



# Analysis Methodology

## Objectives

After reading this chapter, you will have learned

- How to make order-of-magnitude calculations
- The proper use of significant figures
- How to perform an analysis systematically
- The proper method of analysis presentation
- Advantages and disadvantages of using computers for analysis

## 1 INTRODUCTION

One of the most important things an engineering student learns during his or her program of study is how to approach an engineering problem in a systematic and logical fashion. In this respect, the study of engineering is somewhat similar to the study of science in that a science student learns how to think like a scientist by employing the scientific method. The scientific method is a process by which hypotheses about the physical world are stated, theories formulated, data collected and evaluated, and mathematical models constructed. The **engineering method** may be thought of as a problem-solving process by which the needs of society are met through design and manufacturing of devices and systems. Engineering analysis is a major part of this problem-solving process. Admittedly, engineering and science are not the same, because they each play a different role in our technical society. Science seeks to explain how nature works through fundamental investigations of matter and energy. The objective of engineering is more pragmatic. Engineering, using science and mathematics as tools, seeks to design and build products and processes that enhance our standard of living. Generally, the scientific principles underlying the function of any engineering device were derived and established *before* the device was designed. For example, Newton's laws of motion and Kepler's orbital laws were well established scientific principles long before spacecraft orbited the earth or the other planets. Despite their contrasting objectives, both engineering and science employ tried-and-true methodologies that enable people working in each field to solve a variety of problems. To do science, the scientist must know how to employ the scientific method. To do engineering, the engineer must know how to employ the "engineering method."

Engineering analysis is the solution of an engineering problem by using mathematics and principles of science. Because of the close association between analysis and design, analysis is one of the key steps in the design process. Analysis also plays a major role in the study of engineering failures. The engineering method for conducting an analysis is a logical, systematic procedure characterized by a well-defined format. This procedure, when consistently and

correctly applied, leads to the successful solution of any analytical engineering problem. Practicing engineers have been using this analysis procedure successfully for decades, and engineering graduates are expected to know how to apply it upon entering the technical workforce. Therefore, it behooves the engineering student to learn the analysis methodology as thoroughly as possible. The best way to do so is to practice solving analytical problems. As you advance in your engineering course work, you will have ample opportunities to apply the analysis methodology outlined in this chapter. Courses such as statics, dynamics, mechanics of materials, thermodynamics, fluid mechanics, heat and mass transfer, electrical circuits, and engineering economics are analysis intensive. These courses, and others like them, focus almost exclusively on solving engineering problems that are analytical in nature. That is the character of these engineering subjects. The analysis methodology presented here is a *general* procedure that can be used to solve problems in any analytical subject. Clearly, engineering analysis heavily involves the use of numerical calculations.

## 2 NUMERICAL CALCULATIONS

As a college student, you are well aware of the rich diversity of academic programs and courses offered at institutions of higher learning. Because you are an engineering major, you are perhaps more familiar with the genre of engineering, science, and mathematics courses than with liberal arts courses such as sociology, philosophy, psychology, music, and languages. The tenor of liberal arts is vastly different from that of engineering. Suppose for a moment that you are enrolled in a literature class, studying Herman Melville's great book *Moby Dick*. While discussing the relationship between the whale and Captain Ahab, your literature professor asks the class, "What is your impression of Captain Ahab's attitude toward the whale?" As an engineering major, you are struck by the apparent looseness of this question. You are accustomed to answering questions that require a quantitative answer, not an "impression." What would engineering be like if our answers were "impressions"? Imagine an engineering professor asking a thermodynamics class, "What is your impression of the superheated steam temperature at the inlet of the turbine?" A more appropriate question would be, "What *is* the superheated steam temperature at the inlet of the turbine?" Obviously, literature and the other liberal arts disciplines operate in a completely different mode from engineering. By its very nature, engineering is based on specific quantitative information. An answer of "hot" to the second thermodynamics question would be quantitative, but not specific and therefore insufficient. The temperature of the superheated steam at the inlet of the turbine could be calculated by conducting a thermodynamic analysis of the turbine, thereby providing a *specific* value for the temperature, 400°C for example. The analysis by which the temperature was obtained may consist of several numerical calculations involving different thermodynamic quantities. Numerical calculations are mathematical operations on numbers that represent physical quantities such as temperature, stress, voltage, mass, flow rate, etc. In this section, you will learn the proper numerical calculation techniques for engineering analysis.

### 2.1 Approximations

It is often useful, particularly during the early stages of design, to calculate an approximate answer to a given problem when the given information is uncertain or when little information is available. An approximation can be used to establish the cursory aspects of a design and to determine whether a more precise calculation is required. Approximations are usually based on assumptions, which must be modified

or eliminated during the latter stages of the design. Engineering approximations are sometimes referred to as “guesstimates,” “ballpark calculations,” or “back-of-the-envelope calculations.” A more appropriate name for them is **order-of-magnitude calculations**. The term order of magnitude means a *power of 10*. Thus, an order-of-magnitude calculation refers to a calculation involving quantities whose numerical values are estimated to within a factor of 10. For example, if the estimate of a stress in a structure changes from about 1 kPa to about 1 MPa, we say that the stress has changed by three orders of magnitude, because 1 MPa is one thousand ( $10^3$ ) times 1 kPa. Engineers frequently conduct order-of-magnitude calculations to ascertain whether their initial design concepts are feasible. Order-of-magnitude calculations are therefore a useful decision-making tool in the design process. Order-of-magnitude calculations do not require the use of a calculator because all the quantities have simple power-of-10 values, so the arithmetic operations can be done by hand with pencil and paper or even in your head. The example that follows illustrates an order-of-magnitude calculation.

### EXAMPLE 1

A warehouse with the approximate dimensions 200 ft  $\times$  150 ft  $\times$  20 ft is ventilated with 12 large industrial blowers. In order to maintain acceptable air quality in the warehouse, the blowers must provide two air changes per hour, meaning that the entire volume of air within the warehouse must be replenished with fresh outdoor air two times per hour. Using an order-of-magnitude analysis, find the required volume flow rate that each blower must deliver, assuming the blowers equally share the total flow rate.

#### Solution

To begin, we estimate the volume of the warehouse. The length, width, and height of the warehouse is 200 ft, 150 ft, and 20 ft, respectively. These lengths have order-of-magnitude values of  $10^2$ ,  $10^2$ , and  $10^1$ , respectively. Two air changes per hour are required. Thus, the total volume flow rate of air for the warehouse, including the factor of two air changes per hour, is:

$$Q_t \approx (10^2 \text{ ft})(10^2 \text{ ft})(10^1 \text{ ft})(2 \text{ air changes/h}) = 2 \times 10^5 \text{ ft}^3/\text{h}.$$

(Note that “air changes” is not a unit, so it does not appear in the answer.) The number of blowers (12) has an order-of-magnitude value of  $10^1$ . Based on the assumption that each blower delivers the same flow rate, the flow rate per blower is the total volume flow rate divided by the number of blowers:

$$\begin{aligned} Q &= Q_t/N = (2 \times 10^5 \text{ ft}^3/\text{h})(10^1 \text{ blowers}) \\ &= 2 \times 10^4 \text{ ft}^3/\text{h} \cdot \text{blower} \approx 10^4 \text{ ft}^3/\text{h} \cdot \text{blower}. \end{aligned}$$

Our order-of-magnitude calculation shows that each blower must supply  $10^4 \text{ ft}^3/\text{h}$  of outdoor air to the warehouse.

How does our order-of-magnitude answer compare with the exact answer? The exact answer is:

$$Q = (200 \text{ ft})(150 \text{ ft})(20 \text{ ft})(2 \text{ air changes/h})/(12 \text{ blowers}) = 1 \times 10^5 \text{ ft}^3/\text{h} \cdot \text{blower}.$$

By dividing the exact answer by the approximate answer, we see that the approximate answer differs from the exact answer by a factor of ten.

## 2.2 Significant Figures

After order-of-magnitude calculations have been made, engineers conduct more precise calculations to refine their design or to more fully characterize a particular failure mode. Accurate calculations demand more of the engineer than simply keeping track of powers of 10. Final design parameters must be determined with as much precision as possible to achieve the optimum design. Engineers must determine how many digits in their calculations are significant. A **significant figure** or *significant digit* in a number is defined as *a digit that is considered reliable as a result of a measurement or calculation*. The number of significant figures in the answer of a calculation indicates the number of digits that can be used with confidence, thereby providing a way of telling the engineer how precise the answer is. No physical quantity can be specified with infinite precision because no physical quantity is *known* with infinite precision. Even the constants of nature such as the speed of light in a vacuum  $c$ , and the gravitational constant  $G$ , are known only to the precision with which they can be measured in a laboratory. Similarly, engineering material properties such as density, modulus of elasticity, and specific heat are known only to the precision with which these properties can be measured. A common mistake is to use more significant figures in an answer than are justified, giving the impression that the answer is more precise than it really is. No answer can be more accurate than the numbers used to generate that answer.

How do we determine how many significant figures (colloquially referred to as "sig figs") a number has? A set of rules has been established for counting the number of significant figures in a number. (All significant figures are underlined in the examples given for each rule.)

### Rules for Significant Figures

1. All digits *other than zero* are significant. Examples: 8.936, 456, 0.257.
2. All zeroes *between* significant figures are significant. Examples: 14.06, 5.0072.
3. For nondecimal numbers greater than one, all zeroes placed *after* the significant figures are *not* significant. Examples: 2500, 8,640,000. These numbers can be written in scientific notation as  $2.5 \times 10^3$  and  $8.64 \times 10^6$ , respectively.
4. If a decimal point is used *after* a nondecimal number larger than one, the zeroes are significant. The decimal point establishes the precision of the number. Examples: 3200., 550.000.
5. Zeroes placed *after* a decimal point that are *not necessary* to set the decimal point are significant. The additional zeroes establish the precision of the number. Examples: 359.00, 1000.00.
6. For numbers smaller than one, all zeroes placed *before* the significant figures are *not* significant. These zeroes only serve to establish the location of the decimal point. Examples: 0.0254, 0.000609.

Do not confuse the number of significant figures with the number of decimal places in a number. The number of significant figures in a quantity is established by the precision with which a measurement of that quantity can be made. The primary exception to this are numbers such as  $\pi$  and the Napierian base  $e$ , that are derived from mathematical relations. These numbers are precise to an infinite number of significant figures but will be quite adequately approximated by 10-digit decimals.

Let's see how the rules for significant figures are used in calculations.

**EXAMPLE 2**

We wish to calculate the weight of a 25-kg object. Using Newton's second law  $W = mg$ , find the weight of the object in units of N. Express the answer, by using the appropriate number of significant figures.

**Solution**

We have  $m = 25$  kg and  $g = 9.81$  m/s<sup>2</sup>. Suppose that our calculator is set to display six places to the right of the decimal point. We then multiply the numbers 25 and 9.81. In the display of the calculator, we see the number 245.250000. How many digits in this answer are we justified in writing? The number in the calculator's display implies that the answer is precise to six decimal places (i.e., to within one-millionth of a newton). Obviously, this kind of precision is not justified. The rule for significant figures for *multiplication* and *division* is that *the product or quotient should contain the number of significant figures that are contained in the number with the fewest significant figures*.

Another way to state this rule is to say that the quantity with the fewest number of significant figures *governs* the number of significant figures in the answer. The mass  $m$ , contains two significant figures, and the acceleration of gravity  $g$ , contains three. Therefore, we are only justified in writing the weight by using two significant figures, which is the fewest number of significant figures in our given values. Our answer can be written in two ways. First, we can write the weight as 250 N. According to rule 3, the zero is not significant, so our answer contains two significant figures, the "2" and the "5." Second, we can write the weight by using scientific notation as  $2.5 \times 10^2$  N. In this form, we can immediately see that two significant figures are used without referring to the rules. Note that in both cases we *rounded* the answer *up* to the nearest tens place, because the value of the first digit dropped is 5 or greater. If our answer had been lower than 245 N, we would have rounded *down* to 240 N. If our answer had been precisely 245 N, the rules of rounding suggest rounding up, so our answer would again be 250 N.

The preceding example shows how significant figures are used for multiplication or division, but how are significant figures used for *addition* and *subtraction*?

**EXAMPLE 3**

Two collinear forces (forces that act in the same direction) of 875.4 N and 9.386 N act on a body. Add these two forces, expressing the result in the appropriate number of significant figures.

