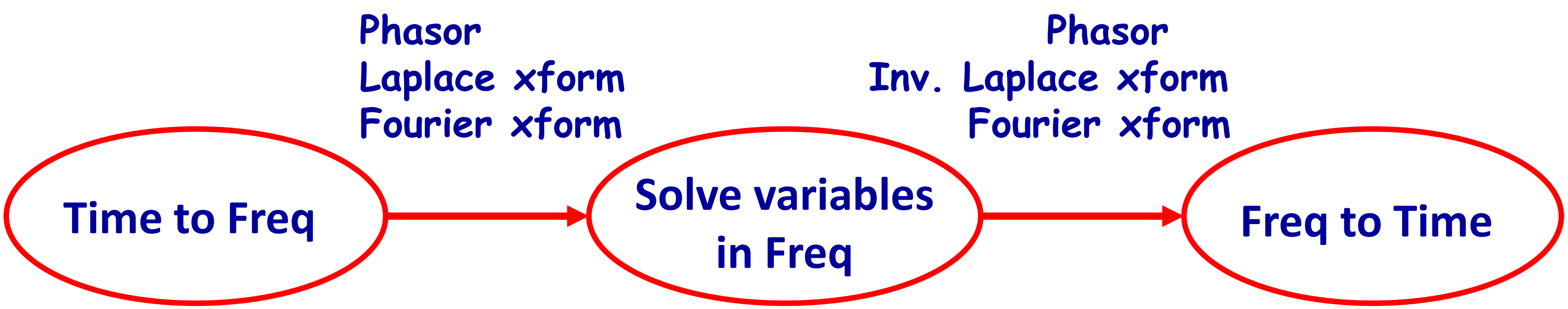
Chapter 10: Sinusoidal Steady-State Analysis

10.1 Basic Approach

- **3 Steps to Analyze AC Circuits:**
  1. **Transform** the circuit to the **phasor or frequency domain**.
  2. **Solve** the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
  3. **Transform** the resulting phasor to the time domain.



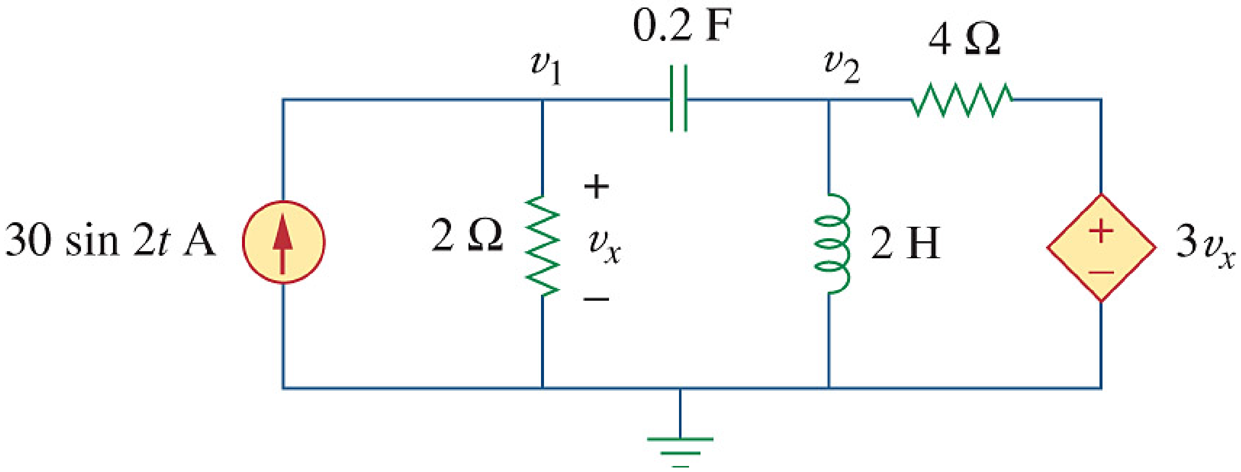
- **Sinusoidal Steady-State Analysis:**

  Frequency domain analysis of AC circuit via phasors is much easier than analysis of the circuit in the time domain.

10.2 Nodal Analysis

The basic of Nodal Analysis is *KCL*.

Example: Using nodal analysis, find v_1 and v_2 in the figure.



10.3 Mesh Analysis

The basic of Mesh Analysis is *KVL*.

Example: Find $\mathbf{I}_o$ in the following figure using mesh analysis.

