 d\tau + RC\frac{dv_0}{dt} + V_{\mathrm{dc}} = 0$
- $v_0 + \frac{R}{L}\int_0^t v_0 d\tau + RC\frac{dv_0}{dt} = V_{\mathrm{dc}}$
- $\frac{R}{L}\int_0^t v_0 d\tau + RC\frac{dv_0}{dt} = V_{\mathrm{dc}}$

Part B

Find $V_o(s)$.

Express your answer in terms of V_{dc}, s, R, L, and C.

ANSWER:

$V_o(s)$ =

Part C

Find $I_o(s)$.

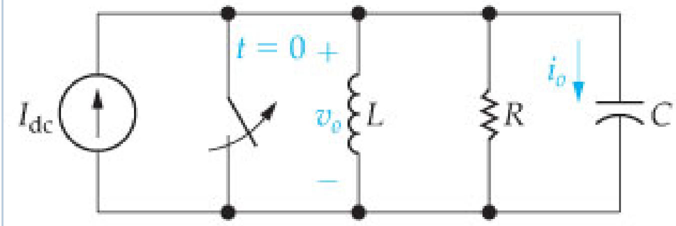
Express your answer in terms of V_{dc}, s, R, L, and C.

ANSWER:

$I_o(s)$ =

Problem 12.34 PSpice|Multisim

The circuit parameters in the circuit shown in have the following values: R = 20 Ω, L = 50 mH, C = 20 $\mu\mathrm{F}$, and I_{dc} = 87 mA . There is no energy stored in the circuit at the time the switch is opened.



Part A

Select the correct expression for $v_o(t)$ for $t \geq 0$, where t is in seconds.

ANSWER:

- $v_o(t) = (11.6e^{500t} + 11.6e^{2000t})u(t)$ V
- $v_o(t) = (2.9e^{-500t} - 2.9e^{-2000t})u(t)$ V
- $v_o(t) = (11.6e^{500t} + 2.9e^{2000t})u(t)$ V
- $v_o(t) = (2.9e^{500t} - 2.9e^{2000t})u(t)$ V
- $v_o(t) = (2.9e^{-500t} - 11.6e^{-2000t})u(t)$ V
- $v_o(t) = (2.9e^{500t} - 11.6e^{2000t})u(t)$ V
- $v_o(t) = (2.9e^{-500t} + 2.9e^{-2000t})u(t)$ V
- $v_o(t) = (11.6e^{-500t} + 2.9e^{-2000t})u(t)$ V

Part B

Select the correct expression for $i_o(t)$ for $t \geq 0$, where t is in seconds.

ANSWER:

- $i_o(t) = (-29e^{-500t} - 116e^{-2000t})u(t)$ mA
- $i_o(t) = (29e^{500t} + 116e^{2000t})u(t)$ mA
- $i_o(t) = (116e^{-500t} - 29e^{-2000t})u(t)$ mA
- $i_o(t) = (-29e^{500t} - 116e^{2000t})u(t)$ mA
- $i_o(t) = (-29e^{-500t} + 116e^{-2000t})u(t)$ mA
- $i_o(t) = (-116e^{-500t} - 29e^{-2000t})u(t)$ mA
- $i_o(t) = (-116e^{500t} - 29e^{2000t})u(t)$ mA
- $i_o(t) = (116e^{500t} - 29e^{2000t})u(t)$ mA

Part C

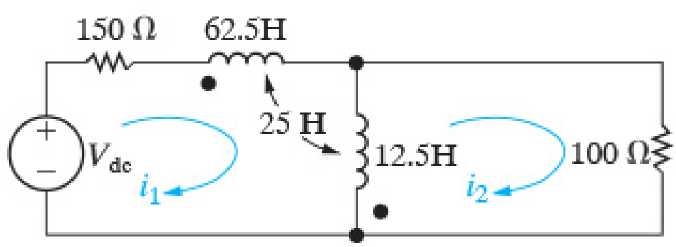
Does your solution for $i_o(t)$ make sense when $t = 0$?