**Solution**

The best way to show how significant figures are used in addition or subtraction is to do the problem by hand. We have:

$$\begin{array}{r} 875.4 \text{ N} \\ + 9.386 \text{ N} \\ \hline 884.786 \text{ N} \end{array}$$

Both forces have four significant figures, but the first force reports one place past the decimal point, whereas the second force reports three places past the decimal



point. The answer is written with six significant figures. Are six significant figures justified? Because addition and subtraction are arithmetic operations that require decimal point alignment, the rule for significant figures for *addition* and *subtraction* is different than for multiplication and division. For addition and subtraction, the answer should show *significant figures only as far to the right as is seen in the least precise number in the calculation*. The least precise number in the calculation is the 875.4-N force, because it reports accuracy to the first decimal place, whereas the second force 9.386 N reports precision to the third decimal place. We are not justified in writing the answer as 884.786 N. We may only write the answer by using the same number of places past the decimal point as seen in the least precise force. Hence, our answer, reported to the appropriate number of significant figures, is 884.8 N. Once again, we rounded the answer up because the value of the first digit dropped is 5 or greater.

In *combined* operations where multiplication and division are performed in the same operation as addition and subtraction, the multiplications and divisions should be performed first, establishing the proper number of significant figures in the intermediate answers, perform the additions and subtractions, and then round the answer to the proper number of significant figures. This procedure, while applicable to operations performed by hand, should not be used in calculator or computer applications, because intermediate rounding is cumbersome and may lead to a serious error in the answer. Perform the entire calculation, letting the calculator or computer software manage the numerical precision, and then express the final answer in the desired number of significant figures:

*It is standard engineering practice to express final answers in three (or sometimes four) significant figures, because the given input values for geometry, loads, material properties, and other quantities are typically reported with this precision.*

Calculators and computer software such as spreadsheets and equation solvers keep track of and can display a large number of digits. How many digits will your calculator display? The number of digits displayed by a scientific calculator can be set by fixing the decimal point or specifying the numerical format. For example, by fixing the number of decimal places to one, the number 28.739 is displayed as 28.7. Similarly, the number 1.164 is displayed as 1.2. Because the first digit dropped is greater than 5, the calculator automatically rounds the answer up. Small and large numbers should be expressed in scientific notation. For example, the number 68,400 should be expressed as  $6.84 \times 10^4$ , and the number 0.0000359 should be expressed as  $3.59 \times 10^{-5}$ . Scientific calculators also have an *engineering notation* display setting because SI unit prefixes are primarily defined by multiples of one thousand ( $10^3$ ). In engineering notation, the number 68,400 may be displayed as  $68.4 \times 10^3$ , and the number 0.0000359 may be displayed as  $35.9 \times 10^{-6}$ . Regardless of how numbers are displayed by calculators or computers, the engineering student who uses these computational tools must understand that significant figures have a physical meaning based on our ability to measure engineering and scientific quantities. The casual or sloppy handling of significant figures in engineering analysis may lead to solutions that are imprecise at best and completely wrong at worst.

**APPLICATION****Calculating Viscosity by Using the Falling-Sphere Method**

You know by experience that some fluids are thicker or more “goeey” than others. For example, pancake syrup and motor oil are thicker than water and alcohol. The technical term we use to describe the magnitude of a fluid’s thickness is *viscosity*. Viscosity is a fluid property that characterizes the fluid’s resistance to flow. Water and alcohol flow more readily than pancake syrup and motor oil under the same conditions. Hence, pancake syrup and motor oil are more viscous than water and alcohol. Gases have viscosities too, but their viscosities are much smaller than those of liquids.

One of the classical techniques for measuring viscosities of liquids is called the *falling-sphere method*. In the falling-sphere method, the viscosity of a liquid is calculated by measuring the time it takes for a small sphere to fall a prescribed distance in a large container of the liquid, as illustrated in Figure 1. As the sphere falls in the liquid under the influence of gravity, it accelerates until the downward force (the sphere’s weight) is exactly balanced by the buoyancy force and drag force that act upward. From this time forward, the sphere falls with a constant velocity, referred to as terminal velocity. The buoyancy force, which is equal to the weight of the liquid allowed for displaced by the sphere, is usually small compared with the drag force, which is caused directly by viscosity. The terminal velocity of the sphere is inversely proportional to viscosity, since the sphere takes longer to fall a given distance in a very viscous liquid, such as motor oil, than in a less viscous liquid, such as water. By employing a force balance on the sphere and invoking some simple relations from fluid mechanics, we obtain the formula

$$\mu = \frac{(\gamma_s - \gamma_f)D^2}{18v}$$

where

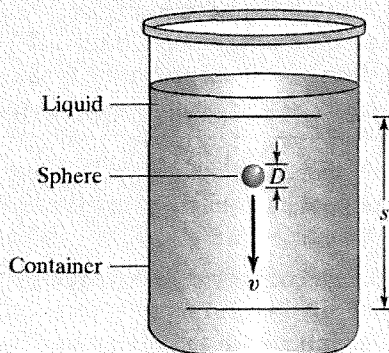
$\mu$  = dynamic viscosity of liquid (Pa · s)

$\gamma_s$  = specific weight of sphere (N/m<sup>3</sup>)

$\gamma_f$  = specific weight of liquid (N/m<sup>3</sup>)

$D$  = sphere diameter

$v$  = terminal velocity of sphere (m/s).



**Figure 1**  
Experimental setup of the falling-sphere method for measuring viscosity.

Note that the quantity *specific weight* is similar to *density*, except that it is a weight per volume, rather than a mass per volume. The word *dynamic* is used to avoid confusion with another measure of viscosity known as *kinematic* viscosity.

Using the falling-sphere method, let's calculate the viscosity of glycerine, a very viscous liquid used to make a variety of chemicals. We set up a large glass cylinder and place two marks, spaced  $s = 200$  mm apart, on the outside surface. The marks are placed low enough on the cylinder to assure that the sphere will achieve terminal velocity before reaching the top mark. For the sphere, we use a steel ( $\gamma_s = 76,800 \text{ N/m}^3$ ) ball bearing with a diameter of 2.381 mm (measured with a micrometer). From a previous measurement, the specific weight of the glycerin is  $\gamma_f = 12,400 \text{ N/m}^3$ . Now, we hold the steel sphere above the surface of the glycerin at the center of the cylinder and release the sphere. As accurately as we can determine with our eye, we start a handheld stopwatch when a part of the sphere reaches the top mark. Similarly, we stop the watch when the same part of sphere reaches the bottom mark. Our stopwatch is capable of displaying hundredths of a second, and it reads 11.32 s. Even though the stopwatch is capable of measuring time to the second decimal place, our crude visual timing method does not justify using a time interval with this precision. Sources of uncertainty such as human reaction time and thumb response do not justify the second decimal place. Thus, our time interval is reported as 11.3 s, which has three significant figures. We know that terminal velocity is distance divided by time:

$$v = \frac{s}{t} = \frac{0.200 \text{ m}}{11.3 \text{ s}} = 0.0177 \text{ m/s}$$

The distance was measured to the nearest millimeter, so the quantity,  $s$  has three significant figures. Thus, terminal velocity may be written to three significant figures. (Remember that the zero, according to rule 6, is not significant.) Values of the given quantities for our calculation are summarized as follows:

$$\gamma_s = 76,800 \text{ N/m}^3 = 7.68 \times 10^4 \text{ N/m}^3$$

$$\gamma_f = 12,400 \text{ N/m}^3 = 1.24 \times 10^4 \text{ N/m}^3$$

$$v = 0.0177 \text{ m/s} = 1.77 \times 10^{-2} \text{ m/s}$$

$$D = 2.381 \text{ mm} = 2.381 \times 10^{-3} \text{ m}.$$

Each quantity, with the exception of  $D$ , which has four significant figures, has three significant figures. Upon substituting values into the equation for dynamic viscosity, we obtain:

$$\begin{aligned} \mu &= \frac{(\gamma_s - \gamma_f)D^2}{18v} \\ &= \frac{(76,800 - 12,400) \text{ N/m}^3 (2.381 \times 10^{-3} \text{ m})^2}{18(0.0177 \text{ m/s})} \\ &= 1.1459 \text{ Pa} \cdot \text{s}. \end{aligned}$$

(Where did the pressure unit Pa come from?) According to the rules of significant figures for multiplication and division, our answer should contain the same number of significant figures as the number with the fewest significant figures. Our answer



should therefore have three significant figures, so the dynamic viscosity of glycerin, expressed in the proper number of significant figures is reported as:

$$\mu = 1.15 \text{ Pa} \cdot \text{s}.$$

Note that because the value of the first digit dropped is 5, we rounded our answer up.

### **Professional Success**

#### *Learn How To Use Your Calculator*

As an engineering student, you need a scientific calculator. If you do not yet own a quality scientific calculator, purchase one as soon as you can and begin learning how to use it. You cannot succeed in school without one. Do not scrimp on cost. You will probably only need one calculator for your entire academic career, so purchase one that offers the greatest number of functions and features. Professors and fellow students may offer advice on which calculator to buy. Your particular engineering department or college may even require that you use a particular calculator because they have heavily integrated calculator usage in the curriculum, and it would be too cumbersome to accommodate several types of calculators. Your college bookstore or local office supply store may carry two or three name brands that have served engineering students and professionals for many years. Today's scientific calculators are remarkable engineering tools. A high-end scientific calculator has hundreds of built-in functions, large storage capacity graphics capabilities, and communication links to other calculators or personal computers.

Regardless of which scientific calculator you own or plan to purchase, *learn how to use it*. Begin with the basic arithmetic operations and the standard mathematical and statistical functions. Learn how to set the number of decimal places in the display and how to display numbers in scientific and engineering notation. After you are confident with performing unit conversions by hand, learn how to do them with your calculator. Learn how to write simple programs on your calculator. This skill will come in handy numerous times throughout your course work. Learn how to use the equation-solving functions, matrix operations and calculus routines. By the time you learn most of the calculator's operations, you will probably have devoted many hours. The time spent mastering your calculator is perhaps as valuable as the time spent attending lectures, conducting experiments in a laboratory, doing homework problems, or studying for exams. Knowing your calculator thoroughly will help you succeed in your engineering program. Your engineering courses will be challenging enough. Do not make them an even bigger challenge by failing to adequately learn how to use your principal computational asset, your calculator.

**Practice!**

1. Using an order-of-magnitude analysis, estimate the surface area of your body in units of  $\text{cm}^2$ .
2. Using an order-of-magnitude analysis, estimate the number of hairs on your head.
3. Use an order-of-magnitude analysis to estimate the number of cell phones in use in the United States.
4. Use an order-of-magnitude analysis to estimate the electrical energy in kWh used by your city in one month.
5. Underline the significant figures in the following numbers (the first number is done for you):
  - a. 0.00254
  - b. 29.8
  - c. 2001
  - d. 407.2
  - e. 0.0303
  - f. 2.006.

*Answer:* b. 29.8 c. 2001 d. 407.2 e. 0.0303 f. 2.006.

6. Perform the following calculations, reporting the answers with the correct number of significant figures:
  - a.  $5.64/1.9$
  - b.  $500./0.0025$
  - c.  $(45.8 - 8.1)/1.922$
  - d.  $2\pi/2.50$
  - e.  $(5.25 \times 10^4)/(100 + 10.5)$
  - f.  $0.0008/(1.2 \times 10^{-5})$ .

*Answer:* a. 3.0 b.  $2.0 \times 10^5$  c. 19.6 d. 2.51 e. 500 f. 70.

7. A ball bearing is reported to have a radius of 3.256 mm. Using the correct number of significant figures, what is the weight of this bearing in units of N if its density is  $\rho = 1675 \text{ kg/m}^3$ ? Use  $g = 9.81 \text{ m/s}^2$ .  
*Answer:*  $2.38 \times 10^{-3} \text{ N}$ .
8. The cylinder of an internal combustion engine is reported to have a diameter of 4.000 in. If the stroke (length) of the cylinder is 6.25 in, what is the volume of the cylinder in units of  $\text{in}^3$ ? Write the answer using the correct number of significant figures.  
*Answer:*  $78.5 \text{ in}^3$

**3 GENERAL ANALYSIS PROCEDURE**

Engineers are problem solvers. In order to solve an engineering analysis problem thoroughly and accurately, engineers employ a solution method that is systematic, logical, and orderly. This method, when consistently and correctly applied, leads the engineer to a successful solution of the analytical problem at hand. The problem-solving method is an integral part of a good engineer's thought process. To the engineer, the procedure is second nature. When challenged by a new analysis, a good

engineer knows precisely how to approach the problem. The problem may be fairly short and simple or extremely long and complex. Regardless of the size or complexity of the problem, the same solution method applies. Because of the *general* nature of the procedure, it applies to analytical problems associated with *any* engineering discipline: chemical, civil, electrical, mechanical, or other. Practicing engineers in all disciplines have been using the **general analysis procedure** in one form or another for a long time, and the history of engineering achievements is a testament to its success. While you are a student, it is vitally important that you learn the steps of the general analysis procedure. After you have learned the steps in the procedure and feel confident that you can use the procedure to solve problems, apply it in your analytical course work. Apply it religiously. Practice the procedure over and over again until it becomes a habit. Establishing good habits while still in school will make it that much easier for you to make a successful transition into professional engineering practice.

