Contemporary Communication Systems

Chapter 7

Noise Performance of Analog Communication Systems

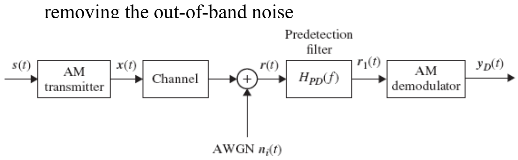
M.F. Mesiya

Performance Metrics

- Two parameters are frequently used to characterize the noise performance of various analog modulation schemes
- **Carrier-to-noise power ratio** (**CNR**) – Defined as the ratio of carrier power to the noise power in a specified bandwidth, usually measured using a spectrum analyzer
  - Predetection measurement on carrier modulated waveforms
- **Signal-to-noise power ratio** (**SNR**) – Defined as the ratio of signal power to noise power made at baseband before modulation or after detection or demodulation
  - Predetection measurement – includes noise contributions from the camera, modulator/ transmitter, in-line amplifiers, and demodulator/receiver
  - Ideal for characterizing end-to-end performance (overall signal quality)

AM Communication System Model

- Message signal $\boldsymbol{s}(t)$ is a stationary, zero mean, low-pass random process whose spectral density is bandlimited to B Hz
- Receiver front end noise $\boldsymbol{n}_i(t)$ is WG with PSD $G_{n_i}(f) = N_o/2$
- We assume that $\boldsymbol{s}(t)$ and $\boldsymbol{n}_i(t)$ are statistically independent
- The receiver front end consists of a BP **predetection** filter with a bandwidth B_T
  - Passes the modulated signal without distortion while removing the out-of-band noise



Noise Performance of DSB-SC

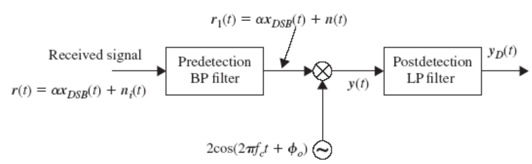
- The input to the predetection filter is

$$\boldsymbol{r}(t) = \alpha \boldsymbol{x}_{DSB}(t) + \boldsymbol{n}_i(t) = \alpha A_c \boldsymbol{s}(t)\cos(2\pi f_c t + \phi) + \boldsymbol{n}_i(t)$$

- The signal power at the predetection filter input is given by

$$P_R = E\left\{\alpha^2 \boldsymbol{x}_{DSB}^2(t)\right\} = \frac{1}{2}\alpha^2 A_c^2 \overline{\boldsymbol{s}^2}$$

- where $\overline{\boldsymbol{s}^2} = E\{\boldsymbol{s}^2(t)\}$ is the average power in the message $\boldsymbol{s}(t)$



Noise Performance of DSB-SC (contd)

- The receiver input noise power, measured in the bandwidth of the baseband message signal, is $P_{n_i} = N_o B$
- The receiver input CNR, defined as the ratio of carrier power to the noise power in the message signal bandwidth, is

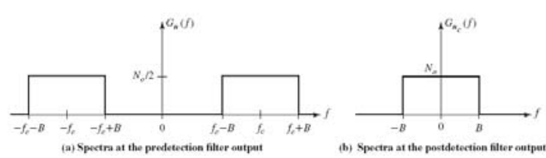
$$CNR_{IN} = \frac{\text{Carrier power at the receiver input}}{\text{Input noise power in the message signal bandwidth}} = \frac{\alpha^2 A_c^2 \overline{\boldsymbol{s}^2}}{2N_o B}$$

- The predetection filter output is

$$\boldsymbol{r}_1(t) = \alpha A_c \boldsymbol{s}(t)\cos(2\pi f_c t + \phi) + \boldsymbol{n}_c(t)\cos(2\pi f_c t + \phi) - \boldsymbol{n}_s(t)\sin(2\pi f_c t + \phi)$$

where $\boldsymbol{n}_c(t)$ and $\boldsymbol{n}_s(t)$ are quadrature components of the narrowband noise $\boldsymbol{n}(t)$ with mean zero and variance $2N_oB$

Noise Performance of DSB-SC (contd)



- To coherently demodulate, $\boldsymbol{r}_1(t)$ is multiplied with carrier $2\cos(2\pi f_c t + \phi)$ and LP filtered to recover the message signal.
- The postdetection LP filter output is given by

$$\boldsymbol{y}_D(t) = \alpha A_c \boldsymbol{s}(t) + \boldsymbol{n}_c(t)$$

Noise Performance of DSB-SC (contd)

- Postdetection signal power: $P_D = (\alpha A_c)^2 \overline{s^2}$
- Noise power at the postdetection filter output: $P_{n_c} = 2N_o B$
- Output (postdetection) SNR

$$SNR_{DSB} = \frac{P_D}{P_{n_c}} = \frac{(\alpha A_c)^2 \overline{s^2}}{2N_o B} = \frac{P_R}{N_o B}$$

$$\Rightarrow SNR_{DSB} = CNR_{IN}$$

- Thus the output SNR of a DSB-SC AM system is equal to the receiver input CNR
- The figure of merit of a DSB-SC AM system is

$$\frac{SNR_{DSB}}{CNR_{IN}} = 1$$

Noise Performance of SSB-AM

- We assume that the transmitted signal is USB-AM. The input to the predetection filter is

$$\boldsymbol{r}(t) = \alpha \boldsymbol{x}_{USB}(t) + \boldsymbol{n}_i(t)$$
$$= \frac{\alpha A_c}{2}\left[\boldsymbol{s}(t)\cos(2\pi f_c t + \phi) - \hat{\boldsymbol{s}}(t)\sin(2\pi f_c t + \phi)\right] + \boldsymbol{n}_i(t)$$