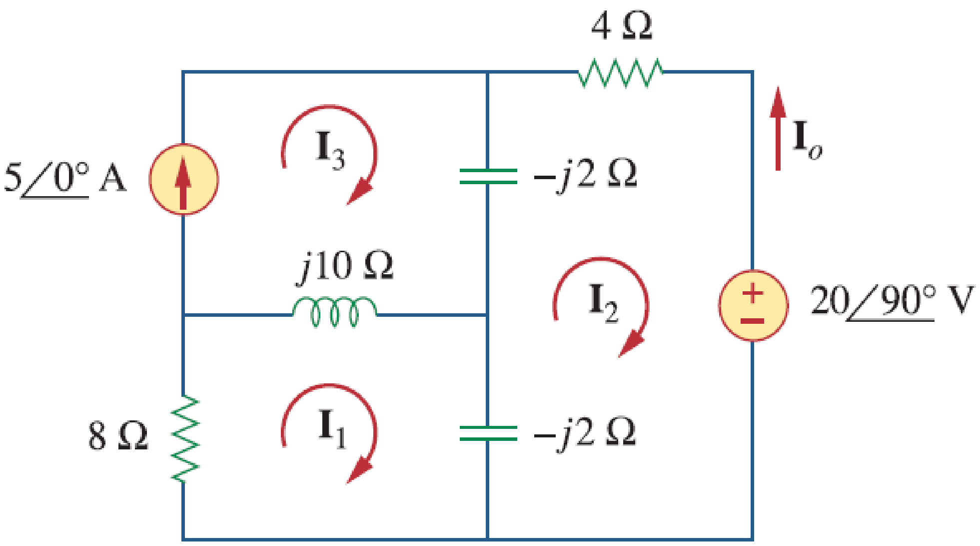


Table Node Voltage Analysis Using the Phasor Concept to Find the Sinusoidal Steady-State Node Voltages

1. Convert the independent sources to phasor form.
2. Select the nodes and the reference node and label the node voltages in the time domain, v_n, and their corresponding phasor voltages, $\mathbf{V}_n$.
3. If the circuit contains only independent current sources, proceed to step 5; otherwise, proceed to step 4.
4. If the circuit contains a voltage source, select one of the following three cases and the associated method:

CASE	METHOD
a. The voltage source connects node q and the reference node.	Set $\mathbf{V}_q = \mathbf{V}_s$ and proceed.
b. The voltage source lies between two nodes.	Create a supernode including both nodes.
c. The voltage source in series with an impedance lies between node d and the ground, with its positive terminal at node d.	Replace the voltage source and series impedance with a parallel combination of an admittance $\mathbf{Y}_1 = 1/\mathbf{Z}_1$ and a current source $\mathbf{I}_1 = \mathbf{V}_S\mathbf{Y}_1$ entering node d.

5. Using the known frequency of the sources, ω, find the impedance of each element in the circuit.
6. For each branch at a given node, find the equivalent admittance of that branch, $\mathbf{Y}_n$.
7. Write KCL at each node.
8. Solve for the desired node voltage $\mathbf{V}_a$, using Cramer's rule.
9. Convert the phasor voltage $\mathbf{V}_a$ back to the time-domain form.

Table Mesh Current Analysis Using the Phasor Concept to Find the Sinusoidal Steady-State Mesh Currents

1. Convert the independent sources to phasor form.
2. Select the mesh currents and label the currents in the time domain, i_n, and the corresponding phasor currents, $\mathbf{I}_n$.
3. If the circuit contains only independent voltage sources, proceed to step 5; otherwise, proceed to step 4.
4. If the circuit contains a current source, select one of the following two cases and the associated method:

CASE	METHOD
a. The current source appears as an element of only one mesh, n.	Equate the mesh current $\mathbf{I}_n$ to the current of the current source, accounting for the direction of the source current.
b. b. The current source is common to two meshes.	Create a supermesh as the periphery of the two meshes. In step 6, write one KVL equation around the periphery of the supermesh. Also record the constraining equation incurred by the current source.

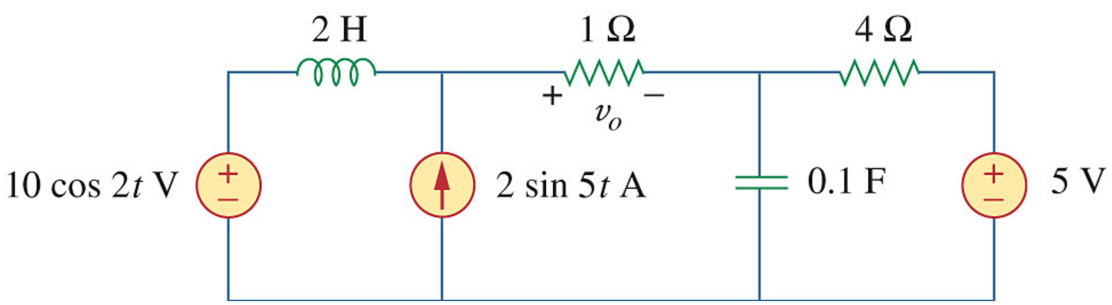
5. Using the known frequency of the sources, ω, find the impedance of each element in the circuit.
6. Write KVL for each mesh.
7. Solve for the desired mesh current $\mathbf{I}_n$, using Cramer's rule.
8. Convert the phasor current $\mathbf{I}_n$ back to the time-domain form.

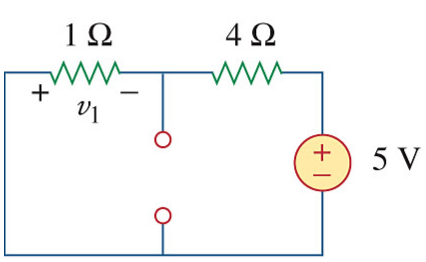
10.4 Superposition Theorem

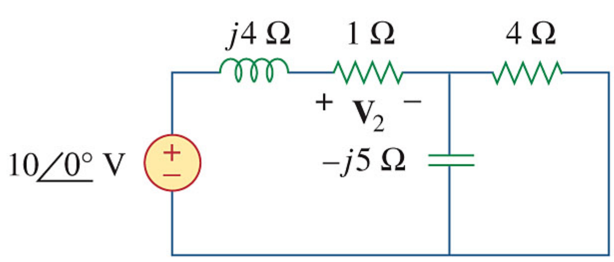
When a circuit has sources operating at **different frequencies**,