### General Analysis Procedure

The general analysis procedure consists of the following seven steps.

1. **Problem Statement** The problem statement is a written description of the analytical problem to be solved. It should be written clearly, concisely, and logically. The problem statement summarizes the given information, including all input data provided to solve the problem. The problem statement also states what is to be determined by performing the analysis.
2. **Diagram** The diagram is a sketch, drawing, or schematic of the system being analyzed. Typically, it is a simplified pictorial representation of the actual system, showing only those aspects of the system that are necessary to perform the analysis. The diagram should show all given information contained in the problem statement such as geometry, applied forces, energy flows, mass flows, electrical currents, temperatures, or other physical quantities as required.
3. **Assumptions** Engineering analysis almost always involves some assumptions. Assumptions are special assertions about the physical characteristics of the problem that simplify or refine the analysis. A very complex analytical problem would be difficult or even impossible to solve without making some assumptions.
4. **Governing Equations** All physical systems may be described by mathematical relations. Governing equations are those mathematical relations that specifically pertain to the physical system being analyzed. These equations may represent physical laws, such as Newton's laws of motion, conservation of mass, conservation of energy, and Ohm's law; or they may represent fundamental engineering definitions such as velocity, stress, moment of force, and heat flux. The equations may also be basic mathematical or geometrical formulas involving angles, lines, areas, and volumes.
5. **Calculations** In this step, the solution is generated. First, the solution is developed algebraically as far as possible. Then numerical values of known physical quantities are substituted for the corresponding algebraic variables. All necessary calculations are performed, using a calculator or computer to produce a numerical result with the correct units and the proper number of significant figures.
6. **Solution Check** This step is crucial. Immediately after obtaining the result, examine it carefully. Using established knowledge of similar analytical solutions and common sense, try to ascertain whether the result is reasonable. However, whether the result seems reasonable or not, double-check every step of the

analysis. Flush out defective diagrams, bad assumptions, erroneously applied equations, incorrect numerical manipulations, and improper use of units.

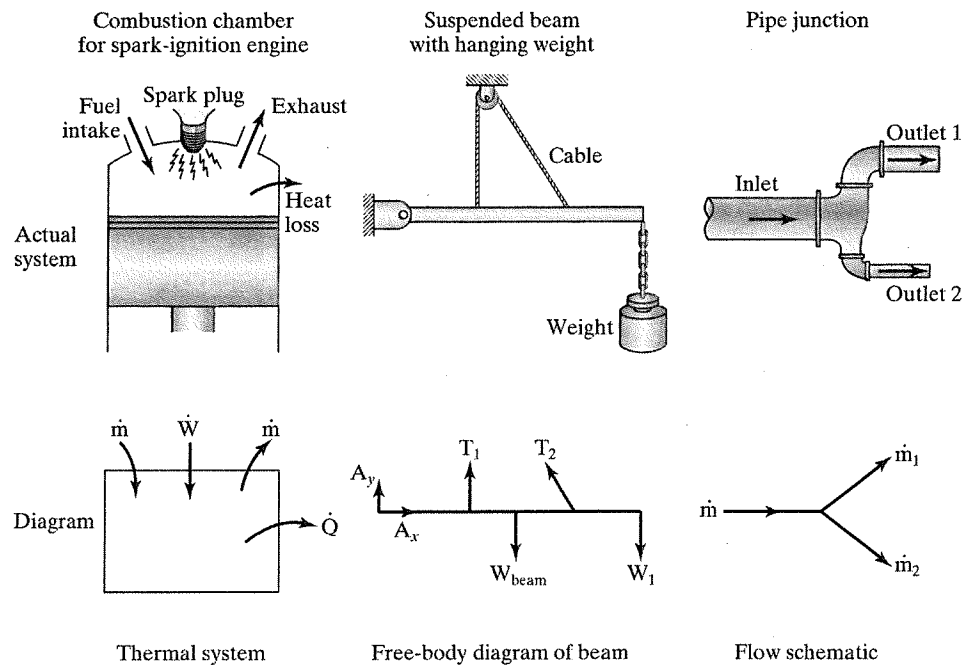
7. **Discussion** After the solution has been thoroughly checked and corrected, discuss the result. The discussion may include an assessment of the assumptions, a summary of the main conclusions, a proposal on how the result may be verified experimentally in a laboratory, or a parametric study demonstrating the sensitivity of the result to a range of input parameters.

Now that the seven-step procedure has been summarized, further discussion of each step is warranted.

**1. Problem statement** In your engineering textbook, the problem statement will generally be supplied to you in the form of a problem or question at the end of each chapter. These problem statements are written by the textbook authors, professors or practicing engineers, who have expertise in the subject area. The great majority of end-of-chapter problems in engineering texts are well organized and well written, so you do not have to fret too much about the problem statement. Alternatively, your engineering professor may give you problem statements from sources outside your textbook or from his or her own engineering experience. In either case, the problem statement should be well posed, contain all the necessary input information, and clearly state what is to be determined by the analysis. What is known and what is unknown in the problem should be clearly identified. If the problem statement is flawed in any way, a meaningful analysis is impossible.

**2. Diagram** The old saying, "One picture is worth a thousand words," is certainly applicable to engineering analysis. A complete diagram of the system being analyzed is critical. A good diagram helps the engineer visualize the physical processes or characteristics of the system. It also helps the engineer identify reasonable assumptions and the appropriate governing equations. A diagram might even reveal flaws in the problem statement or alternative methods of solution. Engineers use a variety of diagrams in their analytical work. One of the most widely used diagrams in engineering is the *free-body diagram*. Free-body diagrams are used to solve engineering mechanics (statics, dynamics, mechanics of materials) problems. These diagrams are called "free-body" diagrams because they represent a specific body, isolated from all other bodies that are in physical contact with, or that may be in the vicinity of, the body in question. The influences of nearby bodies are represented as external forces acting on the body being analyzed. Hence, a free-body diagram is a sketch of the body in question, showing all external forces applied to the body. A free-body diagram is a pictorial representation of a "force balance" on the body. Diagrams are also used in the analysis of thermal systems. Unlike a free-body diagram, which shows forces applied to the body, a diagram of a thermal system shows all the various forms of energy entering and leaving the system. This type of diagram is a pictorial representation of an "energy balance" on the system. Another type of diagram represents a system that transports mass at known rates. Common examples include pipe and duct systems, conveyors, and storage systems. A diagram for these systems shows all the mass entering and leaving the system. This type of diagram is a pictorial representation of a "mass balance" on the system. Still another type of diagram is an electrical circuit schematic. Electrical schematics show how components are connected and the currents, voltages, and other electrical quantities in the circuit. Some examples of diagrams used in analysis are given in Figure 2.

**3. Assumptions** An "atmospheric scientist" who studied various processes that occur in the upper atmosphere once gave a lecture and recounted an accomplishment that seemed truly remarkable. After convincing the audience that atmospheric



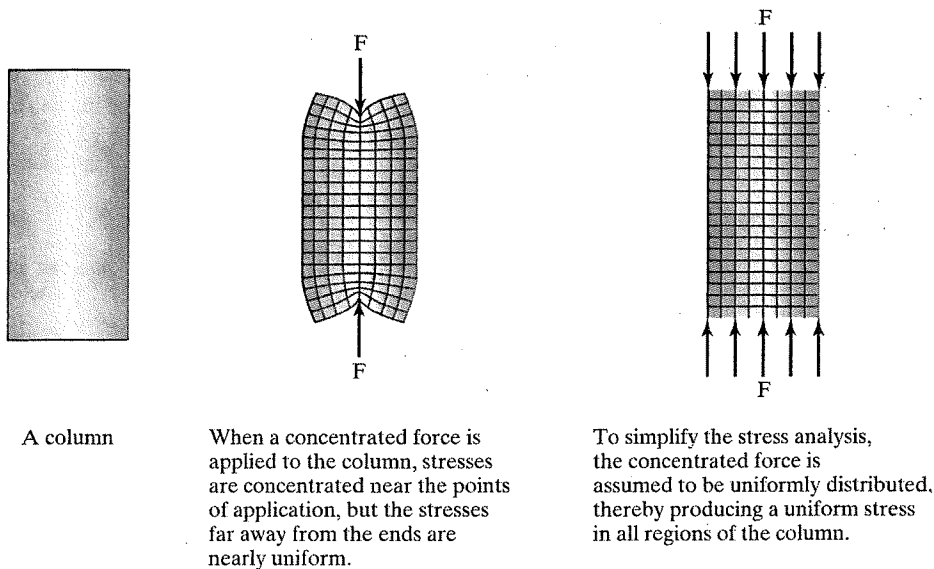
**Figure 2**  
Examples of typical  
diagrams used in  
engineering analysis.

processes are some of the most complex phenomena in physics, he boasted that he had developed, over the space of a few months, an analytical model of the upper atmosphere that contained *no* assumptions. There was only one problem: his model had no solution either. By including every physical mechanism to the minutest detail in the model, his analysis was so mathematically convoluted that it could not generate a solution. Had he made some simplifying assumptions, his atmospheric model could have worked even though the results would have been approximate.

Engineers and scientists routinely employ assumptions to simplify a problem. As this story illustrates, an approximate answer is better than no answer at all. Failure to invoke one or more simplifying assumptions in the analysis, particularly a complex one, can increase the complexity of the problem by an order of magnitude, leading the engineer down a very long road, only to reach a dead end. How do we determine which assumptions to use and whether our assumptions are good or bad? To a large extent, the application of good assumptions is an acquired skill, a skill that comes with engineering experience. However, you can begin to learn this skill in school through repeated application of the general analysis procedure in your engineering courses. As you apply the procedure to a variety of engineering problems, you will gain a basic understanding of how assumptions are used in engineering analysis. Then, after you graduate and accept a position with an engineering firm, you can refine this skill as you apply the analysis procedure to solve problems that are specific to the company. Sometimes, a problem can be overly constrained by assumptions such that the problem is simplified to the point where it becomes grossly inaccurate or even meaningless. The engineer must therefore be able to apply the proper *number* as well as the proper *type* of assumptions in a given analysis. A common assumption made in the stress analysis of a column is shown in Figure 3.

**4. Governing equations** The governing equations are the “workhorses” of the analysis. They must be selected to adequately describe the physical problem at hand.





**Figure 3**  
A common assumption made in the stress analysis of a column.

If the wrong governing equations are used, the analysis may lead to a result that does not reflect the true physical nature of the problem, or the analysis will not be possible at all because the governing equations are not in harmony with the problem statement or assumptions. When using a governing equation to solve a problem, the engineer must ascertain that the equation being used *actually* applies to the specific problem at hand. As an extreme (and probably absurd) example, imagine an engineer attempting to use Newton's second law  $F = ma$  to calculate the heat loss from a boiler. How about trying to apply Ohm's law  $V = IR$  to find the stress in a concrete column that supports a bridge deck? The problem of matching governing equations to the problem at hand is usually more subtle. In thermodynamics, for example, the engineer must determine whether the thermal system is "closed" or "open" (i.e., whether the system allows mass to cross the system boundary). After the type of thermal system has been identified, the thermodynamic equations which apply to that type of system are chosen, and the analysis proceeds. Governing equations must also be consistent with the assumptions. It is counterproductive to invoke simplifying assumptions if the governing equations do not make allowances for them. Some governing equations, particularly those that are experimentally derived, have built-in restrictions that limit the use of the equations to specific numerical values of key variables. A common mistake made in the application of a governing equation in this situation is failing to recognize the restrictions by forcing the equation to accept numerical values that lie outside the equation's range of applicability.

**5. Calculations** A common practice, particularly among beginning students, is to substitute numerical values of quantities into equations *too early* in the calculations. It seems that some students are more comfortable working with *numbers* than *algebraic variables*, so their first impulse is to substitute numerical values for all parameters at the beginning of the calculation. Avoid this impulse. To the extent that it is practical, develop the solution *analytically* prior to assigning physical quantities to their numerical values. Before rushing to "plug" numbers into equations, carefully examine the equations to see if they can be mathematically manipulated to yield simpler expressions. A variable from one equation can often be substituted into another equation to reduce the total number of variables. Perhaps an expression can

be simplified by factoring. By developing the solution analytically first, you might uncover certain physical characteristics about the system or even make the problem easier to solve. The analytical skills you learned in your algebra, trigonometry, and calculus courses are meant to be used for performing mathematical operations on *symbolic* quantities, not numbers. When doing engineering analysis, do not put your mathematics skills on a shelf to collect dust—*use* them.

