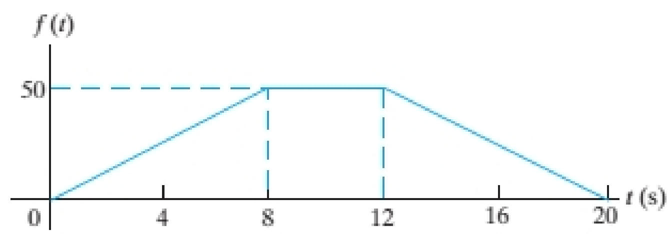
Problem 12.4

Part A

Use step functions to select the correct expression for the function shown in .

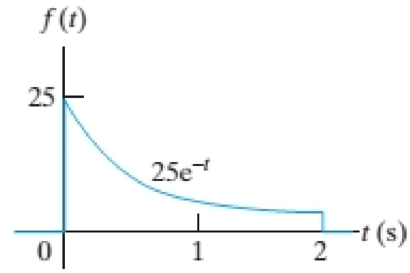


ANSWER:

- $f(t) = 6.25t(u(t) - u(t-9)) + 25(u(t-9) - u(t-12)) + (125 - 6.25t)(u(t-12) - u(t-20)$
- $f(t) = 6.25t(u(t) - u(t-8)) + 50(u(t-8) - u(t-12)) + (125 - 6.25t)(u(t-12) - u(t-20)$
- $f(t) = 6.25t(u(t) - u(t-8)) + 75(u(t-8) - u(t-10)) + (125 - 6.25t)(u(t-10) - u(t-20)$
- $f(t) = 6.25t(u(t) - u(t-8)) + 75(u(t-8) - u(t-12)) + (125 - 6.25t)(u(t-12) - u(t-20)$
- $f(t) = 6.25t(u(t) - u(t-5)) + 50(u(t-5) - u(t-12)) + (125 - 6.25t)(u(t-12) - u(t-20)$
- $f(t) = 6.25t(u(t) - u(t-8)) + 25(u(t-8) - u(t-12)) + (125 - 6.25t)(u(t-12) - u(t-20)$

Part B

Use step functions to select the correct expression for the function shown in .

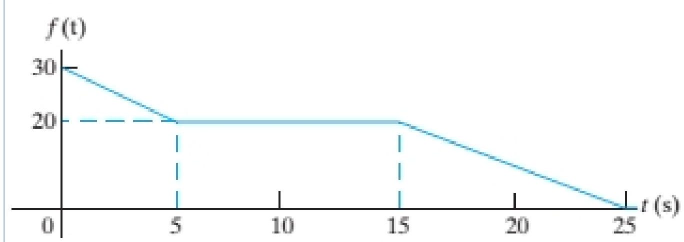


ANSWER:

- $f(t) = 50e^{-t}(u(t) - u(t-4))$
- $f(t) = 25e^{-t}(u(t) - u(t-4))$
- $f(t) = 75e^{-t}(u(t) - u(t-2))$
- $f(t) = 50e^{-t}(u(t) - u(t-2))$
- $f(t) = 75e^{-t}(u(t) - u(t-4))$
- $f(t) = 25e^{-t}(u(t) - u(t-2))$

Part C

Use step functions to select the correct expression for the function shown in .



ANSWER:

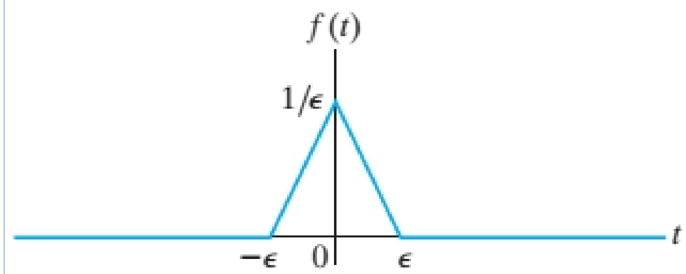
- $f(t) = (30-2t)(u(t)-u(t-3)) + 30(u(t-3)-u(t-15)) + (50-2t)(u(t-15)-u(t-25)$
- $f(t) = (30-2t)(u(t)-u(t-5)) + 40(u(t-5)-u(t-10)) + (50-2t)(u(t-10)-u(t-25)$
- $f(t) = (30-2t)(u(t)-u(t-5)) + 20(u(t-5)-u(t-15)) + (50-2t)(u(t-15)-u(t-25)$
- $f(t) = (30-2t)(u(t)-u(t-5)) + 40(u(t-5)-u(t-15)) + (50-2t)(u(t-15)-u(t-25)$
- $f(t) = (30-2t)(u(t)-u(t-5)) + 30(u(t-5)-u(t-15)) + (50-2t)(u(t-15)-u(t-25)$
- $f(t) = (30-2t)(u(t)-u(t-5)) + 20(u(t-5)-u(t-17)) + (50-2t)(u(t-17)-u(t-25)$

Problem 12.6

Part A

Find the area under the function shown in .

