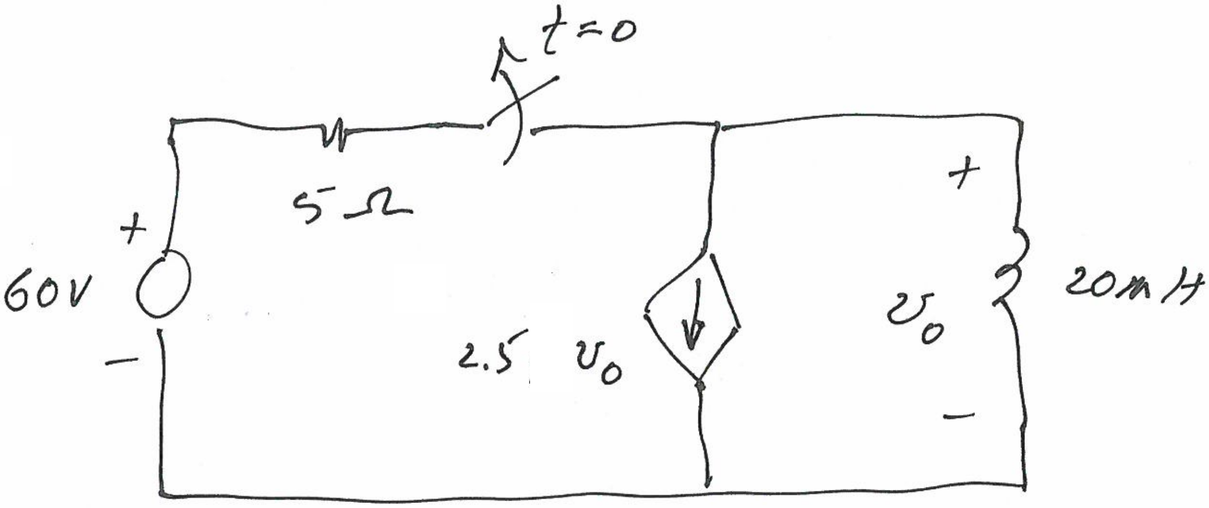
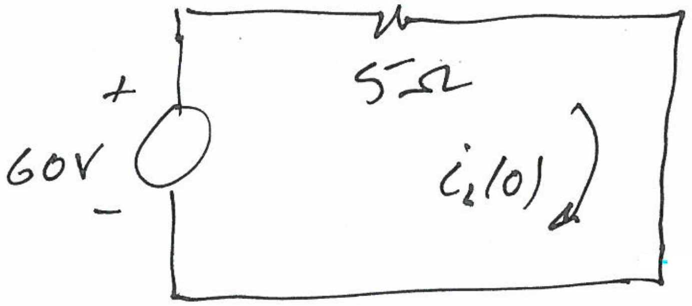
ECE222 Midterm Winter 2016 Solutions

1)



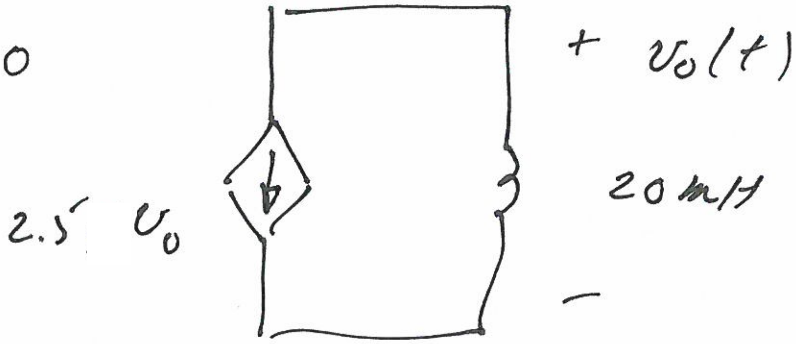
a) $t < 0$ INDUCTOR IS A WIRE $\Rightarrow v_0(0^-)=0$

