SOLUTIONS

Name

ECE222
Midterm May 5. 2016

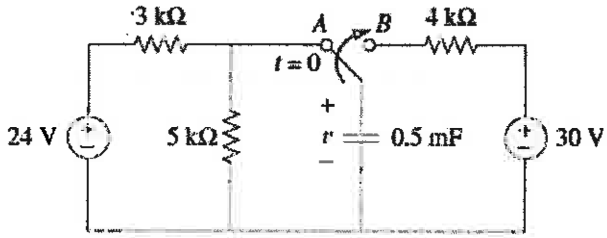
Instructions: This exam is closed book, closed notes. You may have a single sheet of formulas (front and back) with no worked problems that you must turn in with your exam.

Question 1:

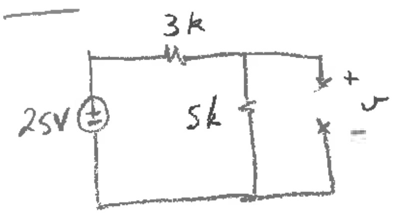
The switch in the circuit shown below has been in position A for a long time. At $t = 0$, the switch moves to B.

a) Find $v(t)$, $t \geq 0$

b) Calculate $v(t)$, at $t = 4$ s.



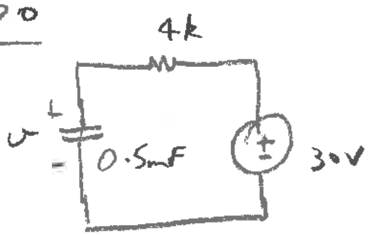
a) $t < 0$



$v(0^-) = \frac{5}{5+3} \times 25 = 15V$

$v(0^+) = v(0^-)$

$t > 0$



$v(\infty) = 30V$

$\tau = RC = 4 \times 10^3 \times 0.5 \times 10^{-3}$

$= 2\,s$

$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$

$= 30 + (15 - 30) e^{-t/2}$

$v(t) = 30 - 15 e^{-0.5t}$ V

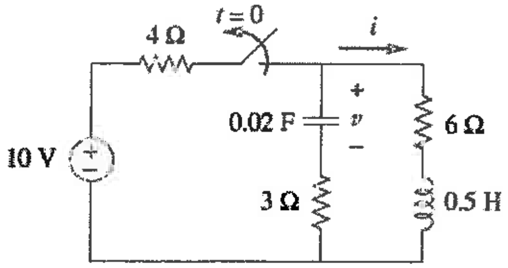
b) $v(4) = 30 - 15 e^{-0.5 \times 4}$

$= 30 - 15 e^{-2}$

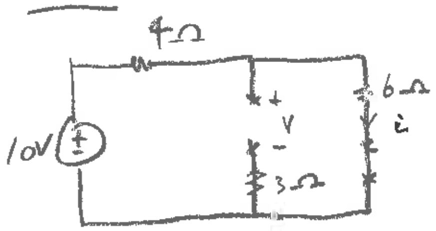
$v(t) = 27.97$ V

Question 2:

The switch in the circuit below has been closed for a long time before opening at $t = 0$. Find $i(t), t \geq 0$.



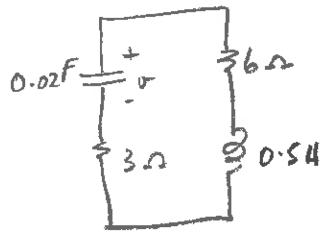
$t < 0$



$$v(0^-) = \frac{6}{4+6}\,10 = 6V$$

$$i(0^-) = \frac{10}{10} = 1A$$

$t > 0$



SERIES RLC NETWORK

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 0.02}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{9}{2(0.5)} = 9$$

$\Rightarrow \alpha < \omega_o \Rightarrow$ UNDERDAMPED RESPONCE

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -9 \pm \sqrt{81-100} = \underbrace{-9}_{-\alpha} \pm \underbrace{j\,4.359}_{\omega_d}$$

$$i(t) = e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right)$$

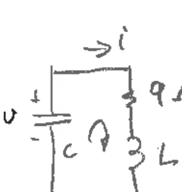
$$i(t) = e^{-9t}\left(B_1 \cos 4.359t + B_2 \sin 4.359t\right)$$

Now WE WILL USE THE INITIAL CONDITIONS to FIND B_1 and B_2

$$i(0) = i(0^+) = 1 = B_1$$

NOW

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{1}{L}\left(v(0) - 9\,i(0)\right) = 2\,(6-9) = -6$$



KVL $\Rightarrow -v(0) + 9i(0) + L\frac{di(0)}{dt} = 0$.

$\Rightarrow \frac{d\,i(0)}{dt} = \frac{1}{L}\left[v(0) - 9i(0)\right]$

ALSO $\frac{di(0)}{dt} = -\alpha B_1 + \omega_d B_2$

$\Rightarrow -6 = -9 + 4.359\, B_2$

$\Rightarrow B_2 = 0.6882$

$$\Rightarrow i(t) = e^{-9t}\left(\cos 4.359t + 0.6882 \sin 4.359t\right) A$$

Question 3:

Use phasors to find the (steady state solution for) current $i(t)$ in a circuit described by the following integro-differential equation:

$$4i(t) + 8\int_{-\infty}^{t} i(\tau)d\tau - 3\frac{di(t)}{dt} = 50\cos(2t + 75^{\circ})$$

Hint: Replace each term in the above equation with its phasor representation and then solve for the current phasor. The time domain solution should follow directly.