ANSWER:

- No, it doesn't. The initial current through the capacitor $i_o(0) = 87$ mA is equal to the source current, the initial inductor current is zero, therefore the current through the resistor is zero, which is unreasonable.
- Yes, it does. The initial current through the capacitor $i_o(0) = 87$ mA is equal to the source current, the initial inductor current is zero, the initial resistor current is zero because the initial capacitor voltage is zero.

Problem 12.39

Consider the circuit shown in . Suppose that $V_{\rm dc} = 625u(t)\ \mathrm{V}$. Assume that the initial energy stored in the circuit is zero.



Part A

Select the correct expression for $i_1(t)$ where t is in seconds and the parameters are rounded to three significant figures.

ANSWER:

- $i_1(t) = (2.50 + 1.67e^{-4t} + 4.17e^{-24t})u(t)$ A
- $i_1(t) = (2.50 + 4.17e^{-4t} - 1.67e^{-24t})u(t)$ A
- $i_1(t) = (4.17 + 2.50e^{-4t} - 1.67e^{-24t})u(t)$ A
- $i_1(t) = (2.50 - 1.67e^{-4t} - 4.17e^{-24t})u(t)$ A
- $i_1(t) = (4.17 + 2.50e^{-4t} + 1.67e^{-24t})u(t)$ A
- $i_1(t) = (2.50 - 4.17e^{-4t} + 1.67e^{-24t})u(t)$ A
- $i_1(t) = (4.17 - 1.67e^{-4t} + 2.50e^{-24t})u(t)$ A
- $i_1(t) = (4.17 - 2.50e^{-4t} - 1.67e^{-24t})u(t)$ A

Part B

Select the correct expression for $i_2(t)$ where t is in seconds and the parameters are rounded to three significant figures.

ANSWER:

- $i_2(t) = (1.25e^{-24t} + 1.25e^{-4t})u(t)\ \mathrm{A}$
- $i_2(t) = (1.25e^{-24t} - 1.25e^{-4t})u(t)\ \mathrm{A}$
- $i_2(t) = (2.50e^{-24t} + 1.25e^{-4t})u(t)\ \mathrm{A}$
- $i_2(t) = (1.25e^{-24t} + 2.50e^{-4t})u(t)\ \mathrm{A}$
- $i_2(t) = (2.50e^{-24t} - 2.50e^{-4t})u(t)\ \mathrm{A}$
- $i_2(t) = (2.50e^{-24t} + 2.50e^{-4t})u(t)\ \mathrm{A}$
- $i_2(t) = (2.50e^{-24t} - 1.25e^{-4t})u(t)\ \mathrm{A}$
- $i_2(t) = (1.25e^{-24t} - 2.50e^{-4t})u(t)\ \mathrm{A}$

Part C

Find $i_1(\infty)$.

Express your answer to three significant figures and include the appropriate units.

ANSWER:

$i_1(\infty)$ =

Part D

Find $i_2(\infty)$.

Express your answer to three significant figures and include the appropriate units.

ANSWER:

$i_2(\infty)$ =

Part E

Do the solutions for i_1 and i_2 make sense?

ANSWER:

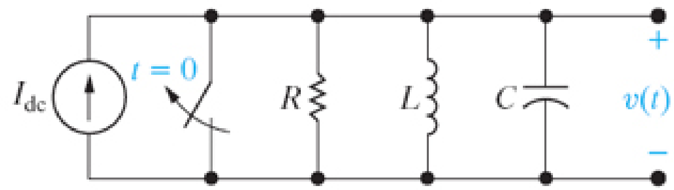
- No, they don't. At $t = \infty$ the current through the resistor is zero, therefore the voltage at 12.5 H is zero, which is unreasonable, because the current through the inductor in not zero.
- Yes, they do. Since i_1 is dc current, at $t = \infty$ there is no voltage on the 12.5 H inductor, it corresponds to $i_2 = 0$. $i_1(0) = 0$, $i_2(0) = 0$ satisfy the condition of no initial energy stored in the circuit.