$\Rightarrow$ DEP SOURCE IS ZERO CURRENT



$i_L(0) = \frac{60V}{5\Omega} = 12A$ //

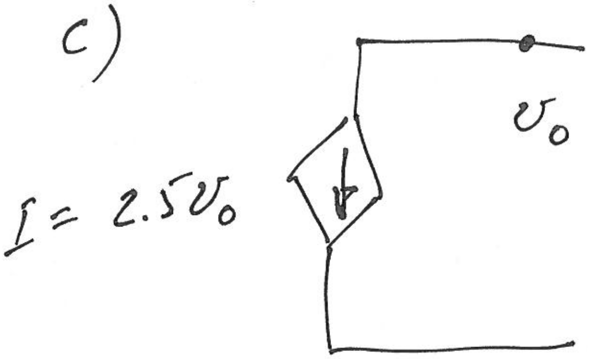
b) $t \geq 0$



NO INDP SOURCE $\Rightarrow i_L(\infty) = 0$ //

THÉVENIN EQ SEEN BY INDUCTOR

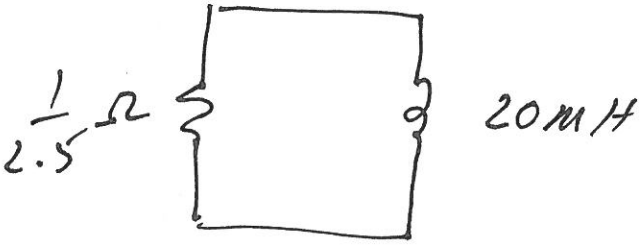
c)



$$I = 2.5 v_o$$

$$v_o = \frac{1}{2.5} I$$

$$\Rightarrow R_{TH} = \frac{1}{2.5}$$



$$\tau = \frac{L}{R} = 0.05 s$$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)] e^{-t/\tau}$$

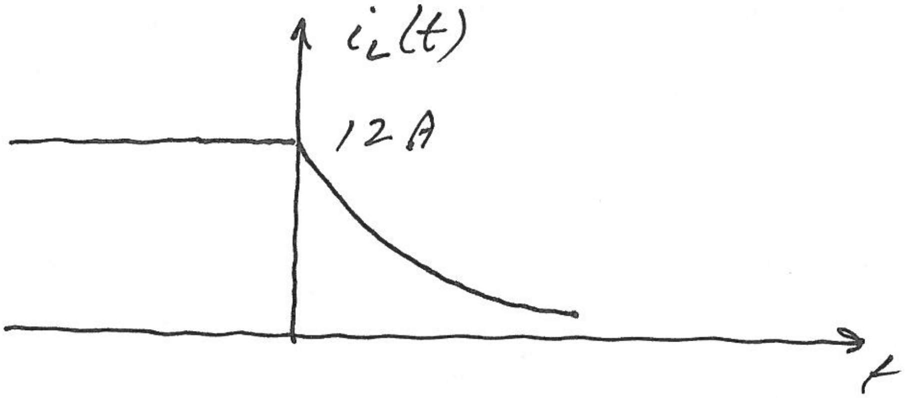
d) $i_L(t) = 12 e^{-20t} A \; ; \; t \geq 0$

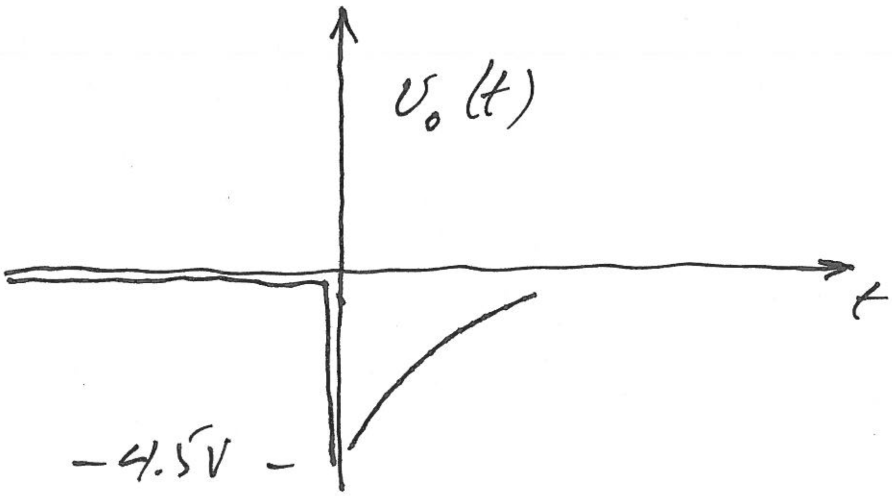
$$v_o(t) = L \frac{di(t)}{dt}$$

e) $= 20mH(-20)(12) e^{-20t}$

$$v_o(t) = -4.8 e^{-20t} \; ; \; t \geq 0^+$$

f)