(Use of another method will result in a maximum score of 50%).

SUBSTITUTING PHASOR $I \Rightarrow$

$$4I + \frac{8I}{j\omega} - 3j\omega I = 50\angle 75^{\circ}$$

$\omega = 2 \Rightarrow$

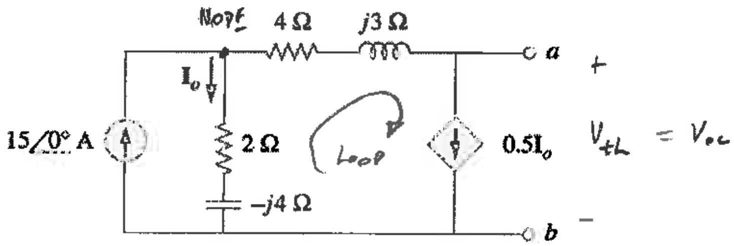
$$4I + \frac{8I}{j2} - 3j(2)I = 50\angle 75^{\circ}$$

$$\Rightarrow (4 - 4j - 6j)I = 50\angle 75^{\circ}$$

$$\Rightarrow I = \frac{50\angle 75^{\circ}}{4 - 10j} = \frac{50\angle 75^{\circ}}{10.77\angle -68.2} = 4.642\angle 143.2^{\circ}$$

$$\Rightarrow i(t) = 4.642\cos(2t + 143.2^{\circ})$$

Question 4:
Determine the Thevenin equivalent circuit of the circuit shown below as seen from terminals *a-b*.



V_{th} is the open circuit voltge, V_{oc}

KCL @ NODE $\Rightarrow$ $15 = I_o + 0.5 I_o$ $\Rightarrow$ $I_o = \frac{15}{1.5} = 10A$

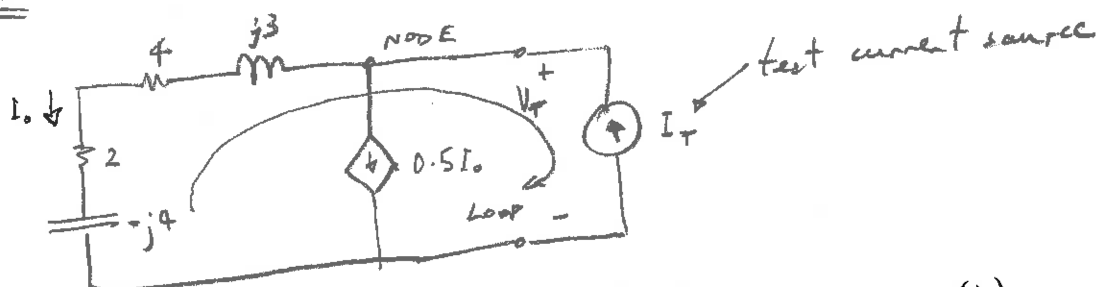
KVL AROUND LOOP $\Rightarrow -I_o(2 - j4) + (4+j3)0.5I_o + V_{th} = 0$

$\Rightarrow (-2+2 + j4 + j\frac{3}{2})I_o + V_{th} = 0$

$\Rightarrow -5.5j\ I_o = V_{th}.$

$\Rightarrow V_{th} = -55j \times 10 = -55j = 55\angle -90°$

Z_{th} : NULL ALL INDEPENDENT SOURCES $\Rightarrow$



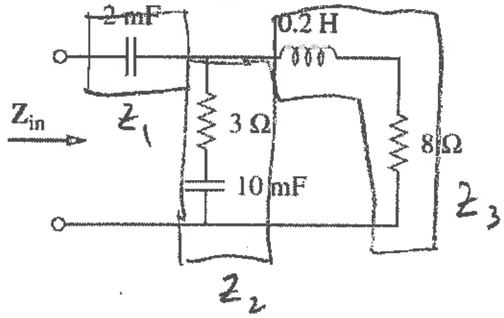
FROM ABOVE CIRCUIT : KCL AT NODE $I_T = I_o + 0.5I_o = 1.5I_o$ (1)

KVL AROUND LOOP : $-I_o(-j4 + 2 + 4 + j3) + V_T = 0$

$\Rightarrow V_T = (6 - j)I_o.$ (2)

(1) & (2) $\Rightarrow$ $Z_{th} = \frac{V_T}{I_T} = \frac{(6-j)I_o}{1.5\ I_o} = \frac{6-j}{1.5} = 4 - j0.6607\ \Omega$

Question 5:
Find the input impedance of the circuit shown below. Assume that the circuit operates at $\omega = 50$ rads/s.



$$Z_{in} = Z_1 + Z_2 \| Z_3 \;; \quad Z_2 \| Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$Z_1 = \frac{1}{j\omega C} = \frac{1}{j\,50 \times 2 \times 10^{-3}} = -j10 \ \Omega$$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$Z_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

$$Z_{in} = Z_1 + Z_2 \| Z_3$$

$$= -j10 + \frac{(3-j2)(8+j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2}$$

$$= -j10 + 3.22 - j1.07 \ \Omega$$

$$\underline{Z_{in} = 3.22 - j11.07 \ \Omega}$$