Express your answer using three significant figures.



ANSWER:

A =

Part B

What is the duration of the function when $\varepsilon = 0$?

ANSWER:

- 0
- 1
- 2
- ∞

Part C

What is the magnitude of $f(0)$ when $\varepsilon = 0$?

ANSWER:

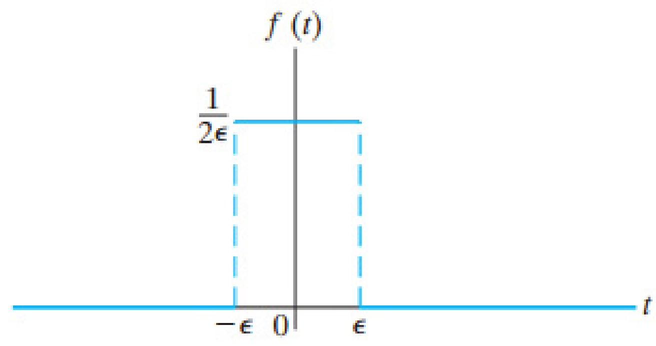
- 0
- 1
- 2
- ∞

Problem 12.9

In Section 12.3 in the textbook, we used the sifting property of the impulse function to show that $\mathscr{L}\{\delta(t)\} = 1$.

Part A

Show that we can obtain the same result by finding the Laplace transform of the rectangular pulse that exists between $\pm\epsilon$ in and then finding the limit of this transform at $\epsilon \to 0$.



ANSWER:

- $F(s) = \int_{-\epsilon}^{\epsilon} \frac{1}{2\epsilon} e^{-st}\,dt = \frac{e^{s\epsilon} + e^{-s\epsilon}}{2\epsilon s} = \frac{1}{2s} \lim_{\epsilon \to 0} \frac{se^{s\epsilon} + se^{-s\epsilon}}{1} = \frac{1}{2s}\frac{2s}{1} = 1$
- $F(s) = \int_{-\epsilon}^{\epsilon} \frac{1}{2\epsilon} e^{-st}\,dt = \frac{e^{s\epsilon} - e^{-s\epsilon}}{2\epsilon s} = \frac{1}{2s} \lim_{\epsilon \to 0} \frac{se^{s\epsilon} - se^{-s\epsilon}}{1} = \frac{1}{2s}\frac{2s}{1} = 1$
- $F(s) = \int_{-\epsilon}^{\epsilon} \frac{1}{2\epsilon} e^{-st}\,dt = \frac{e^{s\epsilon} - e^{-s\epsilon}}{2\epsilon s} = \frac{1}{2s} \lim_{\epsilon \to 0} \frac{se^{s\epsilon} + se^{-s\epsilon}}{1} = \frac{1}{2s}\frac{2s}{1} = 1$
- $F(s) = \int_{-\epsilon}^{\epsilon} \frac{1}{2\epsilon} e^{-st}\,dt = \frac{e^{s\epsilon} + e^{-s\epsilon}}{2\epsilon s} = \frac{1}{2s} \lim_{\epsilon \to 0} \frac{se^{s\epsilon} - se^{-s\epsilon}}{1} = \frac{1}{2s}\frac{2s}{1} = 1$

Problem 12.15

Part A

Find $\mathcal{L}\{e^{-at}f(t)\}$.

Express your answer in terms of $F(s)$ and s, where $F(s)$ is the Laplace tranform of $f(t)$.

ANSWER:

$\mathcal{L}\{e^{-at}f(t)\}$ =

Problem 12.20

Part A

Find the Laplace transform of the function $f(t) = te^{-at}$.

Express your answer in terms of the variables s and a.

ANSWER:

$\mathcal{L}\{te^{-at}\}$ =

Part B

Find the Laplace transform of the function $f(t) = \sin \omega t$.

Express your answer in terms of the variables s and ω.

ANSWER:

$\mathcal{L}\{\sin \omega t\}$ =

Part C

Find the Laplace transform of the function $f(t) = \sin(\omega t + \theta)$.

Express your answer in terms of the variables θ, s and ω.

ANSWER:

$\mathcal{L}\{\sin(\omega t + \theta)\}$ =

Part D

Find the Laplace transform of the function $f(t) = t$.

Express your answer in terms of s.

ANSWER:

$\mathcal{L}\{t\}$ =

Part E

Find the Laplace transform of the function $f(t) = \cosh(t + \theta)$.