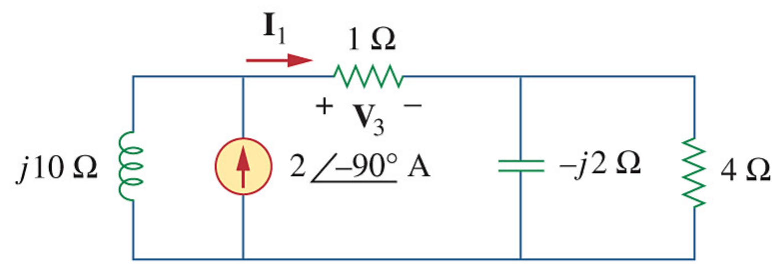
- The **separate** phasor circuit for each frequency must be solved **independently**, and
- The total response is the **sum of time-domain responses** of all the individual phasor circuits.

Example: Calculate v_o in the circuit using the superposition theorem.









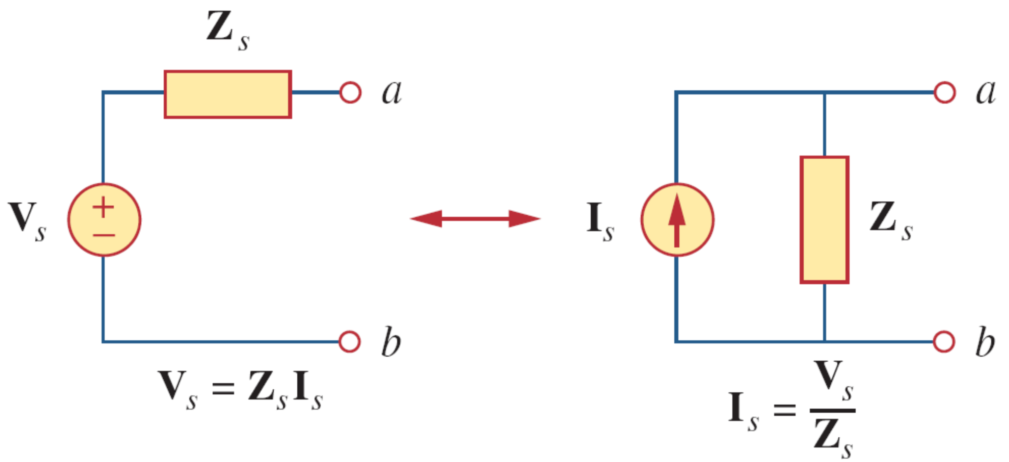
4.3 Superposition Theorem (1)

- **Superposition** states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to *EACH independent source acting alone*.
- The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.
- ***Steps to Apply Superposition Principle***:
  1. Turn off all indep. sources except one source. Find the output (v or i) due to that active source using techniques in Chapters 2 & 3.
  2. Repeat Step 1 for each of the other indep. sources.
  3. Find total contribution by adding all contributions from indep. sources.

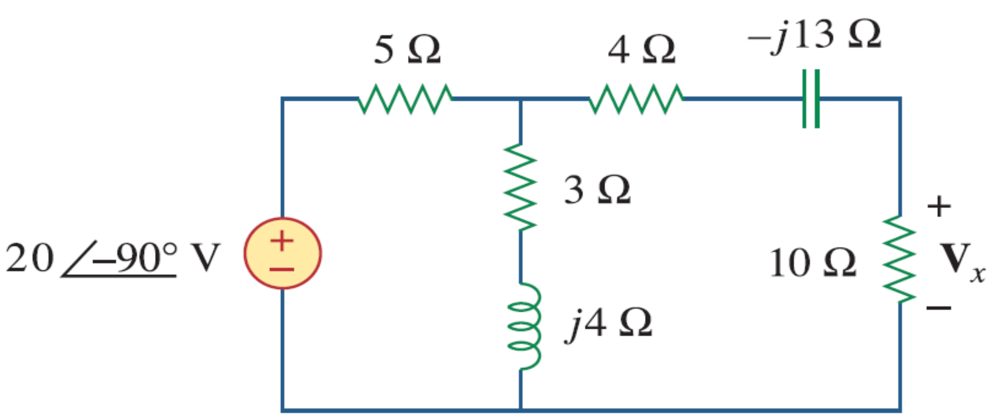
Note: In Step 1, this implies that we replace every **voltage source by $\mathbf{0\ V}$** (or a **short circuit**), and every **current source by $\mathbf{0\ A}$** (or an **open circuit**). Dependent sources are left intact because they are controlled by others.