Problem 12.46

In the circuit shown in the figure, the dc current source is replaced with a sinusoidal source that delivers a current of $1.2\cos t$ A. The circuit components are R= 1 Ω, C= 625 mF, and L= 1.6 H.

Part A

Use the initial-value theorem to find the initial value of v.



ANSWER:

$v(0^+)$ = ______ V

Part B

Can the final-value theorem be used to find the steady-state value of v?

ANSWER:

- yes
- no

Problem 12.52

Part A

Can the initial- and final-value theorems be applied to $F(s) = \frac{320}{s^2(s+8)}$?

ANSWER:

- only the final-value theorem can be applied
- only the initial-value theorem can be applied
- both theorems can be applied
- none of these theorems can be applied

Part B

Apply the initial-value theorem to $F(s) = \frac{320}{s^2(s+8)}$ to find the initial value of $f(t)$.

Express your answer using three significant figures.

ANSWER:

$f(0^+)$ =

Part C

Can the initial- and final-value theorems be applied to $F(s) = \frac{95(s+3)}{s(s+7)^2}$?

ANSWER:

- only the initial-value theorem can be applied
- only the final-value theorem can be applied
- both theorems can be applied
- none of these theorems can be applied

Part D

Apply the final-value theorem to $F(s) = \frac{95(s+3)}{s(s+7)^2}$ to find the final value of $f(t)$.

Express your answer using three significant figures.

ANSWER:

$f(\infty)$ =

Part E

Apply the initial-value theorem to $F(s) = \frac{95(s+3)}{s(s+7)^2}$ to find the initial value of $f(t)$.

Express your answer using three significant figures.

ANSWER:

$f(0^+)$ =

Part F

Can the initial- and final-value theorems be applied to $F(s) = \frac{60(s+5)}{(s+1)^2(s^2+6s+25)}$?

ANSWER:

- only the initial-value theorem can be applied
- only the final-value theorem can be applied
- both theorems can be applied
- none of these theorems can be applied

Part G

Apply the final-value theorem to $F(s) = \frac{60(s+5)}{(s+1)^2(s^2+6s+25)}$ to find the final value of $f(t)$.

Express your answer using three significant figures.

ANSWER:

$f(\infty)$ =

Part H

Apply the initial-value theorem to $F(s) = \frac{60(s+5)}{(s+1)^2(s^2+6s+25)}$ to find the initial value of $f(t)$.

Express your answer using three significant figures.

ANSWER:

$f(0^+)$ =

Part I

Can the initial- and final-value theorems be applied to $F(s) = \frac{25(s+4)^2}{s^2(s+5)^2}$?

ANSWER:

- only the initial-value theorem can be applied
- only the final-value theorem can be applied
- both theorems can be applied
- none of these theorems can be applied

Part J

Apply the initial-value theorem to $F(s) = \frac{25(s+4)^2}{s^2(s+5)^2}$ to find the initial value of $f(t)$.

Express your answer using three significant figures.

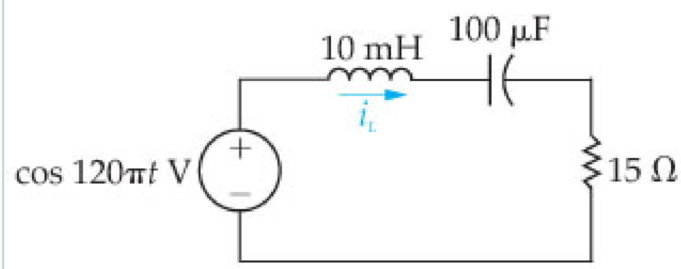
ANSWER:

$f(0^+)$ =

Problem 12.54

Part A

Use phasor circuit analysis techniques from Chapter 9 in the textbook to determine the steady-state expression for the inductor current in the figure.