2)

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

1ST INITIAL COND : $v(0) = V_0$ (VOLTAGE ON CAP.)

$$\boxed{A_1 + A_2 = V_0}$$

2ND INITIAL COND : KCL

$$i_c + i_L + i_R = 0$$

$$(1) \quad C \left.\frac{dv}{dt}\right|_{t=0} + \overset{= I_0 \text{ (CURRENT IN INDUCTOR)}}{i_L(0)} + \frac{\overset{= V_0}{v(0)}}{R} = 0$$

BUT $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$$\frac{dv(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

$$(2) \quad \left.\frac{dv(t)}{dt}\right|_{t=0} = A_1 s_1 + A_2 s_2$$

$$(1) \text{ \& } (2): \quad C(A_1 s_1 + A_2 s_2) + I_0 + \frac{v(0)}{R} = 0$$

$$\boxed{A_1 s_1 + A_2 s_2 = \frac{-(I_0 + V_0/R)}{C}}$$

3) $3x + j2y - jx + 5y = 18 + j5$

(1) $3x + 5y = 18$

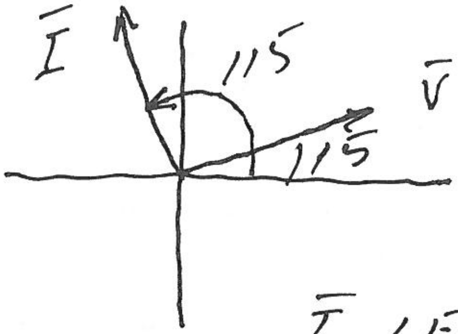
(2) $2y - x = 5 \rightarrow x = 2y - 5$

(1) & (2) : $3(2y-5) + 5y = 18 \rightarrow 6y - 15 + 5y = 18$

$11y = 33 \rightarrow y = 3$ //

From (2): $x = 2 \cdot 3 - 5 \rightarrow x = 1$ //

4) $\bar{V} = 20\angle 15°$ $\bar{I} = 2\angle 105°$



$\bar{I}$ LEADS $\bar{V}$ BY 90°

∴ CAPACITOR

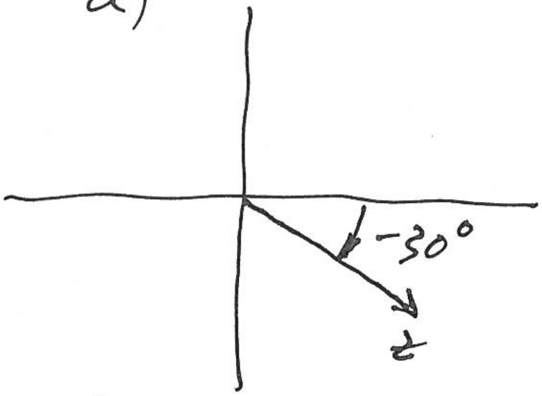
5) $z = 4\angle -30^\circ$

$z = 4\cos(-30^\circ)$

$\quad + 4\sin(-30^\circ)$

$z = 3.46 - j2$

a)