10.5 Source Transformation (1)

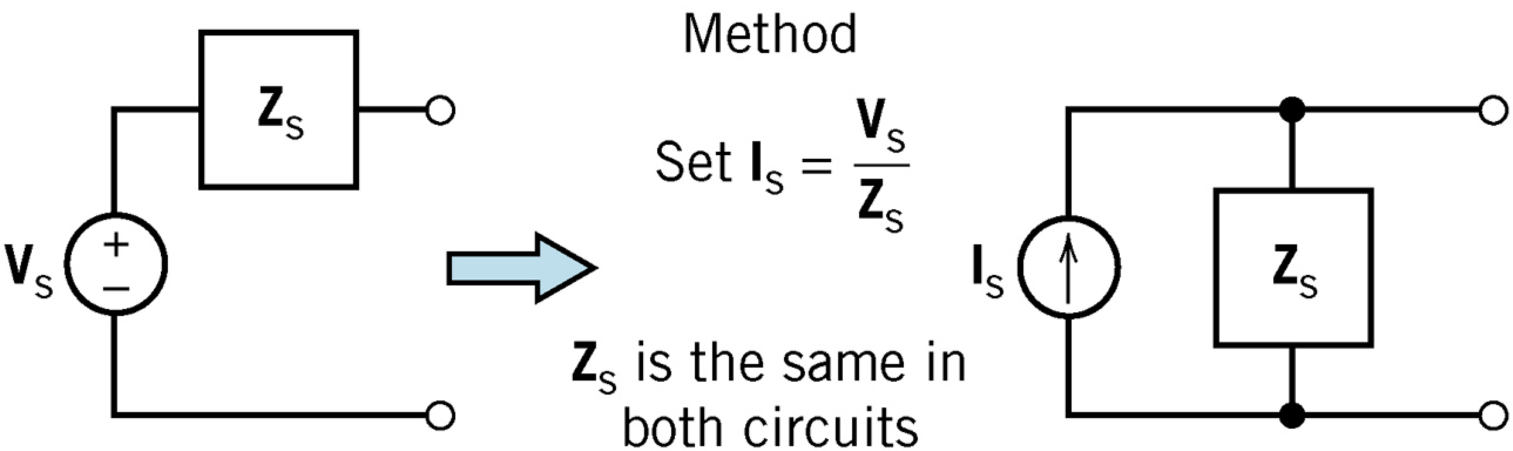
$$\mathbf{V}_s = \mathbf{Z}_s \mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

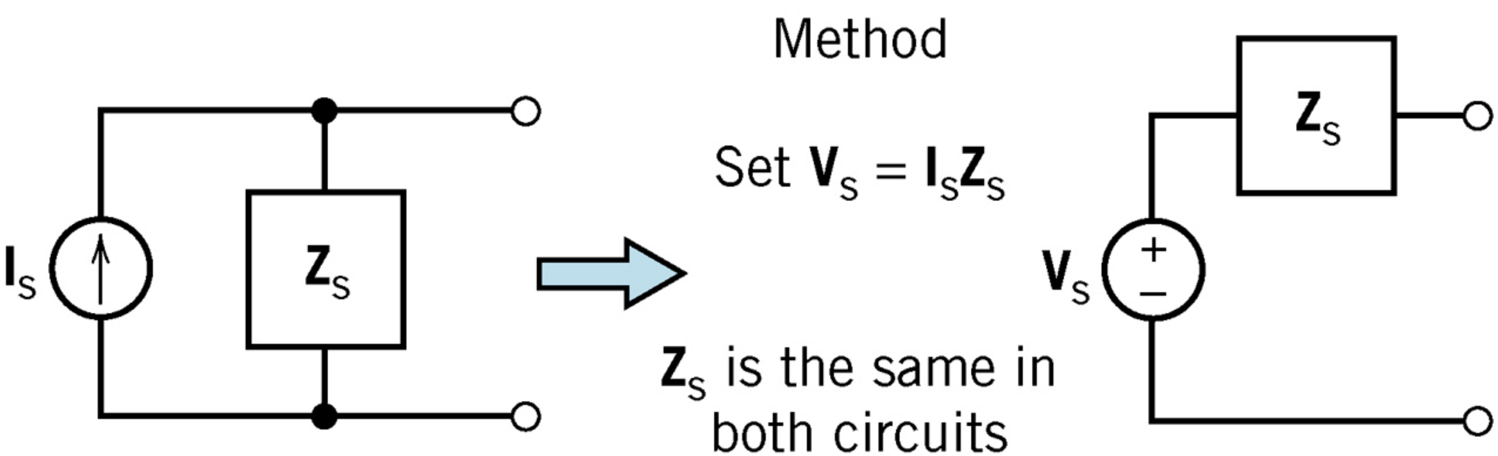


Example: Find $\mathbf{I}_o$ using the concept of source transformation.



10.5 Source Transformation (2)

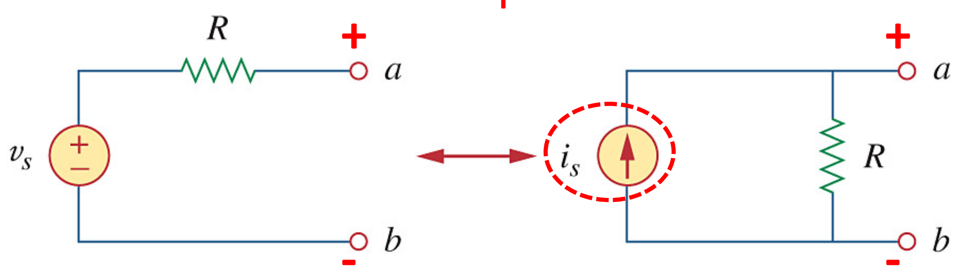




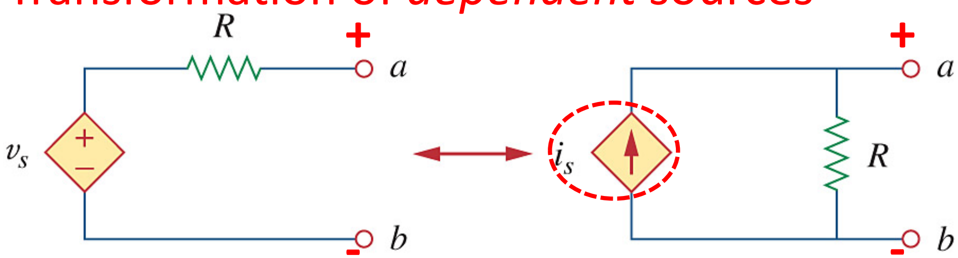
4.4 Source Transformation (1)