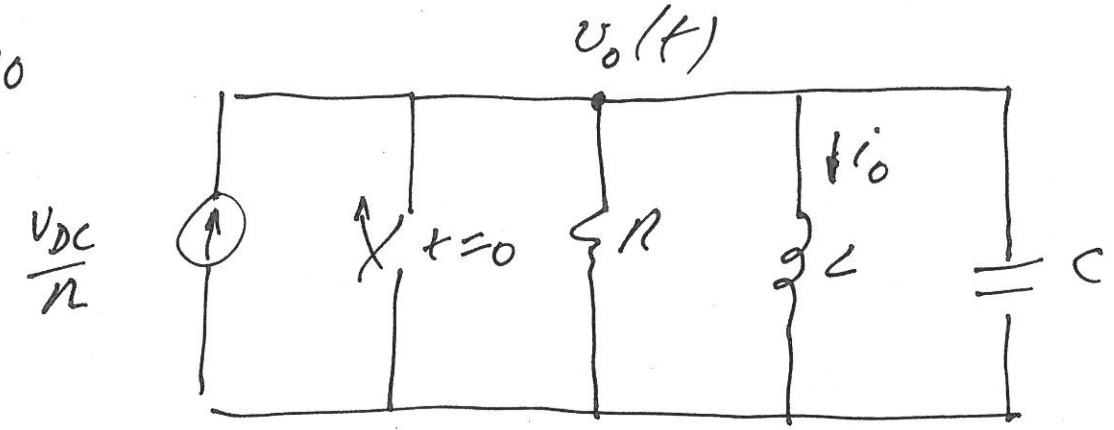


Express your answer in terms of t, where t is in seconds. Enter the phase angle in radians.

ANSWER:

$i_{L-ss}(t)$ = mA

12.30



KCL: $$\frac{v_o(t)}{R} + \frac{1}{L}\int_0^t v_o(x)\,dx + C\frac{d}{dt}v_o(t) - \frac{V_{DC}}{R} = 0$$

a) $$v_o(t) + \frac{R}{L}\int_0^t v_o(x)\,dx + RC\frac{d}{dt}v_o(t) = V_{DC}$$

S-DOMAIN: (NO PRIOR ENERGY)

$$\frac{V_o(s)}{R} + \frac{V_o(s)}{Ls} + Cs\,V_o(s) - \frac{V_{DC}/R}{s} = 0$$

NOTE UNIT STEP

$$V_o(s)\left(\frac{1}{R} + \frac{1}{Ls} + Cs\right) = \frac{V_{DC}/R}{s}$$

$$V_o(s) = \frac{V_{DC}}{s(RCs + R/Ls + 1)}$$

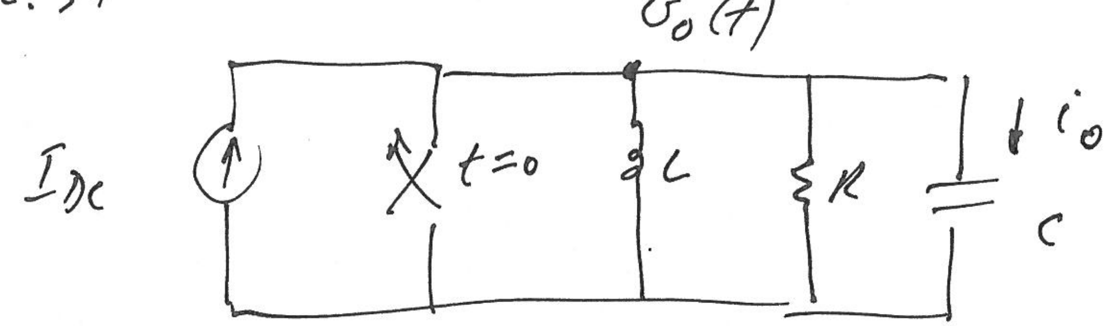
$$V_o(s) = \frac{\cancel{s}\,V_{DC}/RC}{\cancel{s}(s^2 + s/RC + 1/LC)}$$