ANSWER:

- $\frac{\sinh\theta + s[\cosh\theta]}{s^2 - 1}$
- $\frac{\cosh\theta + s[\sinh\theta]}{s^2 - 1}$
- $\frac{s[\cosh\theta]}{s^2 - 1}$
- $s[\sinh\theta] + s^2[\cosh\theta]$
- $s[\cosh\theta]$
- $\frac{\sinh\theta - s[\cosh\theta]}{s^2 + 1}$

12.4

a) $$f(t) = (6.25t)[u(t) - u(t-8)]$$
$$+ (50)[u(t-8) - u(t-12)]$$
$$+ (125 - 6.25t)[u(t-12) - u(t-20)]$$

VALUES WITHIN INTERVALS

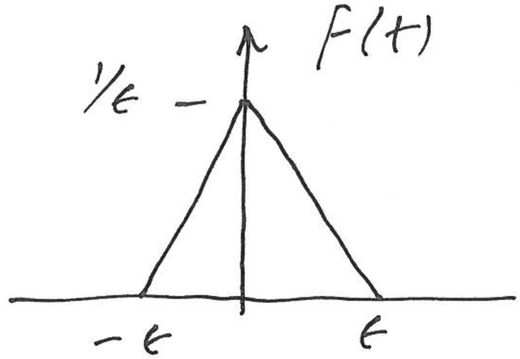
FUNDAMENTAL INTERVALS

b) $$f(t) = 25e^{-t}\underbrace{[u(t) - u(t-2)]}_{\text{INTERVAL}}$$

FUNCTION WITHIN INTERVAL

c) $$f(t) = (30 - 2t)[u(t) - u(t-5)]$$
$$+ (20)[u(t-5) - u(t-15)]$$
$$+ (50 - 2t)[u(t-15) - u(t-25)]$$

12.6

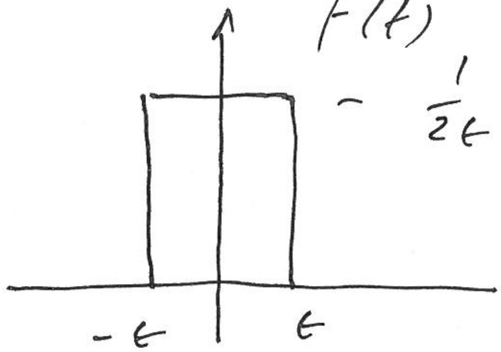


a) AREA = $2 \times \frac{1}{2}\epsilon \times \frac{1}{\epsilon} = 1.00$ //

b) DURATION OF f(t) WHEN $\epsilon = 0$ IS ZERO //

c) MAGNITUDE OF f(t) WHEN $\epsilon = 0$ IS ∞ //

12.9



$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$= \int_{-\epsilon}^{\epsilon} \frac{1}{2\epsilon} e^{-st} dt = \frac{1}{2\epsilon}\left[-\frac{1}{s} e^{-st} \Big|_{-\epsilon}^{\epsilon} \right]$$

$$= \frac{1}{2\epsilon s}\left(e^{+s\epsilon} - e^{-s\epsilon} \right)$$

$$F(s) = \frac{\text{SINCH}(\epsilon s)}{\epsilon s}$$

$$\lim_{\epsilon \to 0} \frac{\text{SINCH}(\epsilon s)}{\epsilon s} = \lim_{\epsilon \to 0} \frac{s \cosh(\epsilon s)}{s} = 1$$

(L'HOSPITAL'S RULE)

$$F(s) = 1$$

12.15 $\mathcal{L}\{e^{-at}f(t)\}$

$$\mathcal{L}\{f(t)\} \equiv \int_0^{\infty} e^{-st} f(t)\,dt \equiv F(s)$$

$$\mathcal{L}\{e^{-at}f(t)\} = \int_0^{\infty} e^{-at} e^{-st} f(t)\,dt$$

$$= \int_0^{\infty} e^{-(a+s)t} f(t)\,dt$$

$$= F(a+s) \;//$$

12.20

a) $f(t) = te^{-\alpha t}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty t e^{-at} e^{-st}\, dt$$

$$= \int_0^\infty t\, e^{-t(s+a)}\, dt$$

BY PARTS: $u = t,\ dv = e^{-t(s+a)}\, dt$

$du = dt,\ v = -\frac{1}{s+a} e^{-t(s+a)}$

$$F(s) = -\frac{t}{s+a} e^{-t(s+a)}\Big|_0^\infty + \frac{1}{s+a}\int_0^\infty e^{-t(s+a)}\, dt$$

$$= \frac{1}{s+a}\left(-\frac{1}{s+a} e^{-t(s+a)}\Big|_0^\infty\right)$$

$$F(s) = \frac{1}{(s+a)^2}$$

ANOTHER WAY: MAKE USE OF

$$e^{-\alpha t} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+\alpha}$$

$$t f(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{d}{ds} F(s)$$

$$-\frac{d}{ds} \frac{1}{s+\alpha} = \frac{1}{(s+\alpha)^2}$$

12.20, CONT'D.