The calculations step demands more of an engineer than the ability to simply “crunch numbers” on a calculator or computer. The numbers have to be meaningful, and the equations containing the numbers must be fully understood and properly used. All mathematical relations must be dimensionally consistent, and all physical quantities must have a numerical value plus the correct units. Here is a tip concerning units that will save you time and help you avoid mistakes: *if the quantities given in the problem statement are not expressed in terms of a consistent set of units, convert all quantities to a consistent set of units before performing any calculations.* If some of the input parameters are expressed as a mixture of SI units and English units, convert all parameters to either SI units or English units, and then perform the calculations. Students tend to make more mistakes when they attempt to perform unit conversions *within* the governing equations. If all unit conversions are done prior to substituting numerical values into the equations, unit consistency is assured throughout the remainder of the calculations, because a consistent set of units is established at the onset. Dimensional consistency should still be verified, however, by substituting all quantities along with their units into the governing equations.

**6. Solution check** This step is perhaps the easiest one to overlook. Even good engineers sometimes neglect to thoroughly check their solution. The solution may “look” good at first glance, but a mere glance is not good enough. Much effort has gone into formulating the problem statement, constructing diagrams of the system, determining the appropriate number and type of assumptions, invoking governing equations, and performing a sequence of calculations. All this work may be for naught if the solution is not carefully checked. Checking the solution of an engineering analysis is analogous to checking the operation of an automobile immediately following a major repair. It’s always a good idea if the mechanic checks to verify that it works before returning the vehicle to its owner.

There are two main aspects of the solution check. First, the result itself should be checked. Ask the question, “Is this result reasonable?” There are several ways to answer this question. The result must be consistent with the information given in the problem statement. For example, suppose you wish to calculate the temperature of a microprocessor chip in a computer. In the problem statement, the ambient air temperature is given as 25°C, but your analysis indicates that the chip temperature is only 20°C. This result is not consistent with the given information because it is physically impossible for a heat-producing component, a microprocessor chip in this case, to have a lower temperature than the surrounding environment. If the answer had been 60°C, it is at least consistent with the problem statement, but it may still be incorrect. Another way to check the result is to compare it with that of similar analysis performed by you or other engineers. If the result of a similar analysis is not available, an alternative analysis that utilizes a different solution approach may have to be conducted. In some cases, a laboratory test may be needed to verify the solution experimentally. Testing is a normal part of engineering design anyway, so a test to verify an analytical result may be customary.

The second aspect of the solution check is a thorough inspection and review of each step of the analysis. Returning to our microprocessor example, if no mathematical or numerical errors are committed, the answer of 60°C may be considered correct insofar as the calculations are concerned, but the answer could still be in error due to bad assumptions. For example, suppose that the microprocessor chip is air cooled by a small fan, so we assert that forced convection is the dominant mechanism by which heat is transferred from the chip. Accordingly, we assume that conduction and radiation heat transfer are negligible, so we do not include these mechanisms in the analysis. A temperature of 60°C seems a little high, so we revise our assumptions. A second analysis that includes conduction and radiation reveals that the microprocessor chip is much cooler, about 42°C. Knowing whether assumptions are good or bad comes through increased knowledge of physical processes and practical engineering experience.

**7. Discussion** This step is valuable from the standpoint of communicating to others what the results of the analysis means. By discussing the analysis, you are in effect writing a “mini-technical report.” This report summarizes the major conclusions of the analysis. In the microprocessor example given earlier, the main conclusion may be that 42°C is below the recommended operating temperature for the chip and therefore, the chip will operate reliably in the computer for a minimum of 10,000 hours before failing. If the chip temperature was actually measured at 45°C shortly after performing the analysis, the discussion might include an examination of why the predicted and measured temperatures differ and particularly why the predicted temperature is lower than the measured temperature. A brief parametric study may be included that shows how the chip temperature varies as a function of ambient air temperature. The discussion may even include an entirely separate analysis that predicts the chip temperature in the event of a fan failure. In the discussion step, the engineer is given one last opportunity to gain additional insights into the problem.

## Professional Success

### *Real-World Problem Statements*

Engineering programs strive to give students a sense of what it is like to actually practice engineering in the “real world.” But *studying* engineering in school and *practicing* engineering in the real world are not the same thing. One difference is amply illustrated by considering the origins of problem statements for analysis. In school, problem statements are typically found at the end of each chapter of your engineering texts. Sometimes your professors obtain problem statements from other texts or invent new ones (especially for exams). In any case, problem statements are supplied to you in a nice, neat little package all ready for you to tackle the problem.

If textbooks and professors supply problem statements to students in school, who or what supplies problem statements to practicing engineers in industry? Real-world engineering problems are not typically found in textbooks and your engineering professors are not going to follow you around after you graduate. So, where do the real-world problem statements come

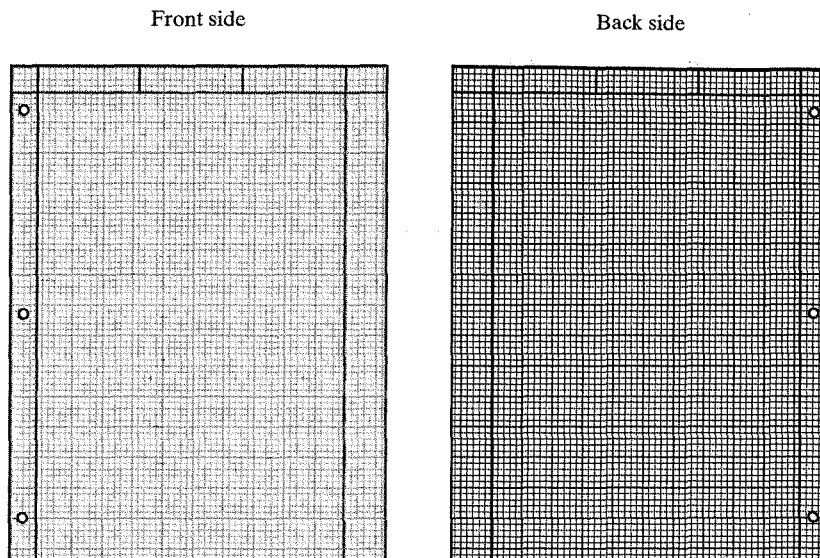
from? They are *formulated* by the engineer who is going to perform the analysis. As stated before, analysis is an integral part of engineering design. As a design matures, quantitative parameters that characterize the design begin to emerge. When an analysis is called for, these parameters are woven into a problem statement from which an analysis may be conducted. The engineer must be able to formulate a coherent, logical problem statement from the design information available. Because engineering design is an intuitive process, the values of some or all of the input parameters may be uncertain. The engineer must therefore be able to write the problem statement in such a way as to allow for these uncertainties. The analysis will have to be repeated several times until the parameters are no longer in a state of flux, at which time the design is complete.

The seven-step procedure for performing an engineering analysis is a time-tested method. In order to effectively communicate an analysis to others, the analysis must be presented in a format that can be readily understood and followed. Engineers are known for their ability to present analyses and other technical information with clarity in a thorough, neat, and careful manner. As an engineering student, you can begin to develop this ability by consistently applying the analysis procedure outlined in this section. Your engineering professors will insist that you follow the procedure, or a procedure similar to it, in your engineering courses. You will probably be graded not only on how well you perform the analysis itself, but how well you *present* the analysis on paper. This grading practice is meant to convince students of the importance of presentation standards in engineering and to assist them in developing good presentation skills. An engineering analysis is of little value to anyone unless it can be read and understood. A good analysis is one that can be easily read by others. If your analysis resembles "hen scratchings" or "alien hieroglyphics" that require an interpreter, the analysis is useless. Apply the presentation guidelines given in this section to the point where they become second nature. Then, after you graduate and begin practicing engineering, you can hone your presentation skills as you gain industrial experience.

The 10 guidelines that follow will help you present an engineering analysis in a clear and complete manner. These guidelines are applicable to analysis work in school as well as industrial engineering practice. It should be noted that the guidelines apply specifically to analyses performed by hand with the use of pencil and paper, as opposed to computer-generated analyses.

### **Analysis Presentation Guidelines**

1. A standard practice of engineers who do analysis is to use a special type of *paper*. This paper is usually referred to as "engineer calculation pad" or "engineer's computation paper." The paper is light green in color, and should be available in your college or university bookstore. The back side of the paper is ruled horizontally and vertically with five squares per inch, with only heading and margin rulings on the front side. The rulings on the back side are faintly visible through the paper to help the engineer maintain the proper position and orientation for lettering, diagrams, and graphs. (See Figure 4.) All work is to be done on the *front* side of the paper. The back side is not used. The paper usually

**Figure 4**

Engineer's computation paper is standard issue for analysis work.

comes pre-punched with a standard three-hole pattern at the left edge for placement in a three-ring binder.

2. No more than *one* problem should be placed on a page. This practice helps maintain clarity by keeping different problems separate. Even if a problem occupies a small fraction of a page, the next problem should be started on a separate page.
3. The *heading* area at the top of the page should indicate your name, date, course number, and assignment number. The upper right corner of the heading area is usually reserved for page numbers. To alert the reader to the total number of pages present, page numbers are often reported for example, as "1/3", which is read as "page 1 of 3." Page 1 is the current page, and there are a total of three pages. When multiple pages are used, they should be stapled in the upper left corner. Each page should nonetheless be identified with your name, in the unlikely event the pages become separated.
4. The *problem statement* should be written out completely, not summarized or condensed. All figures that accompany the problem statement should be shown. If the problem statement originates from a textbook, it should be written *verbatim* so the reader does not have to refer back to the textbook for the full version. One way to do this is to photocopy the problem statement, along with any figures given, and then cut and attach it by using rubber cement or transparent tape directly beneath the heading area on the engineer's computation paper. The problem statement could also be electronically scanned and printed directly onto the paper.
5. Work should be done in *pencil*, not ink. Everyone makes mistakes. If the analysis is written in pencil, mistakes can be easily erased and corrected. If the analysis is written in ink, mistakes will have to be crossed out, and the presentation will not have a neat appearance. To avoid smudges, use a pencil lead with the appropriate hardness. All markings should be dark enough to reproduce a legible copy if photocopies are needed. If you still use a standard wooden pencil, throw it out. Mechanical pencils are superior. They do not require sharpening, contain several months worth of lead, have replaceable



- erasers, produce no waste, and come in a range of lead diameters to suit your own writing needs.
6. Lettering should be *printed*. The lettering style should be consistent throughout.
  7. Correct *spelling* and *grammar* must be used. Even if the technical aspects of the presentation are flawless, the engineer will lose some credibility if the writing is poor.
  8. There are seven steps in the general analysis procedure. These steps should be sufficiently *spaced* so that the reader can easily follow the analysis from problem statement to discussion. A horizontal line drawn across the page is one way of providing this separation.
  9. Good *diagrams* are a must. A straight edge, drawing templates, and other manual drafting tools should be used. All pertinent quantitative information such as geometry, forces, energy flows, mass flows, electrical currents, and pressures, should be shown on the diagrams.
  10. Answers should be *double underlined* or *boxed* for ready identification. To enhance the effect, colored pencils may be used.

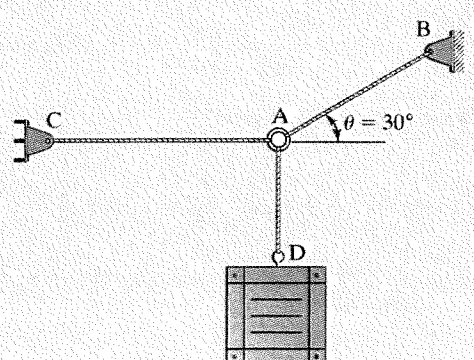
These 10 guidelines for analysis presentation are recommended to the engineering student. You may find that your particular engineering department or professors may advocate guidelines that are slightly different. By all means, follow the guidelines given to you. Your professors may have special reasons for teaching their students certain methods of analysis presentation. Methods may vary somewhat from professor to professor, but should still reflect the major points contained in the guidelines given in this section.

The next four examples illustrate the general analysis procedure and the recommended guidelines for analysis presentation. Each example represents a basic analysis taken from the subject areas of statics, electrical circuits, thermodynamics, and fluid mechanics. You probably have not yet taken courses in these subjects, so do not be overly concerned if you do not understand all the technical aspects of the examples. Therefore, do not focus on the theoretical and mathematical details. Focus instead on how the general analysis procedure is used to solve problems from different engineering areas and the systematic manner in which the analyses are presented.