- Like series-parallel combination and wye-delta transformation, source transformation is another tool for simplifying circuits.
- An **equivalent circuit** is one whose v-i characteristics are identical with the original circuit.
- A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, and vice versa.

• Transformation of independent sources



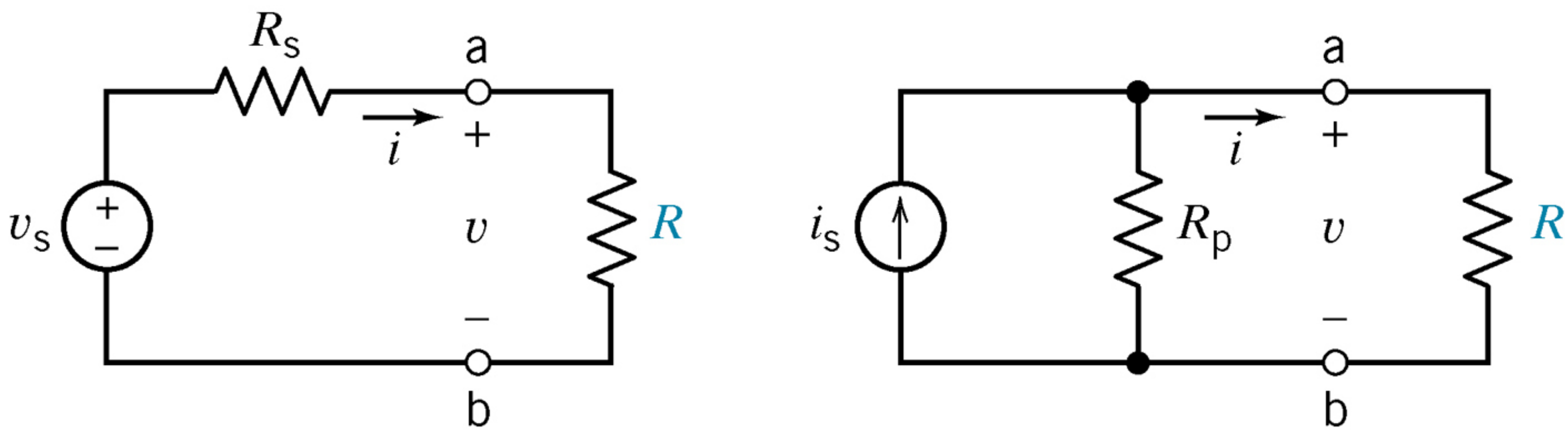
• Transformation of *dependent* sources



✓ The arrow of the current source is directed toward the positive terminal of the voltage source.

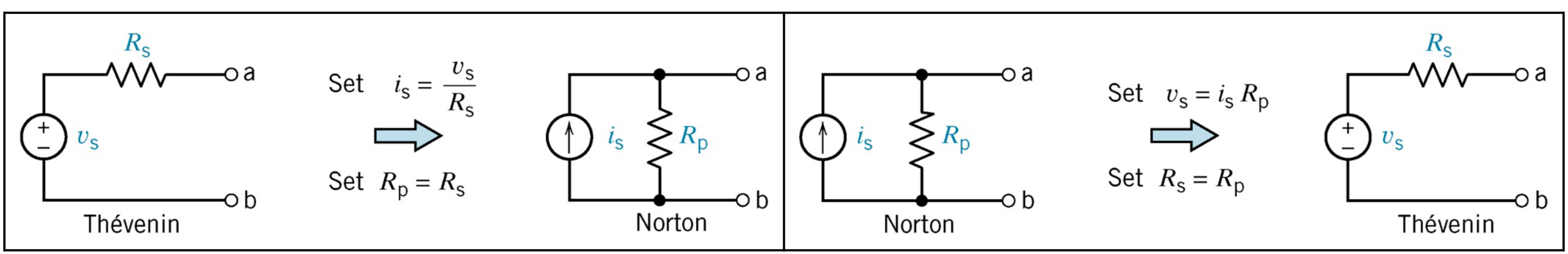
✓ The source transformation is not possible when $R = 0$ for voltage source and $R = \infty$ for current source.