b) $$V_o(s) = \frac{V_{DC}/RC}{s^2 + s/RC + 1/LC}$$

$$I_o(s) = \frac{V_o(s)}{Ls}$$

c) $$I_o(s) = \frac{V_{DC}/RLC}{s(s^2 + s/RC + 1/LC)}$$

12.34



$I_{DC} = 87\,mA$; $R = 20\,\Omega$; $L = 50\,mH$

$C = 20\,\mu F$

SAME PROBLEM AS 12.30, EXCEPT

V_{DC}/R REPLACED BY I_{DC}

$$V_o(s) = \frac{I_{DC}/C}{s^2 + s/RC + 1/LC} = \frac{4{,}350}{s^2 + 2{,}500s + 10^6}$$

ROOTS OF DENOMINATOR

$$s_i = \frac{-2{,}500 \pm \sqrt{(2{,}500)^2 - 4\times 10^6}}{2}$$

$$= -500, -2{,}000$$

$$V_o(s) = \frac{4{,}350}{(s+500)(s+2{,}500)} = \frac{K_1}{s+500} + \frac{K_2}{s + \cancel{2{,}500}\ 2{,}000}$$

$$\left.\frac{4{,}350}{s+2{,}000}\right|_{s=-500} = K_1 = 2.90$$

$$\left.\frac{4{,}350}{s+500}\right|_{s=-2{,}000} = K_2 = -2.90$$

$$V_o(s) = \frac{2.9}{s+500} - \frac{2.9}{s+2{,}000}$$

a) $v_o(t) = 2.9(e^{-500t} - e^{-2{,}000t})u(t)$ V

$$I_o(s) = \frac{V_o(s)}{1/sC} = sC\,V_o(s)$$

$$I_o(s) = \frac{875s}{(s+500)(s+2{,}000)} = \frac{K_1}{s+500} + \frac{K_2}{s+2{,}000}$$

$$\left.\frac{875s}{s+2{,}000}\right|_{s=-500} = K_1 = -29$$

$$\left.\frac{875s}{s+500}\right|_{s=-2{,}000} = K_2 = 116$$

b) $i_o(t) = (-29e^{-500t} + 116e^{-2{,}000t})u(t)$ mA

c) DOES $i_o(0) = 116 - 29 = 87\ mA$

MAKE SENSE?

CONSIDER KCL AT $t=0$

$$-I_{DC} + i_L(0) + i_R(0) + i_o(0) = 0$$

BUT $i_R(0) = \frac{V_o(0)}{R}$; $V_o(0) = 0 \Rightarrow i_R(0) = 0$

MOREOVER, $i_L(0) = 0$

$$\Rightarrow -I_{DC} + i_o(0) = 0$$

$$i_o(0) = I_{DC} = 87\ mA \checkmark$$

12.39

$$V - 150 I_1 - L_1 s I_1 - s L_2 (I_1 - I_2)$$

$$- s M (I_2 - I_1) + s M I_1 = 0$$

1 $$I_1 (150 + sL_1 + sL_2 - 2sM) + I_2 (sM - sL_2) = V$$

$$-sL_2 (I_2 - I_1) - 100 I_2 - sM I_1 = 0$$

2 $$I_1 (sM - sL_2) + I_2 (sL_2 + 100) = 0$$

2' $$I_2 = \frac{I_1 (sL_2 - sM)}{sL_2 + 100}$$

USE 1 & 2':

$$I_1 \left[(150 + sL_1 + sL_2 - 2sM) + \frac{(sM - sL_2)(sL_2 - sM)}{sL_2 + 100} \right] = V$$

$$I_1 \left[(150 + sL_1 + sL_2 - 2sM)(sL_2 + 100) - (sL_2 - sM)^2 \right]$$

$$= V (sL_2 + 100)$$

12.39, CONT'D

TERM IN SQUARE BRACKETS ONLY:

$$[\dots] = 150sL_2 + 15{,}000 + s^2L_1L_2 + 100sL_1$$
$$+ \cancel{s^2L_2^2} + 100sL_2 - \cancel{2s^2ML_2} - 200sM$$
$$- \cancel{s^2L_2^2} + \cancel{2s^2ML_2} - s^2M^2$$
$$= s^2(L_1L_2 - M^2) + s(250L_2 + 100L_1 - 200M) + 15{,}000$$

$$\text{NOW } I_1 = \frac{V(s)(sL_2 + 100)}{s^2(L_1L_2 - M^2) + s(250L_2 + 100L_1 - 200M) + 15{,}000}$$

$$= \frac{V(s)\left(s + 100/L_2\right)}{s^2(L_1 - M^2/L_2) + s(250 + 100L_1/L_2 - 200M/L_2) + 15{,}000/L_2}$$

$$I_1 = \frac{V(s)(s+8)}{s^2(12.5) + s(350) + 1{,}200}$$

$$= \frac{V(s)/12.5\ (s+8)}{s^2 + 28s + 96}$$

$$\text{ROOTS ARE } \frac{-28 \pm \sqrt{(28)^2 - 4\times 96}}{2} = -4, -24$$

12.39, CONT'D.

NOW $v(t) = 625u(t) \longrightarrow V(s) = \frac{625}{s}$

$$I_1(s) = \frac{625}{s}\frac{1}{12.5}\frac{(s+8)}{(s+4)(s+24)}$$

$$= \frac{50(s+8)}{s(s+4)(s+24)} = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+24}$$

$$\left.\frac{50(s+8)}{(s+4)(s+24)}\right|_{s=0} = K_1 = \frac{50(8)}{(4)(24)} = \underline{\underline{4.1667 = K_1}}$$

$$\left.\frac{50(s+8)}{s(s+24)}\right|_{s=-4} = K_2 = \frac{50(8-4)}{(-4)(24-4)} = \underline{\underline{-2.5 = K_2}}$$

$$\left.\frac{50(s+8)}{s(s+4)}\right|_{s=-24} = K_3 = \frac{50(8-24)}{(-24)(4-24)} = \underline{\underline{-1.6667 = K_3}}$$

$$I_1(s) = \frac{4.1667}{s} - \frac{2.5}{s+4} - \frac{1.6667}{s+24}$$

$$i_1(t) = (4.1667 - 2.5e^{-4t} - 1.6667e^{-24t})u(t)$$

A

EQ. 2':

$$I_2(s) = \frac{I_1 s (L_2 - M)}{sL_2 + 100} = I_1 \frac{s(1 - M/L_2)}{s + 100/L_2}$$

$$I_2(s) = \frac{50\,\cancel{(s+8)}}{s(s+4)(s+24)} \frac{s(-1)}{\cancel{(s+8)}}$$

$$= \frac{-50}{(s+4)(s+24)} = \frac{K_1}{s+4} + \frac{K_2}{s+24}$$

$$\left.\frac{-50}{(s+24)}\right|_{s=-4} = K_1 = \frac{-50}{24-4} = \underline{\underline{-2.5 = K_1}}$$

$$\left.\frac{-50}{(s+4)}\right|_{s=-24} = K_2 = \frac{-50}{4-24} = \underline{\underline{2.5 = K_2}}$$

$$I_2(s) = \frac{-2.5}{s+4} + \frac{2.5}{s+24}$$

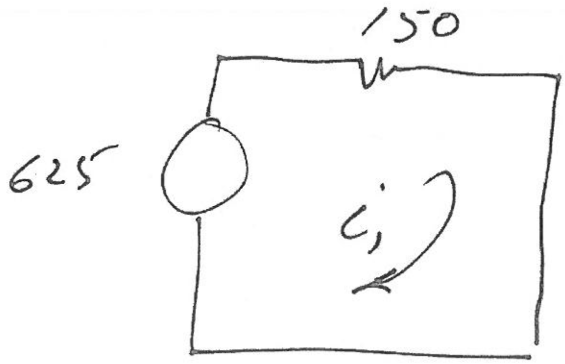
$$i_2(t) = (2.5e^{-24t} - 2.5e^{-4t})u(t) \text{ A}$$