#### EXAMPLE 4

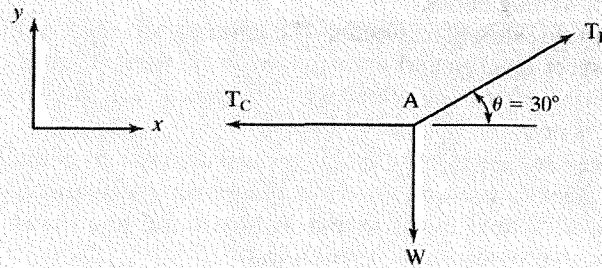
OCT. 12, 2007	EXAMPLE 4	BERT DILLON	1/1
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**Problem Statement**



A 200-kg crate is suspended by ropes as shown. Rope AC is horizontal. Find the tension in ropes AB and AC.

Diagram (Free-Body Diagram)

**Assumptions**

1. Forces in ropes AB, AC and AD are concurrent at point A.
2. Neglect mass of ropes.

**Governing Equations**

$$W = mg$$

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

**Calculations**

$$W = mg = (200 \text{ kg})(9.81 \text{ m/s}^2) = 1962 \text{ N}$$

$$\Sigma F_x = 0 = T_B \cos(30^\circ) - T_C \quad (1)$$

$$\Sigma F_y = 0 = T_B \sin(30^\circ) - W \quad (2)$$

Solving Eq. (2) for  $T_B$  and substituting into Eq. (1) to obtain  $T_C$  gives

$$T_B = 3924 \text{ N} = \underline{3.92 \text{ kN}}, \quad T_C = 3398 \text{ N} = \underline{3.40 \text{ kN}}$$

**Solution Check**

No errors are found. The tensions can be verified by substituting them back into Eqs. (1) and (2):

$$3924 \cos(30^\circ) - 3398 = 0.3 \approx 0$$

$$3924 \sin(30^\circ) - 1962 = 0$$

The negligible nonzero result in Eq. (1) is due to roundoff.

**Discussion**

As  $\theta$  increases,  $T_B$  and  $T_C$  decrease. When  $\theta = 90^\circ$ ,

$T_C = 0$  (rope AC is slack) and  $T_B = W = 1962 \text{ N}$ .

**EXAMPLE 5**

JAN. 03, 2008	EXAMPLE 5	MARIE NORTON	1/2
<b>Problem Statement</b> Two resistors with resistances of $5 \Omega$ and $50 \Omega$ are connected in parallel across a $10 \text{ V}$ battery. Find the current in each resistor.			
<b>Diagram (Electrical Schematic)</b>			
<b>Assumptions</b>			
<ol style="list-style-type: none"> <li>1. Neglect resistance of wires.</li> <li>2. Battery voltage is a constant <math>10 \text{ V}</math>.</li> </ol>			

**Governing Equations (Ohm's law)**

$$V = IR$$

$V$  = Voltage (V)

$I$  = Current (A)

$R$  = Resistance ( $\Omega$ )

**Calculations**

Rearranging Ohm's law:  $I = \frac{V}{R}$ .

Define:  $R_1 = 5 \Omega$ ,  $R_2 = 50 \Omega$

Because resistors are connected in parallel with battery,

$$V = V_1 = V_2 = 10 \text{ V.}$$

$$\therefore I_1 = \frac{V_1}{R_1} = \frac{10 \text{ V}}{5 \Omega} = 2 \text{ A}, I_2 = \frac{V_2}{R_2} = \frac{10 \text{ V}}{50 \Omega} = 0.2 \text{ A}$$

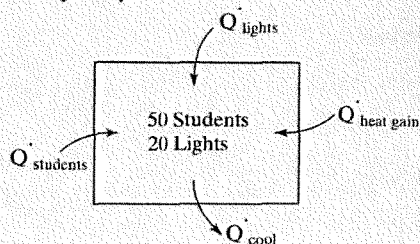
**Solution Check**

The assumptions are reasonable, and there are no errors in the calculations.

JAN. 03, 2008	EXAMPLE 5	MARIE NORTON	2/2
<p><b>Discussion</b>            Current flow in a resistor is inversely proportional to the resistance.            Total current is split according to the ratio of resistances:</p> $\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{2 \text{ A}}{0.2 \text{ A}} = \frac{50 \Omega}{5 \Omega} = 10$ <p>Total current:</p> $I_T = I_1 + I_2 = 2 \text{ A} + 0.2 \text{ A} = 2.2 \text{ A}$ <p>Total current may also be found by finding total resistance and then using Ohm's law.</p> <p>Resistors in parallel as follows:</p> $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{5} + \frac{1}{50}}$ $R_T = 4.5455 \Omega$ $I_T = \frac{V}{R_T} = \frac{10 \text{ V}}{4.5455 \Omega} = 2.2 \text{ A}$			

**EXAMPLE 6**

MAR. 24, 2007	EXAMPLE 6	CY BRAYTON	1/2
<p><b>Problem Statement</b>            A classroom occupied by 50 students is to be air conditioned with window-mounted air conditioning units with a 4 kW rating. There are 20 fluorescent lights in the room, each rated at 60W. While sitting at their desks, each student dissipates 100W. If the heat transfer to the classroom through the roof, walls, and windows is 5 kW, how many air conditioning units are required to maintain the classroom at a constant temperature of 22°C?</p>			

**Diagram (Thermodynamic System)****Assumptions**

1. Classroom is a closed system, i.e., no mass flows.
2. All heat flows are steady.
3. No other heat sources in classroom such as computers, TVs, etc.

**Governing Equations (Conservation of Energy)**

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system}$$

**Calculations**

$$\begin{aligned}\dot{E}_{in} &= \dot{Q}_{students} + \dot{Q}_{lights} + \dot{Q}_{heat\ gain} \\ &= (50)(100\text{ W}) + (20)(60\text{ W}) + 5000\text{ W} = 11,200\text{ W} = 11.2\text{ kW}\end{aligned}$$

$$\Delta \dot{E}_{system} = 0 \text{ (Classroom is maintained at constant temperature)}$$

$$\dot{E}_{out} = \dot{Q}_{cool}$$

Thus,

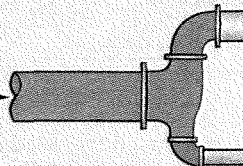
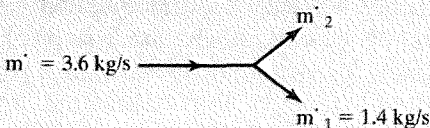
$$\dot{E}_{in} = \dot{Q}_{cool}$$

$$\text{Number of A.C. units required} = \frac{\dot{Q}_{cool}}{4\text{ kW}} = \frac{11.2\text{ kW}}{4\text{ kW}} = 2.8$$

Fractions of A.C. units are impossible, so round up answer to next integer.

MAR. 24, 2007	EXAMPLE 6	CY BRAYTON	2/2
<p>Number of A.C. units required = <u>3</u>.</p> <p><b>Solution check</b></p> <p>In order to maintain a constant temperature, the net heat transfer into the classroom must be equivalent to the heat removed by the air conditioner. Reviewing the assumptions, equations, and calculation, no errors are found.</p> <p><b>Discussion</b></p> <p>The classroom temperature of 22°C was not used in the calculation because this temperature, as well as the outdoor air temperature, are inferred in the given heat gain by a prior heat transfer analysis.</p> <p>Suppose that the classroom was a computer lab containing 30 computers each dissipating 250 W. We eliminate assumption 3 by including heat input by the computers.</p> $\begin{aligned}\dot{Q}_{cool} &= \dot{Q}_{students} + \dot{Q}_{lights} + \dot{Q}_{heat\ gain} + \dot{Q}_{computers} \\ &= 11,200\text{ W} + 30(250\text{ W}) = 18,700\text{ W} = 18.7\text{ kW}\end{aligned}$ $\text{Number of A.C. units required} = \frac{\dot{Q}_{cool}}{4\text{ kW}} = \frac{18.7\text{ kW}}{4\text{ kW}} = 4.7$ <p>Number of A.C. units required = <u>5</u>.</p> <p>This example illustrates the effect computers have on air conditioning requirements.</p>			

## EXAMPLE 7

JULY 2, 2007	EXAMPLE 7	MAX POWER	1/2
<p><b>Problem Statement</b></p> <p>Water enters a pipe junction at a mass flow rate of 3.6 kg/s. If the mass flow rate in the small branch is 1.4 kg/s, what is the mass flow rate in the large pipe branch? If the inside diameter of the large pipe branch is 5 cm, what is the velocity in the large pipe branch?</p> <div style="text-align: center;">  </div> <p><b>Diagram (Flow Schematic)</b></p> <div style="text-align: center;">  </div> <p><b>Assumptions</b></p> <ol style="list-style-type: none"> <li>1. Steady, incompressible flow</li> <li>2. Density of water: <math>\rho = 1000 \text{ kg/m}^3</math></li> </ol> <p><b>Governing Equations</b></p> <div style="display: flex; justify-content: space-between;"> <div style="width: 30%;"> <p>Conservation of mass:</p> <p>Mass flow rate:</p> </div> <div style="width: 30%;"> <p><math>m_{in} = m_{out}</math></p> <p><math>\dot{m} = \rho A v</math></p> </div> <div style="width: 30%;"> <p><math>\dot{m}</math> = mass flow rate (kg/s)</p> <p><math>\rho</math> = fluid density (kg/m<sup>3</sup>)</p> <p><math>A</math> = flow cross-sectional area (m<sup>2</sup>)</p> <p><math>v</math> = velocity (m/s)</p> </div> </div> <p><b>Calculations</b></p> <div style="margin-top: 20px;"> <p><math>\dot{m} = \dot{m}_1 + \dot{m}_2</math></p> <p><math>\dot{m}_2 = \dot{m} - \dot{m}_1 = 3.6 \text{ kg/s} - 1.4 \text{ kg/s}</math></p> <p style="padding-left: 40px;"><math>= 2.2 \text{ kg/s}</math></p> </div>			

JULY 12, 2007	EXAMPLE 7	MAX POWER	2/2
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$$\dot{m}_2 = \rho A_2 v_2 = \rho \frac{\pi D_2^2}{4} v_2$$

$$v_2 = \frac{4 \dot{m}_2}{\pi \rho D_2^2} = \frac{4 (2.2 \text{ kg/s})}{\pi (1000 \text{ kg/m}^3) (0.05 \text{ m})^2}$$

$$= \underline{\underline{1.12 \text{ m/s}}}$$

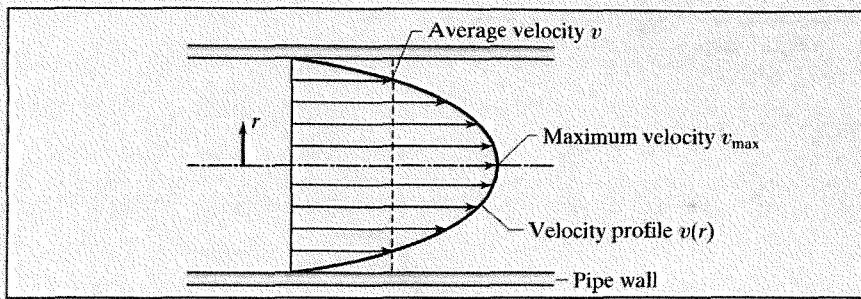
**Solution check**

The calculated value for the velocity in the large branch seems reasonable for a typical plumbing system. No errors are found in the calculations.

**Discussion**

The velocity is an average value because there is a velocity profile across the pipe. The velocity profile is caused by viscosity. If the flow condition is laminar, the velocity profile is parabolic, as shown in the following sketch.





In contrast to Examples 4 through 7 that illustrated hand calculations, the final example incorporates a computer analysis tool, TK Solver. This software is an equation solver (see Section 4.2) that is useful for performing the calculations step of the general analysis procedure. The other steps of the procedure are done in the usual manner.

### EXAMPLE 8

JANUARY 2, 2007

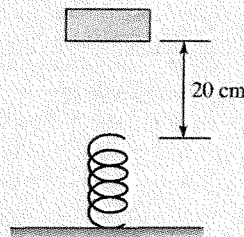
EXAMPLE 8

FRANK GRIMES

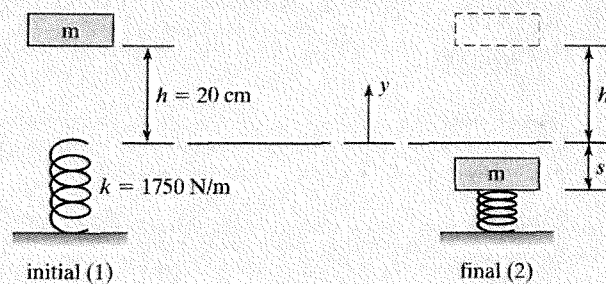
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#### Problem Statement

A 6-kg block falls 20 cm into a spring with a spring constant of 1750 N/m. When the block comes into contact with the spring, it sticks to the spring. If the block falls from rest, what is the deformation of the spring when the block momentarily comes to rest?



