Problem 12.13

Part A

Find the unilateral Laplace transform of the following function:
$f(t) = 20e^{-500(t-10)}u(t-10)$

Express your answer in terms of s.

ANSWER:

$\mathscr{L}\{f(t)\}$ =

Part B

Find the unilateral Laplace transform of the following function:
$f(t) = (5t+20)[u(t+4) - u(t+2)] - 5t[u(t+2) - u(t-2)] + (5t-20)[u(t-2) - u(t-4)]$

Express your answer in terms of s.

ANSWER:

$\mathscr{L}\{f(t)\}$ =

Problem 12.22

Part A

Find $\mathcal{L}\left\{\frac{d}{dt}\sin\omega t\right\}$.

Express your answer in terms of s and ω.

ANSWER:

$\mathcal{L}\left\{\frac{d}{dt}\sin\omega t\right\}$ =

Part B

Find $\mathcal{L}\left\{\frac{d}{dt}\cos\omega t\right\}$.

Express your answer in terms of s and ω.

ANSWER:

$\mathcal{L}\left\{\frac{d}{dt}\cos\omega t\right\}$ =

Part C

Find $\mathcal{L}\left\{\frac{d^3}{dt^3}t^2u(t)\right\}$.

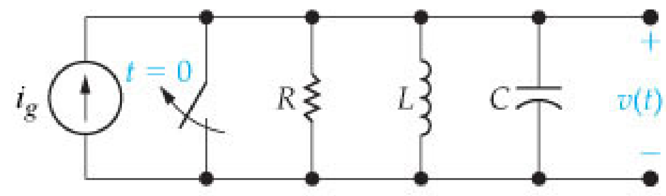
Express your answer in terms of s.

ANSWER:

$\mathcal{L}\left\{\frac{d^3}{dt^3}t^2u(t)\right\}$ =

Problem 12.26

In the circuit shown in , the source delivers a current of $i_g = 5\cos 20t \;\; \mathrm{A}$. The circuit components are R = 1.25 Ω, C = 50 mF, and L = 200 mH . No initial energy is stored in the circuit at the instant when the switch, which is shorting the current source, is opened.



Part A

Find the numerical expression for $V(s)$.

Express your answer in terms of s.

ANSWER:

$V(s)$ =

12.13

a) $f(t) = 20e^{-500(t-10)}u(t-10)$

USE $e^{-at} \longleftrightarrow \frac{1}{s+a}$

AND SHIFT THM $f(t-a)u(t-a) \longleftrightarrow e^{-as}F(s)$

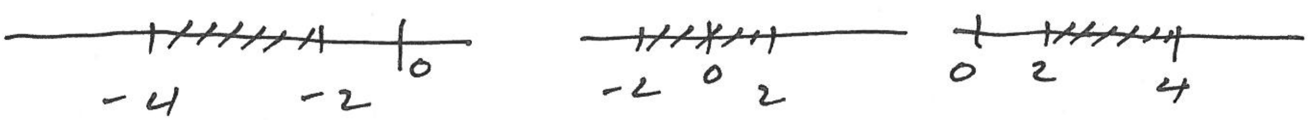
$$20e^{-500(t-10)}u(t-10) \longleftrightarrow \frac{20e^{-10s}}{s+500}$$

b) $f(t) = (5t+20)[u(t+4) - u(t+2)]$ (a)

$-5t[u(t+2) - u(t-2)]$ (b)

$+(5t-10)[u(t-2) - u(t-4)]$ (c)

TERM(a) TERM(b) TERM(c)