b) ⇒ CAPACITIVE

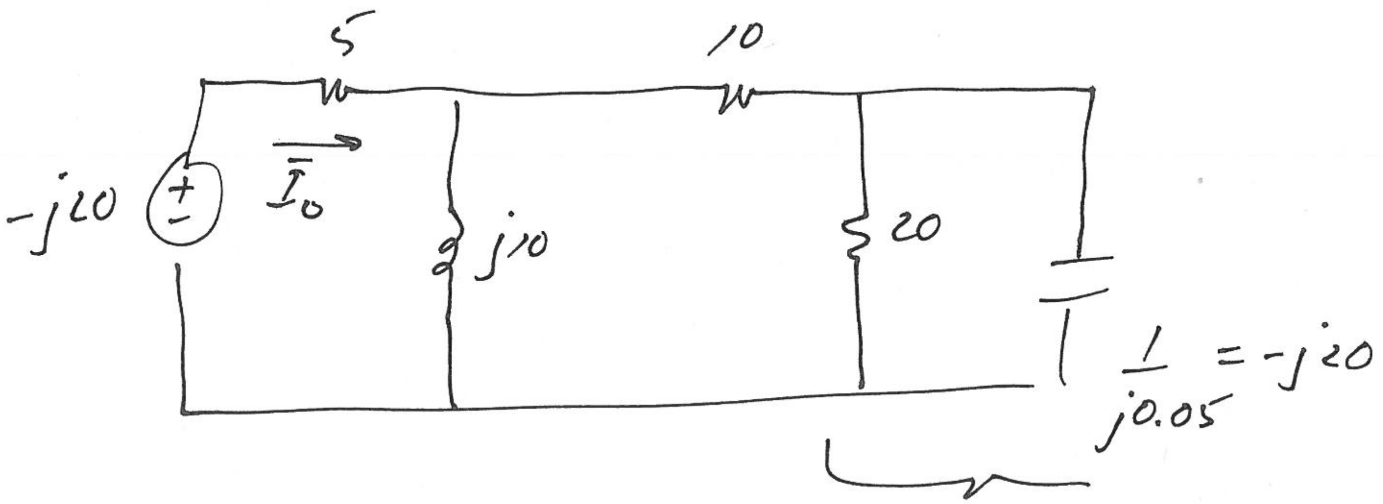
c) REACTANCE $= -2\ \Omega$

d) CURRENT WILL LEAD VOLTAGE BY 30°

6)

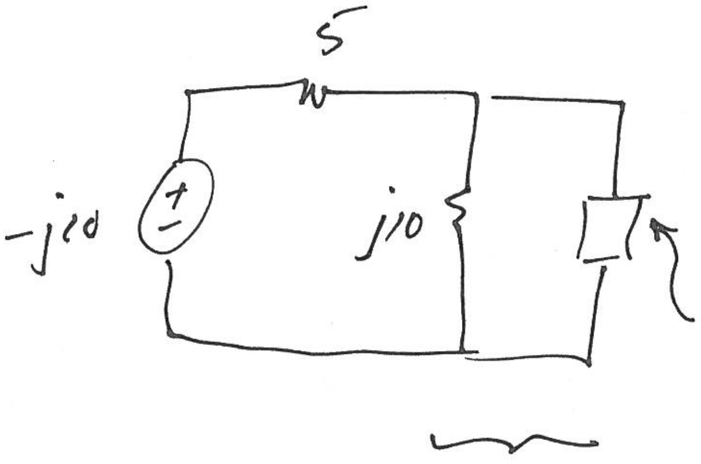
$$v_g(t) = 20\sin(4{,}000t) = 20\cos(4{,}000t - 90^\circ)$$

$$\bar{V}_g = 20\angle{-90^\circ} = -j20$$



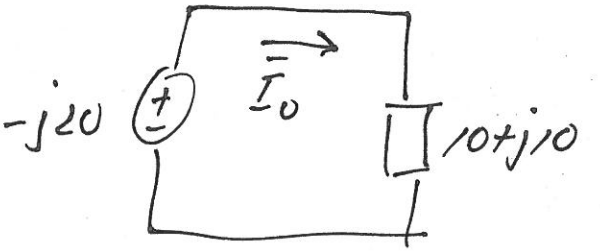
$$\frac{20(-j20)}{20-j20} = \frac{-j20}{1-j}$$