#### Diagram



**Assumptions**

1. Linear spring, i.e., follows Hooke's law  $F = ks$
2. Block sticks to spring (inelastic collision)
3. Spring deforms along its axis, i.e., no bending

**Governing Equations**

Conservation of energy:  $V_1 + T_1 = V_2 + T_2$

Gravitational potential energy:  $V = mgh$

Spring potential energy:  $V = \frac{1}{2}ks^2$

**Calculations**

The *rule sheet* shows the governing equations, and the *variable sheet* shows the inputs and outputs for all the physical quantities. TK Solver does not require the user to perform any algebraic manipulations; the software is capable of solving the governing equations in their original form. Due to the spring potential energy term, the conservation of energy equation becomes a quadratic equation, which has two roots. To generate these roots, a "guess" value for the spring deformation  $s$  is entered in the input column of the variable sheet. Then, a G (for guess) is entered in the *status* column beside the output variable. The iterative solver is initiated, generating *one* of the two roots. The root that is calculated depends on how close the guess value is to that root.

**Rule sheet**

Status	Rule
Satisfied	$V_1 + T_1 = V_2 + T_2$
Satisfied	$V_1 = m \cdot g \cdot h$
Satisfied	$V_2 = -m \cdot g \cdot s + 0.5 \cdot k \cdot s^2$

**Variable sheet**

Status	Input	Name	Output	Unit	Comment
		V1	11.772	J	initial potential energy
	0	T1		J	initial kinetic energy
		V2	11.772	J	final potential energy
	0	T2		J	final kinetic energy
	6	m		kg	mass of block
	9.81	g		m/s <sup>2</sup>	gravitational acceleration
	.2	h		m	initial height of block
	1750	k		N/m	spring constant
G		s	.15440257	m	spring deformation

As shown in the variable sheet, the spring deformation is:

$$s = 0.1544 \text{ m} \approx \underline{15.4 \text{ cm}}$$

**Solution check**

The solution may be checked by substituting values into the conservation of energy equation. Rearranging the equation, we have:

$$V_1 + T_1 - (V_2 + T_2) = 0$$

$$mgh + 0 - (-mgs + \frac{1}{2}ks^2 + 0) = 0$$

$$(6)(9.81)(0.20) + 0 - [-(6)(9.81)(0.1544) + \frac{1}{2}(1750)(0.1544)^2 + 0] = 0$$

$$11.7720 - [-9.0880 + 20.8594] = 0$$

$$11.7720 - 11.7714 = 5.84 \times 10^{-4} \approx 0$$

The small nonzero answer is due to round-off, so our answer is verified.

### Discussion

The second root is  $s = -0.0871$ , which is obtained by using a guess value of zero or less. Because the spring deformation  $s$  is defined as a positive quantity in the spring potential energy equation, the second root is nonphysical, i.e., has no physical significance.

It can be easily shown that the location of the origin is arbitrary.

### Professional Success

#### *Avoiding A "Cookbook" Learning Approach To Engineering Analysis*

A good engineer is a person who solves an engineering analysis problem by reasoning through it, rather than simply following a prepared "recipe" consisting of step-by-step instructions written by someone else. Similarly, a good engineering student is a person who learns engineering analysis by thinking conceptually about each problem, rather than simply memorizing a collection of disjointed solution sequences and mathematical formulas.

This "cookbook" learning approach is a detour on the road of engineering education. Furthermore, the cookbook learning style promotes fragmented rather than integrative learning. A student who embraces this type of learning method will soon discover that it will be difficult and take a long time to solve new engineering problems, unless identical or very similar problems have been previously solved by using an established recipe. An analogy may be drawn from the familiar maxim "Give a man a fish, and you have fed him for a day. Teach a man to fish, and you have fed him for a lifetime."

A recipe enables a student to solve only one specific type of problem, whereas a more general conceptual-based learning approach enables a student to solve many engineering problems.

### Practice!

Use the general analysis procedure to solve the following problems (present the analysis by using the guidelines for analysis presentation covered in this section):

1. Radioactive waste is to be permanently encased in concrete and buried in the ground. The vessel containing the waste measures 30 cm  $\times$  30 cm  $\times$  80 cm. Federal regulations dictate that there must be a minimum concrete thickness of 50 cm surrounding the vessel on all sides. What is the minimum volume of concrete required to safely encase the radioactive waste?

Answer: 2.97 m<sup>3</sup>

2. An elevator in an office building has an operating capacity of 15 passengers with a maximum weight of 180 lb<sub>f</sub> each. The elevator is suspended by a special pulley system with four cables, two of which support 20 percent of the total load and two of which support 80 percent of the total load. Find the maximum tension in each elevator cable.

*Answer:* 270 lb<sub>f</sub>, 1080 lb<sub>f</sub>

3. A technician measures a voltage drop of 25 V across a 100-Ω resistor by using a digital voltmeter. Ohm's law states that  $V = IR$ . What is the current flow through the resistor? How much power is consumed by the resistor? (*Hint:  $P = I^2R$ .*)

*Answer:* 250 mA, 6.25 W

4. Air flows through a main duct at a mass flow rate of 4 kg/s. The main duct enters a junction that splits into two branch ducts, one with a cross section of 20 cm × 30 cm and one with a cross section of 40 cm × 60 cm. If the mass flow rate in the large branch is 2.8 kg/s, what is the mass flow rate in the small branch? If the density of air is  $\rho = 1.16 \text{ kg/m}^3$ , what is the velocity in each branch?

*Answer:* 1.2 kg/s, 10.1 m/s, 17.2 m/s

#### 4 THE COMPUTER AS AN ANALYSIS TOOL

Computers are an integral part of the civilized world. They affect virtually every aspect of our everyday lives, including communications, transportation, financial transactions, information processing, food production, and health care. The world is a much different place today than it was prior to the advent of computers. People use computers for accessing and processing information, word processing, electronic mail, entertainment, and on-line shopping. Like everyone else, engineers use computers in their personal lives in the same ways just mentioned, but they also depend heavily on computers in their professional work. To the engineer, the computer is an indispensable tool. Without the computer, engineers would not be able to do their work as accurately or efficiently. The primary advantage of the computer to engineers is its ability to perform various functions extremely rapidly. For example, a complex sequence of calculations that would take days with a slide rule can be carried out in a few seconds by a computer. Furthermore, the numerical precision of the computer enables engineers to make calculations that are much more accurate. Engineers use computers for computer-aided design (CAD), word processing, communications, information access, graphing, process control, simulation, data acquisition, and of course, analysis.

The computer is one of the most powerful analysis tools available to the engineer, but the computer does not replace the engineer's thinking. When faced with a new analysis, the engineer must reason through the problem by using sound scientific principles, applied mathematics, and engineering judgement. A computer is only a machine, and as yet, no machine has been developed that can out-think a human being (except at playing chess, perhaps). A computer can only carry out the instructions supplied to it, but it does so with remarkable speed and efficiency. A computer yields wrong answers just as quickly as it yields right ones. The burden is upon the engineer to supply the computer with correct input. An often-used engineering acronym is *GIGO* (Garbage In, Garbage Out), which refers to a situation in which erroneous input data is supplied to a computer, thereby producing erroneous output. When *GIGO* is at work, the calculations are numerically correct, but the results of those calculations are meaningless, because the engineer supplied the computer with bad input, or the computer program that the engineer wrote is flawed. The computer is

capable of accurately performing enormous numbers of computations in a very short time, but it is incapable of composing a problem statement, constructing a diagram of the engineering system, formulating assumptions, selecting the appropriate governing equations, checking the reasonableness of the solution, or discussing and evaluating the results of the analysis. Thus, the only step in the analysis procedure for which a computer is perfectly suited is step 5: calculations. This is not to say that a computer cannot be used to write problem statements, assumptions, and equations as well as draw diagrams. These steps may also be performed by using the computer, but as directed by the engineer, whereas calculations are performed automatically once the equations and numerical inputs are supplied.

Engineers use analysis primarily as a design tool and as a means of predicting or investigating failures. Specifically, how does an engineer use the computer to perform an analysis? Steps 1 through 4 and steps 6 and 7 of the analysis procedure are largely unchanged, whether a computer is employed or not. So, exactly how are the calculations in step 5 carried out on a computer? There are basically five categories of computer tools for doing engineering analysis work:

1. Spreadsheets
2. Equation solvers and mathematics software
3. Programming languages
4. Specialty software
5. Finite element software.

#### 4.1 Spreadsheets

The term **spreadsheet** originally referred to a special tabulation of rows and columns for doing financial calculations. The computer-based spreadsheet is a modern electronic version of the paper spreadsheet and was initially used for business and accounting applications. By virtue of their general structure, spreadsheets are useful not only for doing financial calculations, but can also be used for performing a variety of scientific and engineering calculations. Like the original paper version, the computer-based spreadsheet consists of any array of rows and columns. The intersection of a row with a column is called a *cell*. Cells serve as locations for input and output data such as text, numbers, or formulas. For example, a cell may contain an equation representing Newton's second law of motion,  $F = ma$ . A nearby cell would contain a number for the mass  $m$ , while another cell would contain a number for the acceleration  $a$ . Immediately after entering these two input values in their respective cells, the spreadsheet automatically evaluates the formula, inserting the numerical value of the force  $F$  in the cell containing the formula for Newton's second law. If the values of the mass or acceleration are changed, the spreadsheet automatically updates the value of the force. This example is very simple, but spreadsheets are capable of doing calculations that involve hundreds or even thousands of variables. Suppose that our analysis involves 100 variables and that we want to know how changing only *one* of those variables affects the solution. We simply change the variable of interest and the entire spreadsheet automatically updates all calculations to reflect the change. The spreadsheet is an excellent analysis tool for rapidly answering "what if" questions. Numerous design alternatives can be efficiently investigated by performing the analysis on a spreadsheet. In addition to numerical functions, spreadsheets also have graphics capabilities. Excel,<sup>1</sup> Quattro Pro,<sup>2</sup> and Lotus 1-2-3<sup>3</sup> are popular spreadsheet products. Figure 5 shows a simple example of calculating force using Newton's second law by using Excel.

<sup>1</sup>Excel is a registered trademark of Microsoft® Corporation.

<sup>2</sup>Quattro® Pro is a registered trademark of Corel® Corporation.

<sup>3</sup>Lotus 1-2-3 is a registered trademark of Lotus® Development Corporation part of IBM®.



	A	B	C	D	E	F	G	H	I
1	Mass	Acceleration	Force						
2	(kg)	(m/s^2)	(N)						
3									
4	100	9.81	981						
5									
6									
7									
8									
9									
10									
11									
12									
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21									
22									

**Figure 5**

A calculation of Newton's second law using Excel. Note the formula for the force, +A4\*B4, entered in cell C4.

## 4.2 Equation Solvers and Mathematics Software

**Equation solvers** and **mathematics software** packages are general-purpose scientific and engineering tools for solving equations and performing symbolic mathematical operations. Equation solvers are primarily designed for solving problems that involve *numerical* inputs and outputs, whereas mathematics packages are primarily suited for performing *symbolic* mathematical operations much as you would do in a mathematics course. Equation solvers accept a set of equations that represent the mathematical model of the analytical problem. The equations can be linear or nonlinear. The equations may be written in their usual form without prior mathematical manipulation to isolate the unknown quantities on one side of the equals sign. For example, Newton's second law would be written in its usual form as  $F = ma$  even if the unknown quantity was the acceleration  $a$ . Solving this problem by hand, however, we would have to write the equation as  $a = F/m$  because we are solving for the acceleration. This is not necessary when we use equation solvers. After we supply the numerical values for the known quantities, equation solvers solve for the remaining unknown values. Equation solvers have a large built-in library of functions for use in trigonometry, linear algebra, statistics, and calculus. Equation solvers can perform a variety of mathematical operations, including differentiation, integration, and matrix operations. In addition to these mathematical features, equation solvers also do unit conversions. Equation solvers also have the capability of displaying results in graphical form. Programming can also be done within equation solvers. Although all equation solvers have some symbolic capabilities, some have the capacity for data acquisition, image analysis, and signal processing. Popular equation solvers are TK Solver,<sup>4</sup> Mathcad,<sup>5</sup> and Matlab.<sup>6</sup>

<sup>4</sup>TK Solver is a registered trademark of Universal Technical Systems, Incorporated.