LAPLACE TRANSFORM (UNILATERAL) DEFINED ONLY FOR $t \geq 0$

$\Rightarrow$ ACTUAL FUNCTION WE CAN TRANSFORM IS

$$f(t) = -5t[u(t) - u(t-2)]$$
$$+(5t-20)[u(t-2) - u(t-4)]$$

$$f(t) = -5t\,u(t) + 5t\,u(t-2)$$

$$+ (5t-20)u(t-2) - (5t-20)u(t-4)$$

$$= -5t\,u(t) + 10(t-2)\,u(t-2)$$

$$-5(t-4)u(t-4)$$

NOW WE CAN USE

$$f(t-a)u(t-a) \longleftrightarrow e^{-as}F(s)$$

AND $\quad t\,f(t) \longleftrightarrow \frac{d}{ds}F(s)$

$$F(s) = -\frac{5}{s^2} + \frac{10e^{-2s}}{s^2} - \frac{5e^{-4s}}{s^2}$$

$$F(s) = \frac{-5\left(1 - 2e^{-2s} + e^{-4s}\right)}{s^2}$$

12.22 $\mathcal{L}\left\{\frac{d}{dt}\sin\omega t\right\}$

$$f(t) = \sin\omega t \longleftrightarrow \frac{\omega}{s^2+\omega^2}$$

$$\frac{d}{dt}f(t) \longleftrightarrow sF(s) - f(0^-)$$

$$f(0^-) = 0$$

a) $$\frac{d}{dt}\sin\omega t \longleftrightarrow \frac{s\omega}{s^2+\omega^2}$$

b) $\mathcal{L}\left\{\frac{d}{dt}\cos\omega t\right\}$

$$\cos\omega t \longleftrightarrow \frac{s}{s^2+\omega^2}$$

$$\cos(\omega t)\Big|_{t=0^-} = 1$$

$$\frac{d}{dt}\cos\omega t \longleftrightarrow \frac{s^2}{s^2+\omega^2} - 1$$

$$\longleftrightarrow \frac{-\omega^2}{s^2+\omega^2}$$

c) $\frac{d^3}{dt^3} t^2 u(t) \longleftrightarrow$

$$t\,u(t) \longleftrightarrow \frac{1}{s^2}$$

$$t\,f(t) \longleftrightarrow -\frac{d}{ds} F(s)$$

$$\therefore\ t^2 u(t) \longleftrightarrow \frac{2}{s^3}$$

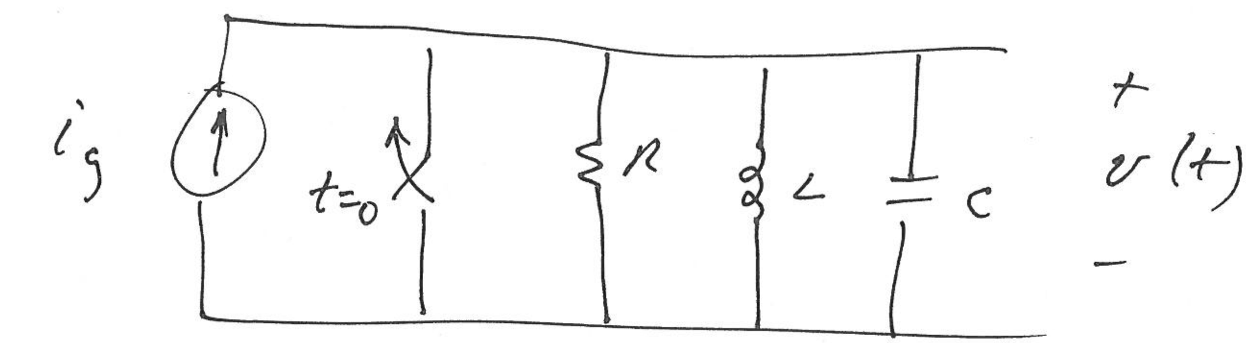
$f(t) = t^2 u(t)$

$f(0^-) = 0 \ ; \ f'(0^-) = 0 \ ; \ f''(0^-) = 0$

$$\therefore\ \frac{d^3}{dt^3} f(t) = s^3 F(s)$$

$$\frac{d^3}{dt^3} t^2 u(t) \longleftrightarrow 2$$

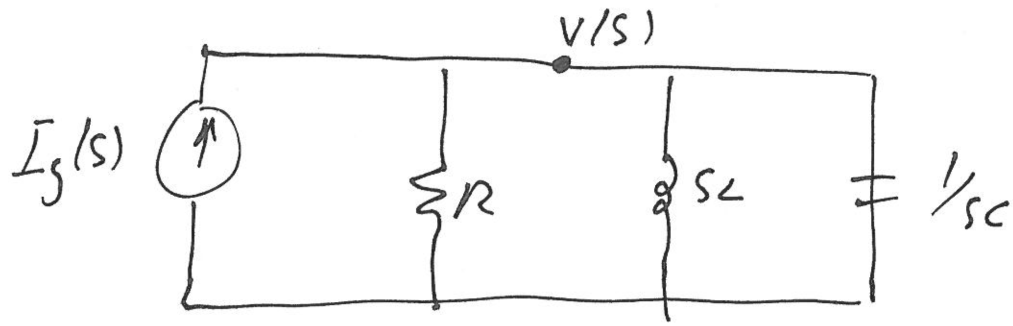
12.26



$$i_g(t) = 5\cos 20t\, u(t)\ A$$

$R = 1.25\ \Omega$
$L = 200\ mH$ NO INITIAL ENERGY
$C = 50\ mF$

$$I_g(s) = \frac{5s}{s^2+400}$$



KCL: $\dfrac{V(s)}{R} + \dfrac{V(s)}{sL} + \dfrac{V(s)}{1/sC} - I_g(s) = 0$

$$V(s)\left(\frac{1}{R} + \frac{1}{sL} + sC\right) = \frac{5s}{s^2+400}$$

$$V(s)\left(\frac{sL + R + s^2RLC}{sRL}\right) =$$

$$V(s)\left(\frac{s^2 + s/RC + 1/LC}{s/C}\right) = \frac{5s}{s^2+400}$$

$$V(s) = \frac{s/C}{s^2+s/RC + 1/LC} \cdot \frac{5s}{s^2+400}$$

$$V(s) = \frac{100s^2}{(s^2+16s+100)(s^2+400)}$$

$$V(s) = \frac{100s^2}{(s+8-j12)(s+8+j12)(s-j20)(s+j20)}$$