$$= \frac{-j20}{1-j}\left(\frac{1+j}{1+j}\right) = 10-j10$$



$10-j10+10$
$= 20-j10$

$$\frac{j10(20-j10)}{j10+20-j10} = 5+j10$$



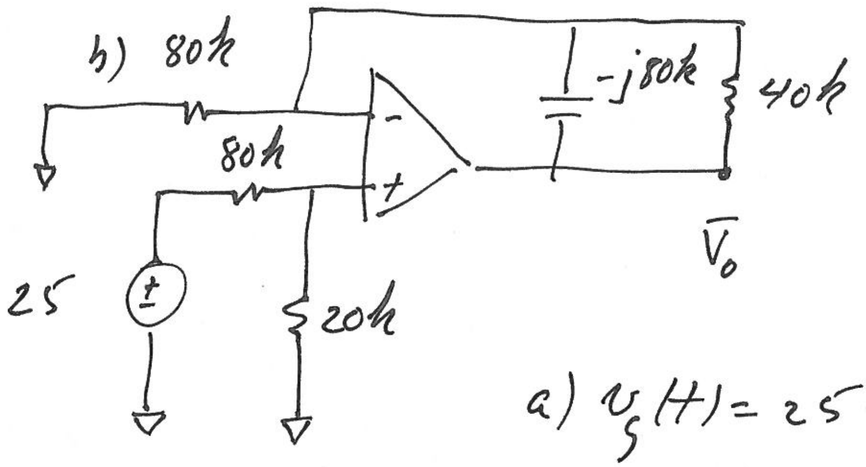
$$\bar{I}_o = \frac{-j20}{10+j10}\left(\frac{10-j10}{10-j10}\right)$$

$$= -1-j = \sqrt{2}\angle{-135^\circ}$$

$$i_o(t) = \sqrt{2}\cos(4{,}000t - 135^\circ)$$

7)

b)



a) $v_g(t) = 25\cos(50{,}000t)$ V

$\bar{V}_g = 25\angle 0^\circ = 25$

c)

$$\frac{\bar{V}_n}{80k} + \frac{\bar{V}_n - \bar{V}_o}{-j\,80k} + \frac{\bar{V}_n - \bar{V}_o}{40k} = 0 \qquad (1)$$

$$\frac{V_p - 25}{80k} + \frac{\bar{V}_p}{20k} = 0 \qquad (2)$$

$$\bar{V}_n + j(\bar{V}_n - \bar{V}_o) + 2(\bar{V}_n - \bar{V}_o) = 0 \qquad (1')$$

$$\bar{V}_p - 25 + 4\bar{V}_p = 0 \Rightarrow \bar{V}_p = 5V \qquad (2')$$

d) $\bar{V}_n = \bar{V}_p = 5V$

$$(1'') \quad 5 + j(5 - \bar{V}_o) + 2(5 - \bar{V}_o) = 0$$

$$5 + j5 + 10 - j\bar{V}_o - 2\bar{V}_o = 0$$

$$15 + j5 = \bar{V}_o(2 + j)$$

$$\bar{V}_o = \frac{15 + j5}{2 + j}$$

$$\bar{V}_0 = \frac{15+j5}{2+j}\left(\frac{2-j}{2-j}\right) = \frac{5(3+j)(2-j)}{5}$$

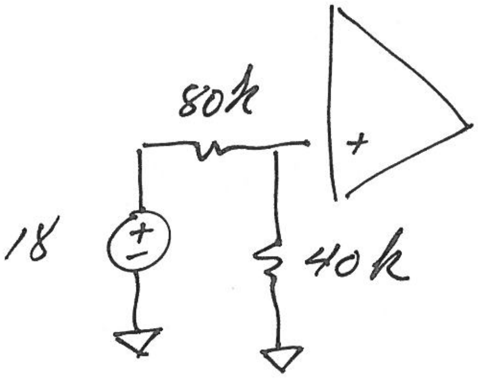
$$= 6-3j+2j+1 = 7-j$$

$$\bar{V}_0 = \sqrt{50}\angle -8.13^\circ$$

e) $v_0(t) = \sqrt{50}\cos(50{,}000t - 8.13^\circ)$ V

f) THE TWO RESISTORS AT THE NO-INVERTING TERMINAL CONSTITUTE A VOLTAGE DIVIDER

g) THIS CIRCUIT AMPLIFIES THE INPUT AND SHIFTS THE PHASE



$\bar{V}_p = 6V$

$v_0(t) = \frac{6}{5}\sqrt{50}\cos(50{,}000t - 8.13^\circ)$ V