<sup>5</sup>Mathcad® is a registered trademark of Mathsoft™, Incorporated.

<sup>6</sup>MATLAB® is a registered trademark of The MathWorks, Incorporated.

The strength of mathematics packages is their ability to perform symbolic mathematical operations. A symbolic mathematical operation is one that involves the manipulation of symbols (variables), using mathematical operators such as the vector product, differentiation, integration, and transforms. These packages are capable of performing very complex and sophisticated mathematical procedures. They also have extensive graphical capabilities. Even though mathematics packages are primarily designed for symbolic operations, they can also perform numerical computations. Mathematica<sup>7</sup> and Maple<sup>8</sup> are popular mathematics software products.

#### 4.3 Programming Languages

Spreadsheets, equation solvers, and mathematics software packages may not always meet the computational demands of every engineering analysis. In such cases, engineers may choose to write their own computer programs with the use of a programming language. **Programming languages** refer to sequential instructions supplied to a computer for carrying out specific calculations. Computer languages are generally categorized according to their level. *Machine language* is a low-level language, based on a binary system of “zeroes” and “ones.” Machine language is the most primitive language, because computers are digital devices whose rudimentary logic functions are carried out by using solid state switches in the “on” or “off” positions. *Assembly language* is also a low-level language, but its instructions are written in English-like statements rather than binary. Assembly language does not have many commands, and it must be written specifically for the computer hardware. Computer programs written in low-level languages run very fast because these languages are tied closely to the hardware, but writing the programs is very tedious.

Due to the tediousness of writing programs in low-level languages, engineers usually write programs in high-level languages that consist of straightforward, English-like commands. The most commonly used high-level languages by engineers are Fortran, C, C++, Pascal, Ada, and BASIC. Fortran is the patriarch of all scientific programming languages. The first version of Fortran (FORmula TRANslation) was developed by IBM between 1954 and 1957. Since its inception, Fortran has been the workhorse of scientific and engineering programming languages. It has undergone several updates and improvements and is still in widespread use today. The C language evolved from two languages, BCPL and B, which were developed during the late 1960s. In 1972, the first C program was compiled. The C++ language grew out of C and was developed during the early 1980s. Both C and C++ are popular programming languages for engineering applications because they use powerful commands and data structures. Pascal was developed during the early 1970s and is a popular programming language for beginning computer science students who are learning programming for the first time. The U.S. Department of Defense prompted the development of Ada during the 1970s in order to have a high-level language suitable for embedded computer systems. BASIC (Beginner’s All-purpose Symbolic Instruction Code) was developed during the mid-1960s as a simple learning tool for secondary school students as well as college students. BASIC is often included as part of the operating software for personal computers.

Writing programs in high-level languages is easier than writing programs in low-level languages, but the high-level languages utilize a larger number of commands. Furthermore, high-level languages must be written with specific grammatical rules, referred to as *syntax*. Rules of syntax govern how punctuation, arithmetic

<sup>7</sup>MATHEMATICA® is a registered trademark of Wolfram Research, Incorporated.

<sup>8</sup>Maple™ is a registered trademark of Maplesoft™, a division of Waterloo Maple, Incorporated.

operators, parentheses, and other characters are used in writing commands. To illustrate the syntactical differences between programming languages, equation solvers, and mathematics packages, Table 1 shows how a simple equation is written. Note the similarities and differences in the equals sign, the constant  $\pi$ , and the operator for exponentiation.

**Table 1 Comparison of Computer Statements for the Equation,  
 $V = 4/3\pi R^3$ , the Volume of a Sphere**

Computer Tool	Statement
Mathcad	$V := 4/3 * \pi * R^3$
TK Solver	$V = 4/3 * \text{pi}() * R^3$
MATLAB	$V = 4/3 * \text{pi} * R^3;$
MATHEMATICA	$V = 4/3 * \text{Pi} * R^3$
Maple	$V := 4/3 * \text{pi} * R^3;$
Fortran	$V = 4/3 * 3.141593 * R^{**}3$
C, C++	$V = 4/3 * 3.141593 * \text{pow}(R, 3);$
Pascal	$V := 4/3 * 3.141593 * R * R * R;$
Ada	$V := 4/3 * 3.141593 * R^{**}3;$
BASIC	$V = 4/3 * 3.141593 * R^3$

#### 4.4 Specialty Software

Considered as a whole, engineering is a broad field that covers a variety of disciplines and careers. Some of the main engineering disciplines are chemical engineering, civil engineering, electrical and computer engineering, environmental engineering, and mechanical engineering. Primary engineering career fields include research, development, design, analysis, manufacturing, and testing. Given the variety of specific problems that engineers who work in these fields encounter, it comes as no surprise that numerous *specialty software* packages are available to help the engineer analyze specific problems relating to a particular engineering system. For example, specialty software packages are available to electrical engineers for analyzing and simulating electrical circuits. Mechanical and chemical engineers can take advantage of software packages designed specifically for calculating flow parameters in pipe networks. Special software is available to civil and structural engineers for calculating forces and stresses in trusses and other structures. Other specialty software packages are available for performing analysis of heat exchangers, machinery, pressure vessels, propulsion systems, turbines, pneumatic and hydraulic systems, manufacturing processes, mechanical fasteners, and many others too numerous to list. After you graduate and begin working for a company that produces a specific product or process, you will probably become familiar with one or more of these specialty software packages.

#### 4.5 Finite Element Software

Some engineering analysis problems are far too complex to solve using any of the aforementioned computer tools. *Finite element* software packages enable the engineer to analyze systems that have irregular configurations, variable material properties,

complex conditions at the boundaries, and nonlinear behavior. The finite element method originated in the aerospace industry during the early 1950s when it was used for stress analysis of aircraft. Later, as the method matured, it found application in other analysis areas such as fluid flow, heat transfer, vibrations, impacts, acoustics, and electromagnetics. The basic concept behind the finite-element method is to subdivide a continuous region (i.e., the system to be analyzed is divided into a set of simple geometric shapes called "finite elements"). The elements are interconnected at common points called "nodes." Material properties, conditions at the system boundaries, and other pertinent inputs are supplied. With the use of an advanced mathematical procedure, the finite element software calculates the value of parameters such as stress, temperature, flow rate, or vibration frequency at each node in the region. Hence, the engineer is provided with a set of output parameters at discrete points that approximates a continuous distribution of those parameters for the entire region. The finite element method is an advanced analysis method and is normally introduced in colleges and universities at the senior level or the first-year graduate level.

## Professional Success

### *Pitfalls of Using Computers*

The vital role that computers play in engineering analysis cannot be overstated. Given the tremendous advantages of using computers for engineering analysis, however, it may be difficult to accept the fact that there are also pitfalls. A common hazard that entangles some engineers is the tendency to treat the computer as a "black box," a wondrous electronic device whose inner workings are largely unknown, but that nonetheless provides output for every input supplied. Engineers who treat the computer as a black box are not effectively employing the general analysis procedure and in so doing are in danger of losing their ability to systematically reason their way through a problem. The computer is a remarkable computational machine, but it does not replace the engineer's thinking, reasoning, and judgment. Computers, and the software that runs on them, produce output that *precisely* reflects the input supplied to them. If the input is good, the output will be good. If the input is bad, the output will be bad. Computers are not smart enough to compensate for an engineer's inability to make good assumptions or employ the correct governing equations. Engineers must have a thorough understanding of the physical aspects of the problem at hand and the underlying mathematical principles *before* implementing the solution on the computer. A good engineer understands *what* the computer does when it "crunches the numbers" in the analysis. A good engineer is confident that the input data will result in reasonable output because a lot of sound thinking and reasoning has gone into the formulation of that input.

Can the computer be used too much? In a sense, it can. The tendency of some engineers is to use the computer to analyze problems that may not require a computer at all. Upon beginning a new problem, their first impulse is to set up the problem on the computer without even checking to see whether the problem can be solved by hand. For example, a problem in engineering statics may be represented by the quadratic equation,

$$x^2 + 4x - 12 = 0.$$

This problem can be solved analytically by factoring,  $(x + 6)(x - 2) = 0$ , which yields the two roots,  $x = -6$  and  $x = 2$ . To use the computer in a situation like this is to rely on the computer as a “crutch” to compensate for weak analytical skills. Continued reliance on the computer to solve problems that do not require a computer will gradually dull your ability to solve problems with pencil and paper. Do not permit this to happen. Examine the equations carefully to see whether a computer solution is justified. If it is, use one of the computer tools discussed earlier. If not, solve the problem by hand. Then, if you have time and wish to check your solution with the use of the computer, by all means do so.

## APPLICATION

### Computers for Numerical Analysis

Most of the equations that you will encounter in school can be solved analytically; that is, they can be solved by employing standard algebraic operations to isolate the desired variable on one side of the equation. Some equations, however, cannot be solved analytically with standard algebraic operations. These equations are referred to as *transcendental* equations because they contain one or more transcendental functions such as a logarithm or trigonometric function. Transcendental equations occur often in engineering analysis work, and techniques for solving them are known as *numerical methods*. For example, consider the transcendental equation:

$$e^x - 3x = 0$$

This equation looks straightforward enough, but try solving it by hand. If we add  $3x$  to both sides and take the natural logarithm of both sides to undo the exponential function, we obtain:

$$x = \ln(3x) \quad (a)$$

which, unfortunately, does not isolate the variable  $x$  because we still have the term  $\ln(3x)$  on the right side of the equation. If we add  $3x$  to both sides and then divide both sides by 3, we obtain:

$$\frac{e^x}{3} = x \quad (b)$$

The variable  $x$  is still not isolated without leaving a transcendental function in the equation. Clearly, this equation cannot be solved analytically, so it must be solved numerically. To solve it numerically, we utilize a method called *iteration*, a process by which we repeat the calculation until an answer is obtained.

Before solving this problem by using the computer, we will work it manually to illustrate how iteration works. To begin, we rewrite Equation (a) in the iterative form:

$$x_{i+1} = \ln(3x_i)$$

The “ $i$ ” and the “ $i + 1$ ” subscripts refer to “old” and “new” values of  $x$ , respectively. The iteration process requires that we begin the calculation by immediately substituting a number into the iteration formula. This first number constitutes an estimate for the root (or roots) of the variable  $x$ , that satisfy the formula. To keep track of the iterations, we use an iteration table, illustrated in Table 2. To start the iterations, we estimate a value of  $x$  by letting  $x_1 = 1$ . We now substitute this number into the right

Table 2 Iteration Table for Finding One Root of the Equation  $e^x - 3x = 0$ 

Iteration	$x_i$	$x_{i+1}$
1	1	1.098612
2	1.098612	1.192660
3	1.192660	1.274798
4	1.274798	1.341400
5	1.341400	1.392326
.		
.		
.		1.512134
41	1.512134	1.512135

side of the formula, yielding a new value of  $x_2 = 1.098612$ . We again substitute into the right side of the formula, yielding the next new value,  $x_3 = 1.192660$ . This process is repeated until the value of  $x$  stops changing significantly. At this point, we say that the calculation has *converged* to an answer. Table 2 shows the first five iterations and indicates that 41 iterations are required for the calculation to converge to an answer that is accurate to the sixth decimal place. Upon substituting  $x = 1.512135$  into the original equation, we see that the equation is satisfied. As this example illustrates, numerous iterations may be required to obtain an accurate solution. The accuracy of the answer depends on how many iterations are taken. Some equations converge to a precise answer in a few iterations but others, like this one, require several iterations. It is important to note that 1.512135 is not the only root of this equation. The equation has a second root at  $x = 0.619061$ . If we attempt to find this root by using Equation (a), we discover that our calculation either converges again to 1.512135 or does not converge at all by leading us to an illegal operation, that is, taking the logarithm of a negative number. To find the second root, we iterate on Equation (b), writing it in the iterative form:

$$x_{i+1} = \frac{e^{x_i}}{3}.$$

With numerical methods, there are often no guarantees that a certain iteration formula will converge rapidly or even converge at all. The success of the iteration formula may also depend on the initial estimate chosen to start the iterations. If our initial estimate for Equation (a) is less than  $\frac{1}{3}$ , the new value of  $x$  immediately goes negative, leading to an illegal operation. If our initial estimate for Equation (b) is too large, the new value of  $x$  grows large very rapidly, leading to an exponential overflow. These and other kinds of numerical difficulties can occur whether the iterations are performed by hand or by using a computer.