4.4 Source Transformation (2)



A voltage source v_s connected in series with a resistor R_s and a current source i_s is connected in parallel with a resistor R_p are equivalent circuits provided that

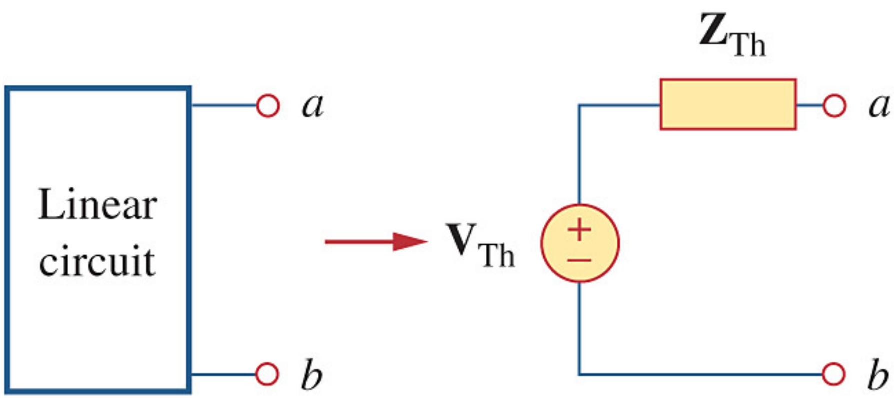
$$R_p = R_s \;\; \& \;\; v_s = R_s i_s$$



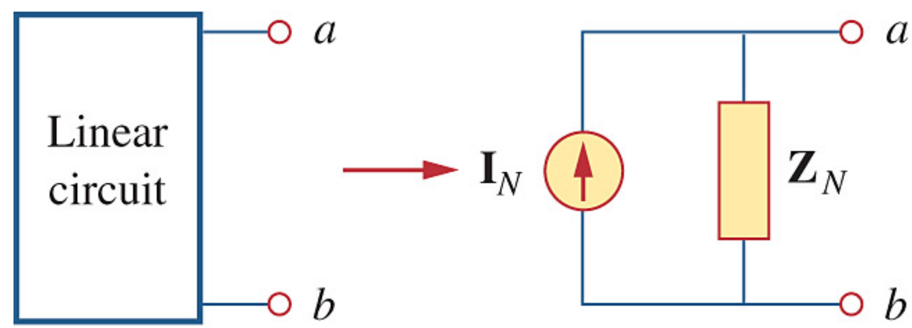
10.6 Thevenin & Norton Equivalent Circuits

$$\mathbf{V}_{\text{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\text{Th}} = \mathbf{Z}_N$$

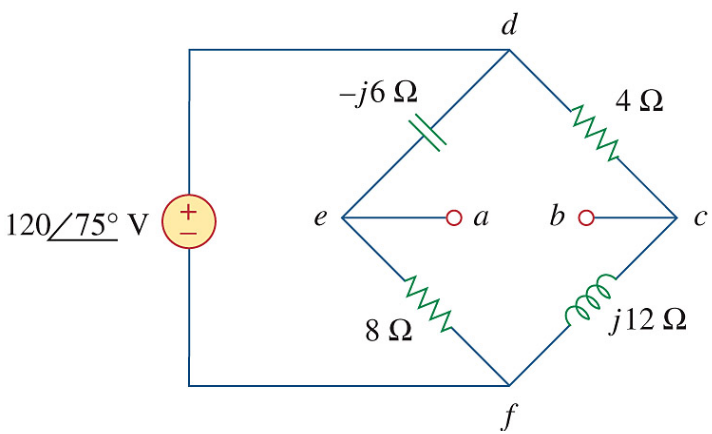
Thevenin Equivalent

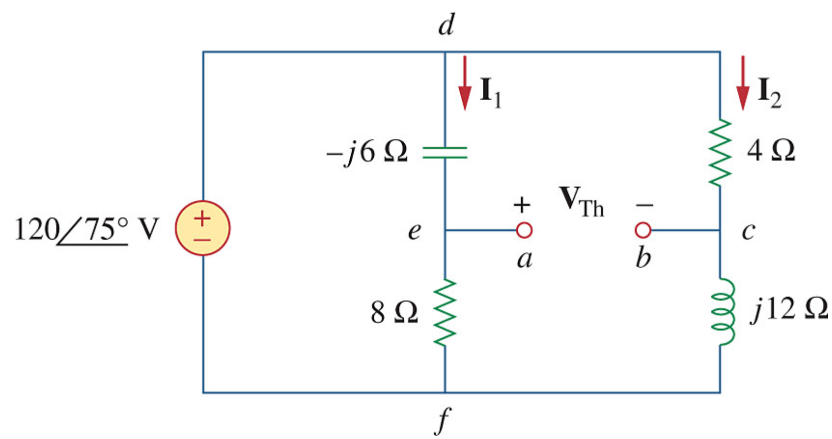


Norton Equivalent



Example: Find the Thevenin equivalent at terminals *a*–*b*.