12.39, CONT'D.

$$i_1(t) \xrightarrow[t \to \infty]{} 4.1667\ A$$

$$i_2(t) \xrightarrow[t \to \infty]{} 0\ A$$

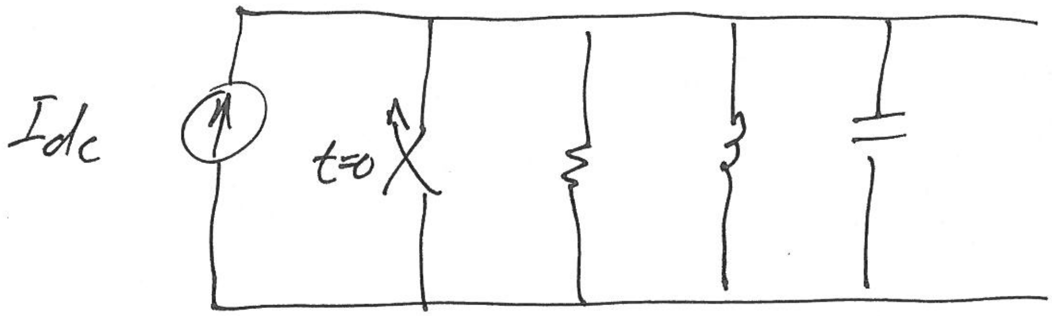
THIS MAKE SENSE AS CKT BECOMES



$$i_1 \to \frac{625}{150}$$

$$= 4.1667\ A$$

12.46



$I_{dc} = 1.2 \cos t \ A$

$R = 1\,\Omega$, $C = 625\,mF$, $L = 1.6\,H$

SAME GENERAL SOL'N AS 12.26

$$V(s) = \frac{s/C}{s^2 + s/RC + 1/LC} \quad \frac{1.2s}{(s^2+1)}$$

$$V(s) = \frac{1.92 s^2}{(s^2+1.6s+1)(s^2+1)}$$

INITIAL VALUE THM: $v(0^+) = \lim_{s\to\infty} sF(s)$

$$\lim_{s\to\infty} \frac{1.92 s^3}{(s^2+1.6s+1)(s^2+1)} = 0 = \underline{\underline{v(0^+)}}$$

AGREES WITH WHAT WE KNOW ABOUT INSTANTANEOUS VOLTAGE CHANGE FOR A CAPACITOR

12.46

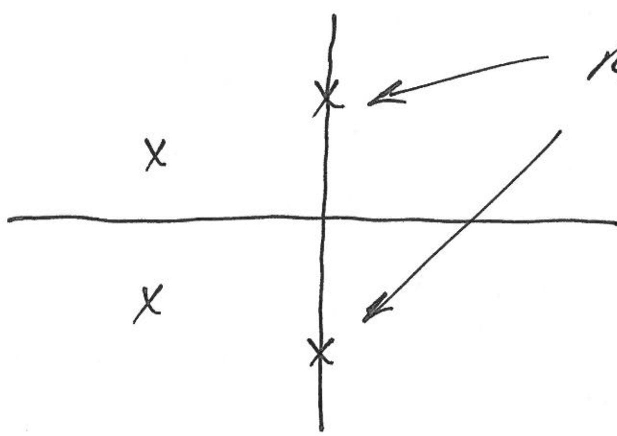
FINAL VALUE THM: $\lim_{t \to \infty} f(t) = \lim_{s \to 0^+} sF(s)$

IFF POLES LIE IN LHP (WITH EXCEPTION OF SIMPLE POLE AT $s=0$)