As Table 2 suggests, performing iterations by hand can be a long and tedious task. The computer is tailor-made for performing repetitive calculations. The roots of our transcendental equation can readily be found by using one of the computer tools discussed earlier. Figure 6 shows a computer program, written in the BASIC language for finding the first root  $x = 1.512135$ . In the first line the user inputs an initial estimate, which is assigned the variable name XOLD. The program then executes what is referred to as a DO loop that performs the iterations. Each time



**Figure 6**

BASIC computer program for finding one root of the equation  $e^x - 3x = 0$ .

```

INPUT "ESTIMATE = ", XOLD
DO
  XNEW = LOG (3*XOLD)
  DIFF = ABS (XNEW - XOLD)
  XOLD = XNEW
LOOP WHILE DIFF > 0.0000001
PRINT XNEW
END

```

through the loop, a new value of  $x$  is calculated from the old value and an absolute value of the difference between the old and new values is calculated. This value is called DIFF. While DIFF is larger than a preselected convergence tolerance of 0.0000001, the new value of  $x$ , XNEW, is reset to the old value XOLD and looping continues. When DIFF is less than or equal to the convergence tolerance, convergence has been achieved, and looping is halted. The root is then printed. The same program, with the third line replaced with  $XNEW = \text{EXP}(XOLD)/3$ , could be used to find the second root. There are more sophisticated numerical methods for finding roots than the simple iteration technique illustrated here, and you will study them in your engineering or mathematics courses.

### Practice!

Using one of the computer tools discussed in this section, work the following problems:

(Note: These problems are identical to those in Section 3.)

1. Radioactive waste is to be permanently encased in concrete and buried in the ground. The vessel containing the waste measures 30 cm  $\times$  30 cm  $\times$  80 cm. Federal regulations dictate that there must be a minimum concrete thickness of 50 cm surrounding the vessel on all sides. What is the minimum volume of concrete required to safely encase the radioactive waste?

*Answer:* 2.97 m<sup>3</sup>

2. An elevator in an office building has an operating capacity of 15 passengers with a maximum weight of 180 lb<sub>f</sub> each. The elevator is suspended by a special pulley system with four cables, two of which support 20 percent of the total load and two of which support 80 percent of the total load. Find the maximum tension in each elevator cable.

*Answer:* 270 lb<sub>f</sub>, 1080 lb<sub>f</sub>

3. A technician measures a voltage drop of 25 V across a 100- $\Omega$  resistor by using a digital voltmeter. Using Ohm's law, we find that  $V = IR$ . What is the current flow through the resistor? How much power is consumed by the resistor? (*Hint:*  $P = I^2R$ .)

*Answer:* 250 mA, 6.25 W

4. Air flows through a main duct at a mass flow rate of 4 kg/s. The main duct enters a junction that splits into two branch ducts, one with a

cross section of  $20\text{ cm} \times 30\text{ cm}$  and one with a cross section of  $40\text{ cm} \times 60\text{ cm}$ . If the mass flow rate in the large branch is  $2.8\text{ kg/s}$ , what is the mass flow rate in the small branch? If the density of air is  $\rho = 1.16\text{ kg/m}^3$ , what is the velocity in each branch?  
 Answer:  $1.2\text{ kg/s}$ ,  $10.1\text{ m/s}$ ,  $17.2\text{ m/s}$

engineering method  
 equation solver  
 general analysis  
 procedure

mathematics software  
 order of magnitude  
 programming language

significant figure  
 spreadsheet

## KEY TERMS

## REFERENCES

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## PROBLEMS

### Order of magnitude analysis

- Using an order-of-magnitude analysis, estimate the number of gallons of gasoline used by all automobiles in the United States each year.
- Using an order-of-magnitude analysis, estimate the number of  $4\text{ ft} \times 8\text{ ft}$  plywood sheets required for the floor, roof, and exterior sheathing of a  $6000\text{-ft}^2$  house.
- Using an order-of-magnitude analysis, estimate the number of basketballs (fully inflated) that would fit in the Grand Canyon.
- Using an order-of-magnitude analysis, estimate the number of spam e-mail messages received by residents of the United States each year.
- Using an order-of-magnitude analysis, estimate the number of breaths you will take during your lifetime.
- Use an order-of-magnitude analysis to estimate the number of tons of human waste produced worldwide each year.
- The earth has a mean radius of about  $6.37 \times 10^6\text{ m}$ . Assuming the earth is made of granite ( $\rho = 2770\text{ kg/m}^3$ ), estimate the mass of the earth using an order-of-magnitude analysis.
- The solar radiation flux just outside the earth's atmosphere is about  $1350\text{ W/m}^2$ . Using an order-of-magnitude analysis, estimate the amount of solar energy that is intercepted by the Pacific Ocean each year.
- Using an order-of-magnitude analysis, estimate the total textbook expenditure incurred by all engineering majors at your school per year.

**Significant figures**

- 10** Underline the significant figures in the following numbers (the first number is done for you):
- (a) 3450
  - (b) 9.807
  - (c) 0.00216
  - (d) 9000
  - (e) 7000.
  - (f) 12.00
  - (g) 1066
  - (h) 106.07
  - (i) 0.02880
  - (j) 163.07
  - (k)  $1.207 \times 10^{-3}$
- 11** Perform the following calculations, reporting the answers with the correct number of significant figures:
- (a)  $(8.14)(260)$
  - (b)  $456/4.9$
  - (c)  $(6.74)(41.07)/4.13$
  - (d)  $(10.78 - 4.5)/300$
  - (e)  $(10.78 - 4.50)/300.0$
  - (f)  $(65.2 - 13.9)/240.0$
  - (g)  $(1.2 \times 10^6)/(4.52 \times 10^3 + 769)$
  - (h)  $(1.764 - 0.0391)/(8.455 \times 10^4)$
  - (i)  $1000/(1.003 \times 10^9)$
  - (j)  $(8.4 \times 10^{-3})/5000$
  - (k)  $(8.40 \times 10^3)/5000.0$
  - (l)  $8\pi$
  - (m)  $(2\pi - 5)/10.$
- 12** A 125.5-kg mass hangs by a cable from the ceiling. Using the standard value of gravitational acceleration  $g = 9.81 \text{ m/s}^2$ , what is the tension in the cable? Express your answer with the correct number of significant figures.
- 13** A 9-slug mass hangs by a rope from the ceiling. Using the standard value of gravitational acceleration  $g = 32.2 \text{ ft/s}^2$ , what is the tension in the rope? Express your answer with the correct number of significant figures. Redo the problem, using a mass of 9.00 slug. Is the answer different? Why?
- 14** A 175 mA current flows through a 47- $\Omega$  resistor. Using Ohm's law  $V = IR$ , what is the voltage across the resistor? Express your answer with the correct number of significant figures.
- 15** A rectangular building lot is reported to have the dimensions 200 ft  $\times$  300 ft. Using the correct number of significant figures, what is the area of this lot in units of acre?

**General analysis procedure**

For problems 16 through 31, use the general analysis procedure of (1) problem statement, (2) diagram, (3) assumptions, (4) governing equations, (5) calculations, (6) solution check, and (7) discussion.

- 16** An excavation crew digs a hole in the ground measuring 60 yd  $\times$  50 yd  $\times$  8 yd to facilitate a basement for an office building. Five dump trucks, each with a

capacity of  $20 \text{ yd}^3$ , are used to haul the material away. How many trips must each truck make to remove all the material?

- 17 Find the current in each resistor and the total current for the circuit shown in Figure P17.

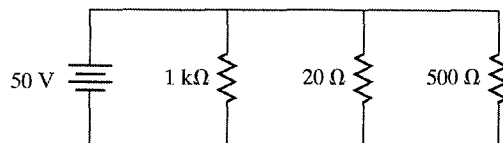


Figure P17

- 18 For easy handling, long sheets of steel for manufacturing automobile body panels are tightly rolled up into a cylinder-shaped package. Consider a roll of steel with an inside and outside diameter of 45 cm and 1.6 m, respectively, that is suspended by a single cable. If the length of the roll is 2.25 m and the density of steel is  $\rho = 7850 \text{ kg/m}^3$ , what is the tension in the cable?
- 19 In a chemical processing plant, glycerin flows toward a pipe junction as shown in Figure P19. If the mass flow rate in the small pipe branch is 8 kg/s, find the velocity in both branches. The density of glycerin is  $\rho = 1260 \text{ kg/m}^3$ .

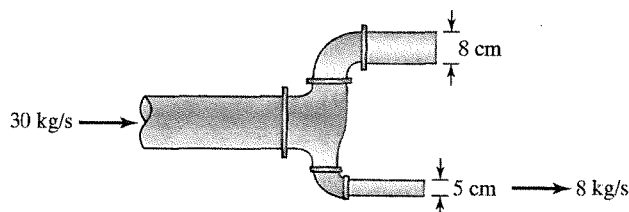


Figure P19

- 20 A portable classroom is heated with small propane heating units with a capacity of 3 kW each. The portable classroom is occupied by 45 students, each dissipating 120 W, and is lighted by 10 light fixtures that dissipate 60 W each. If the heat loss from the portable classroom is 15 kW, how many heating units are required to maintain the classroom at a temperature of  $20^\circ\text{C}$ ?
- 21 A man pushes on a barrel with a force of  $P = 30 \text{ N}_f$  as shown. Assuming that the barrel does not move, what is the friction force between the barrel and the floor? (*Hint:* The friction force acts parallel to the floor toward the man. See Figure P21.)

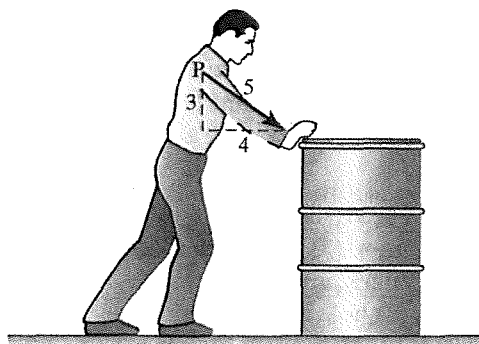


Figure P21

- 22 The total resistance for resistors connected in series is the arithmetic sum of the resistances. Find the total resistance for the series circuit shown in Figure P22. Because the resistors are connected in series, the current is the same in each resistor. Using Ohm's law, find this current. Also, find the voltage drop across each resistor.

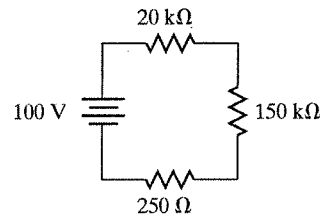


Figure P22

- 23 The pressure exerted by a static liquid on a vertical submerged surface is calculated from the relation:

$$P = \rho gh$$

where

$P$  = pressure

$\rho$  = density of the liquid

$g$  = gravitational acceleration =  $9.81 \text{ m/s}^2$

$h$  = height of vertical surface that is submerged.

Consider the dam shown in Figure P23. What is the pressure exerted on the dam's surface at depths of 1 m, 5 m, and 25 m? For the density of water, use  $\rho = 1000 \text{ kg/m}^3$ .

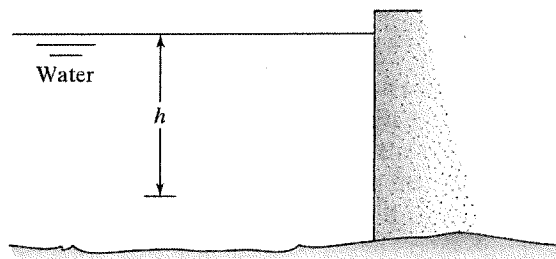


Figure P23

- 24 Work Problem 16, using one of the computer tools discussed in this chapter.  
 25 Work Problem 17, using one of the computer tools discussed in this chapter.  
 26 Work Problem 18, using one of the computer tools discussed in this chapter.  
 27 Work Problem 19, using one of the computer tools discussed in this chapter.  
 28 Work Problem 20, using one of the computer tools discussed in this chapter.  
 29 Work Problem 21, using one of the computer tools discussed in this chapter.  
 30 Work Problem 22, using one of the computer tools discussed in this chapter.  
 31 Work Problem 23, using one of the computer tools discussed in this chapter.

## ANSWERS TO SELECTED PROBLEMS

**10** b. 9.807, c. 0.00216, d. 9000 e. 7000. f. 12.00 g. 1066 h. 106.07  
i. 0.02880 j. 163.07 k.  $1.207 \times 10^{-3}$

**12** 1230 N

**14** 8.2 V

**16** 240

**18** 0.32 MN

**20** 3

**22** 170 k $\Omega$ , 0.587 mA

20 k $\Omega$ :  $V = 11.7$  V; 150 k $\Omega$ :  $V = 88.1$  V; 250  $\Omega$ :  $V = 0.147$  V