b) $f(t) = \sin \omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j}$

$$F(s) = \frac{1}{2j}\int_0^{\infty}\left(e^{j\omega t} - e^{-j\omega t}\right)e^{-st}\,dt$$

$$= \frac{1}{2j}\left[\int_0^{\infty} e^{-t(s-j\omega)}\,dt - \int_0^{\infty} e^{-t(s+j\omega)}\,dt\right]$$

$$= \frac{1}{2j}\left[-\frac{1}{s-j\omega}e^{-t(s-j\omega)}\Big|_0^{\infty} + \frac{1}{s+j\omega}e^{-t(s+j\omega)}\Big|_0^{\infty}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{2j}\left[\frac{s+j\omega-(s-j\omega)}{s^2+\omega^2}\right]$$

$$F(s) = \frac{\omega}{s^2+\omega^2}$$

12.20, CONT'D.

c) $f(t) = \sin(\omega t + \theta)$

$$= \frac{e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}}{j2}$$

$$F(s) = \frac{1}{j2}\left[\int_0^\infty e^{j(\omega t+\theta)} e^{-st} dt - \int_0^\infty e^{-j(\omega t+\theta)} e^{-st} dt\right]$$

$$= \frac{1}{j2}\left[e^{j\theta}\int_0^\infty e^{-t(s-j\omega)} dt - e^{-j\theta}\int_0^\infty e^{-t(s+j\omega)} dt\right]$$

$$= \frac{1}{j2}\left[e^{j\theta}\left(-\frac{1}{s-j\omega} e^{-t(s-j\omega)}\Big|_0^\infty\right) - e^{-j\theta}\left(-\frac{1}{s+j\omega} e^{-t(s+j\omega)}\Big|_0^\infty\right)\right]$$

$$= \frac{1}{j2}\left[e^{j\theta}\left(\frac{1}{s-j\omega}\right) - e^{-j\theta}\left(\frac{1}{s+j\omega}\right)\right]$$

$$= \frac{1}{j2}\left[\frac{e^{j\theta}(s+j\omega) - e^{-j\theta}(s-j\omega)}{s^2+\omega^2}\right]$$

$$= \frac{1}{j2}\left[\frac{s(e^{j\theta} - e^{-j\theta}) + j\omega(e^{j\theta} + e^{-j\theta})}{s^2+\omega^2}\right]$$

c) $F(s) = \dfrac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$

d) $f(t) = t$

$$F(s) = \int_0^\infty t e^{-st}\, dt$$

BY PARTS: $u = t$, $dv = e^{-st} dt$

$du = dt$, $v = -\frac{1}{s} e^{-st}$

$$F(s) = -\frac{t}{s} e^{-st} \Big|_0^\infty + \frac{1}{s} \int_0^\infty e^{-st} dt$$

$$F(s) = \frac{1}{s^2}$$

ANOTHER WAY: MAKE USE OF

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s}$$

$$t f(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{d}{ds} F(s)$$

$$-\frac{d}{ds} \frac{1}{s} = \frac{1}{s^2}$$

12.20, CONT'D.

e) $f(t) = \cosh(t+\theta) = \dfrac{e^{(t+\theta)} + e^{-(t+\theta)}}{2}$

$$F(s) = \frac{1}{2}\left[e^{\theta}\int_0^{\infty} e^{t} e^{-st}\,dt \right.$$

$$\left. + e^{-\theta}\int_0^{\infty} e^{-t} e^{-st}\,dt \right]$$

$$= \frac{1}{2}\left[e^{\theta}\int_0^{\infty} e^{-t(s-1)}\,dt + e^{-\theta}\int_0^{\infty} e^{-t(s+1)}\,dt \right]$$

$$= \frac{1}{2}\left[e^{\theta}\left(-\frac{1}{s-1} e^{-t(s-1)}\Big|_0^{\infty}\right) \right.$$

$$\left. + e^{-\theta}\left(-\frac{1}{s+1} e^{-t(s+1)}\Big|_0^{\infty}\right) \right]$$

$$= \frac{1}{2}\left[e^{\theta}\left(\frac{1}{s-1}\right) + e^{-\theta}\left(\frac{1}{s+1}\right) \right]$$

$$= \frac{1}{2}\left[\frac{e^{\theta}(s+1) + e^{-\theta}(s-1)}{(s^2-1)} \right]$$

$$= \frac{1}{2}\left[\frac{s(e^{\theta}+e^{-\theta}) + (e^{\theta}-e^{-\theta})}{s^2-1} \right]$$

$$F(s) = \frac{s\cosh(\theta) + \sinh(\theta)}{s^2-1}$$