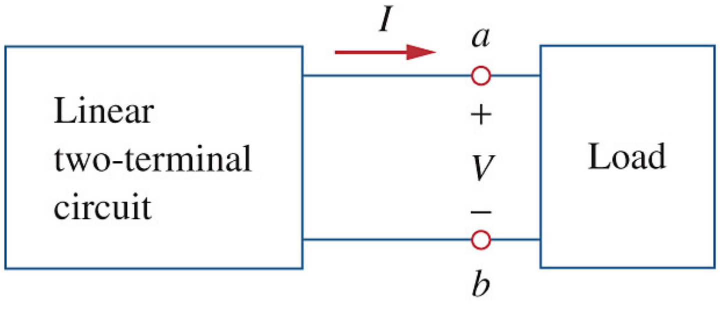




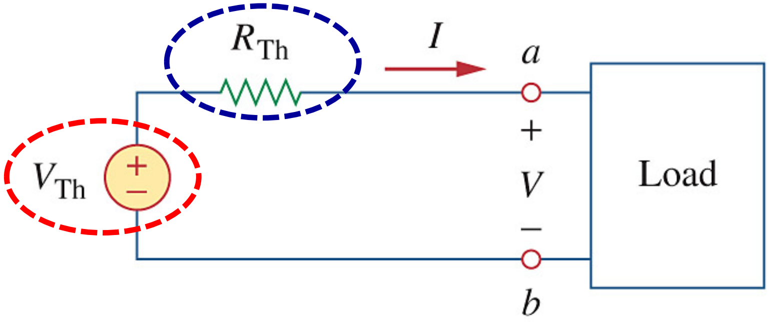
4.5 Thevenin's Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where

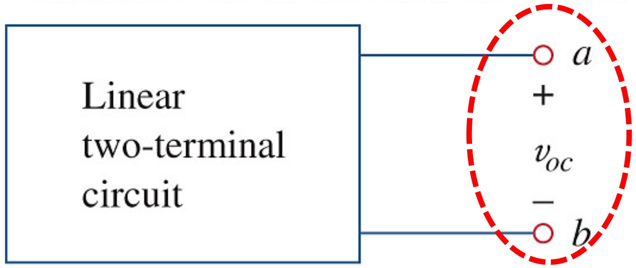
- ✓ V_{Th} is the open-circuit voltage at the terminals.
- ✓ R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



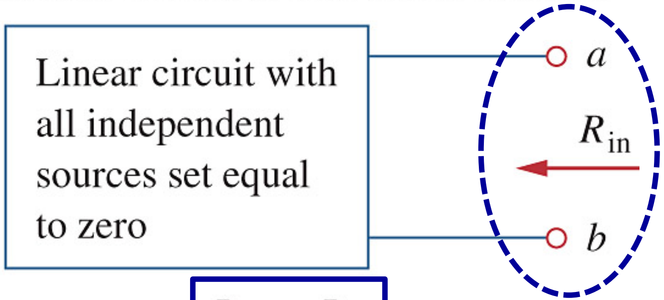
(a)



(b)



$V_{Th} = v_{oc}$



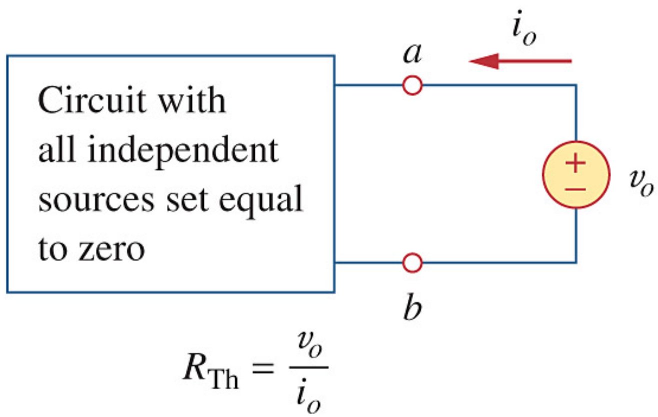
$R_{Th} = R_{in}$

4.5 Thevenin's Theorem (2)

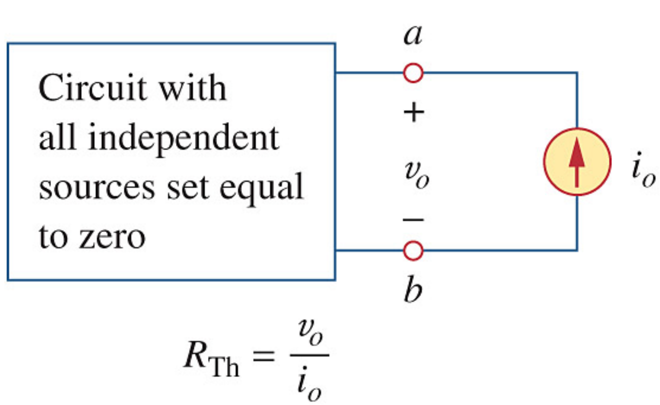
To find R_{Th} :

- ✓ ***Case 1***: If the network has no dependent sources, we turn off all indep. Source. R_{Th} is the input resistance of the network looking btw terminals a & b.
- ✓ ***Case 2***: If the network has depend. Sources. Depend. sources are not to be turned off because they are controlled by circuit variables. (a) Apply v_o at a & b and determine the resulting i_o. Then $R_{Th} = v_o/i_o$. Alternatively, (b) insert i_o at a & b and determine v_o. Again $R_{Th} = v_o/i_o$.