POLES ARE $\dfrac{-1.6 \pm \sqrt{(1.6)^2 - 4}}{2}$

$= \dfrac{-1.6 \pm j1.2}{2} = -0.8 \pm j0.6$

AND AT $\pm j$



REQUIREMENTS OF FVT ARE VIOLATED

12.52

a) $F(s) = \dfrac{320}{s^2(s+8)}$

FVT CANNOT BE APPLIED (DOUBLE POLE AT ORIGIN)

b) IVT: $f(0^+) = \lim_{s \to \infty} \dfrac{s\,320}{s^2(s+8)} = 0$

c) $F(s) = \dfrac{95(s+3)}{s(s+7)^2}$ IVT, FVT OK

d) FVT: $f(\infty) = \lim_{s \to 0} \dfrac{95(s+3)}{(s+7)^2} = \dfrac{95 \times 3}{49} = 5.82$

e) IVT: $f(0^+) = \lim_{s \to \infty} \dfrac{95(s+3)}{(s+7)^2} = 0$

f) $F(s) = \dfrac{60(s+5)}{(s+1)^2(s^2+6s+25)}$ IVT, FVT OK

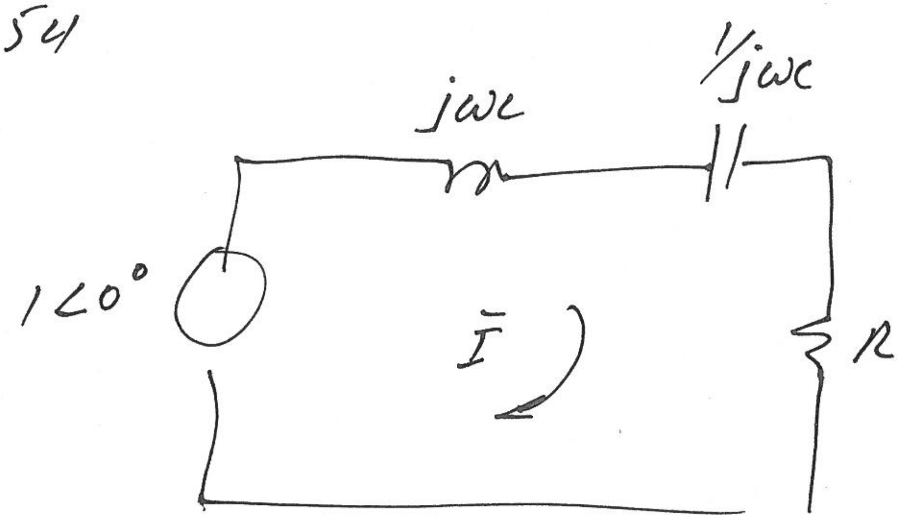
g) FVT: $f(\infty) = \lim_{s \to 0} \dfrac{s\,60(s+5)}{(s+1)^2(s^2+6s+25)} = 0$

h) IVT: $f(0^+) = \lim_{s \to \infty} \dfrac{s\,60(s+5)}{(s+1)^2(s^2+6s+25)} = 0$

i) $F(s) = \dfrac{25(s+4)^2}{s^2(s+5)^2}$ ONLY IVT

j) IVT: $f(0^+) = \lim_{s \to \infty} \dfrac{25(s+4)^2}{s(s+5)^2} = 0$

12.54



$$1 = \bar{I}(j\omega L + 1/j\omega C + R)$$

$$\bar{I} = \frac{1}{j\omega L + 1/j\omega C + R} = \frac{j\omega}{-\omega^2 L + \frac{1}{C} + j\omega R}$$

$$\bar{I} = \frac{j\omega/L}{(\frac{1}{LC} - \omega^2) + j\omega R/L}$$

$$= \frac{\omega/L \angle 90^\circ}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + (\omega R/L)^2} \angle \theta}$$

$$\theta = \tan^{-1}\left(\frac{\omega R/L}{\frac{1}{LC} - \omega^2}\right)$$

$$\bar{I} = 36.69 \angle 90^\circ - 33.39^\circ \text{ mA}$$

$$i_L(t) = i(t) = 36.69 \cos(120\pi t - 0.988) \text{ mA}$$

IN RADIANS