(a)



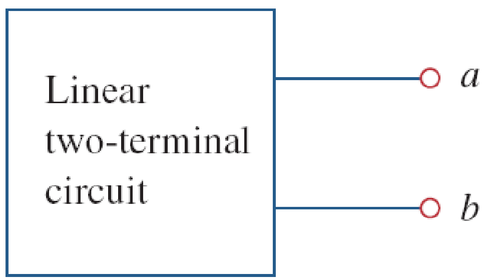
(b)



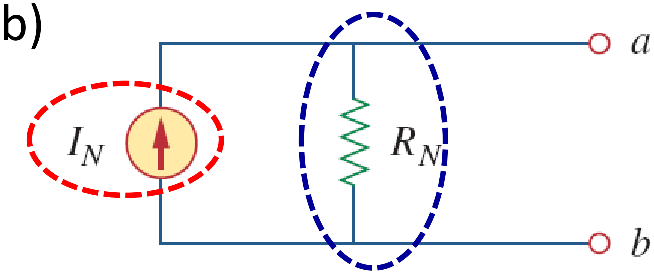
4.6 Norton's Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a current source I_N in parallel with a resistor R_N,

(a)

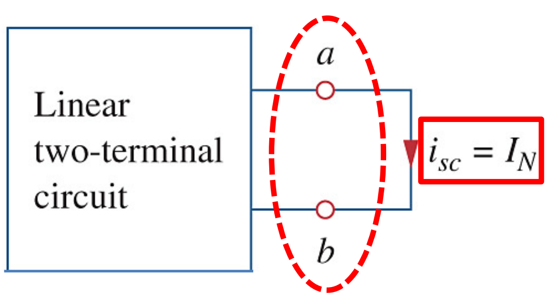


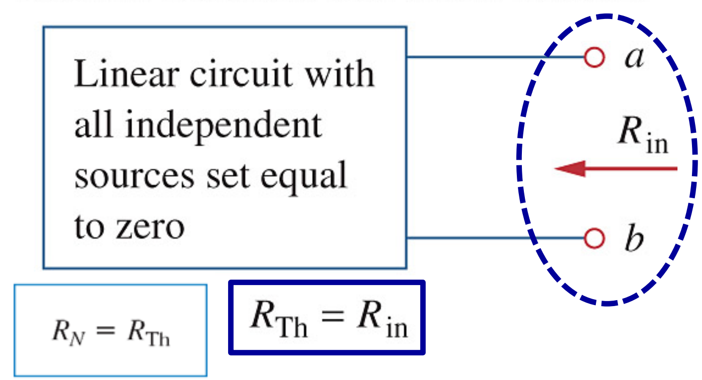
(b)



where

- ✓ I_N is the short-circuit current through the terminals.
- ✓ R_N is the input or equivalent resistance at the terminals when the indepen. sources are turned off.



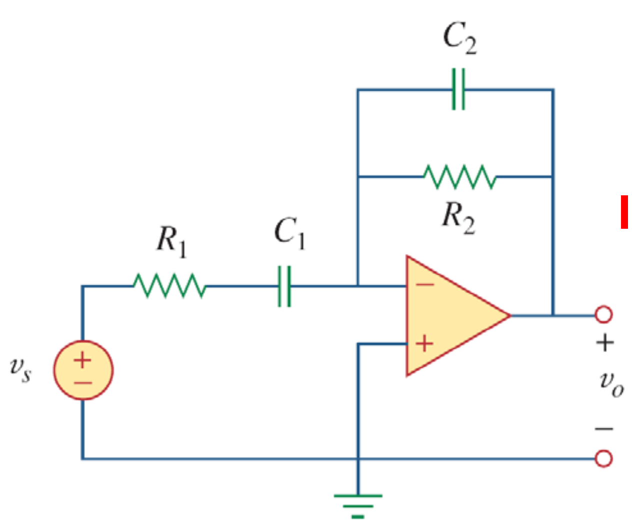


10.7 Op Amp AC Circuits (1)

The key to analyzing op amp circuits is to keep two important properties of an ideal op amp in mind:

- No current enters either of its input terminals.
- The voltage across its input terminals is zero.

Example: Compute the closed-loop gain and phase shift. Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$, and $\omega = 200$ rad/s.



$$\mathbf{Z}_f = R_2 \left\| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}\right.$$

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

Inverting Amplifier:

$$\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

Substituting the given values of R_1, R_2, C_1, C_2, and ω, we obtain

$$\mathbf{G} = \frac{-j4}{(1 + j4)(1 + j2)} = 0.434\underline{/130.6^\circ}$$

Thus the closed-loop gain is 0.434 and the phase shift is 130.6°.

10.9 Summary (1)

- With the pervasive use of ac electric power in the home and industry, it is important for engineers to analyze circuits with sinusoidal independent sources.
- The steady-state response of a linear circuit to a sinusoidal input is itself a sinusoid having the same frequency as the input signal.
- Circuits that contain inductors and capacitors are represented by differential equations. When the input to the circuit is sinusoidal, the phasors and impedances can be used to represent the circuit in the frequency domain. In the frequency domain, the circuit is represented by algebraic equations.
- The steady-state response of a linear circuit with a sinusoidal input is obtained as follows:
  1. Transform the circuit into the frequency domain, using phasors and impedances.

10.9 Summary (2)

2. Represent the frequency-domain circuit by algebraic equation, for example, mesh or node equations.
3. Solve the algebraic equations to obtain the response of the circuit.
4. Transform the response into the time domain, using phasors.

- A circuit contains several sinusoidal sources, two cases:
  - ✓ When all of the sinusoidal sources have the same frequency, the response will be a sinusoid with that frequency, and the problem can be solved in the same way that it would be if there was only one source.
  - ✓ When the sinusoidal sources have different frequencies, superposition is used to break the time-domain circuit up into several circuits, each with sinusoidal inputs all at the same frequency. Each of the separate circuits is analyzed separately and the responses are summed in the time domain.