- The signal power at the predetection filter input is given by

$$P_R = \frac{\alpha^2 A_c^2}{8}\left[E\{\boldsymbol{s}^2(t)\} + E\{\hat{\boldsymbol{s}}^2(t)\}\right] = \frac{\alpha^2 A_c^2 \overline{\boldsymbol{s}^2}}{4}$$

- The receiver input noise power measured in the bandwidth of the baseband message signal equals $P_{n_i} = N_o B$

Noise Performance of SSB-AM (contd)

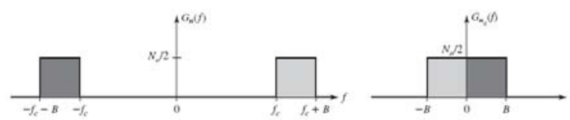
- The receiver input CNR is given by

$$CNR_{IN} = \frac{P_R}{P_{n_i}} = \frac{P_R}{N_o B} = \frac{\alpha^2 A_c^2 \overline{\boldsymbol{s}^2}}{4N_o B}$$

- The ideal predetection BP filter (Bandwidth = B Hz) in the SSB-AM receiver passes the upper (or lower) sideband signal without any distortion while rejecting the out-of-band noise
  - The center frequency of the predetection filter is $f_c \pm B/2$, where the sign depends on the choice of sideband
- The predetection filter output is

$$\boldsymbol{r}_1(t) = \left[\frac{\alpha A_c}{2}\boldsymbol{s}(t) + \boldsymbol{n}_c(t)\right]\cos(2\pi f_c t + \phi)$$
$$- \left[\frac{\alpha A_c}{2}\hat{\boldsymbol{s}}(t) + \boldsymbol{n}_s(t)\right]\sin(2\pi f_c t + \phi)$$

Noise Performance of SSB-AM (contd)



(a) Spectra at the predetection filter output (b) Spectra at the postdetection filter output

- $\boldsymbol{n}_c(t)$ and $\boldsymbol{n}_s(t)$ are quadrature components of the narrowband noise $\boldsymbol{n}(t)$ with mean zero and variance $N_o B$
- To coherently demodulate, $\boldsymbol{r}_1(t)$ is multiplied with carrier $2\cos(2\pi f_c t + \phi)$ and LP filtered to recover the message signal.
- The postdetection LP filter output is given by

$$\boldsymbol{y}_D(t) = \frac{\alpha A_c}{2}\boldsymbol{s}(t) + \boldsymbol{n}_c(t)$$

Noise Performance of SSB-AM (contd)

- Postdetection signal power: $P_D = \left(\frac{\alpha A_c}{2}\right)^2 \overline{\boldsymbol{s}^2}$
- Noise power at the postdetection filter output: $P_{n_c} = N_o B$
- Output (postdetection) SNR

$$SNR_{SSB} = \frac{P_D}{P_{n_c}} = \frac{\alpha^2 A_c^2 \overline{\boldsymbol{s}^2}}{4N_o B} = \frac{P_R}{N_o B}$$

$$\Rightarrow SNR_{SSB} = CNR_{IN}$$

- Thus the output SNR of a SSB-AM system is equal to the receiver input CNR
- The figure of merit of a SSB-AM system is

$$\frac{SNR_{SSB}}{CNR_{IN}} = 1$$

Noise Performance of Conventional AM

- In the conventional amplitude modulation scheme, the input to the predetection filter is

$$\boldsymbol{r}(t) = \alpha \boldsymbol{x}_{AM}(t) + \boldsymbol{n}_i(t) = \alpha A_c\left[1 + m_a \boldsymbol{s}_n(t)\right]\cos(2\pi f_c t + \phi) + \boldsymbol{n}_i(t)$$

- The signal power at the predetection filter input is given by

$$P_R = E\left\{\alpha^2 \boldsymbol{x}_{AM}^2(t)\right\} = \frac{\alpha^2 A_c^2}{2}\left(1 + m_a^2 \overline{\boldsymbol{s}_n^2}\right)$$

- The receiver input noise power, measured in the bandwidth of the baseband message signal equals $P_{n_i} = N_o B$
- The receiver input CNR is given by

$$CNR_{IN} = \frac{P_R}{P_{n_i}} = \frac{P_R}{N_o B} = \frac{\alpha^2 A_c^2\left(1 + m_a^2 \overline{\boldsymbol{s}_n^2}\right)}{2N_o B}$$

Noise Performance of Conventional AM

- The predetection filter output is given by

$$r_1(t) = \alpha A_c\left[1 + m_a s_n(t)\right]\cos\left(2\pi f_c t + \phi\right) + n_c(t)\cos\left(2\pi f_c t + \phi\right) - n_s(t)\sin\left(2\pi f_c t + \phi\right)$$

- Conventional AM signal can be either
  - Coherently detected or
  - Envelope detected
- **Coherent Detection**
- The demodulated output is given by

$$y_D(t) = \alpha A_c\left[1 + m_a s_n(t)\right] + n_c(t) = \underbrace{\alpha A_c}_{\text{DC term; blocked out}} + \alpha A_c m_a s_n(t) + n_c(t)$$

Noise Performance of Conventional AM (contd)

- Postdetection signal power: $P_D = \alpha^2 A_c^2 m_a^2 \overline{s_n^2}$
- Noise power at the postdetection filter output: $P_{n_c} = 2N_o B$
- Output (postdetection) SNR

$$SNR_{AM} = \frac{P_D}{P_{n_c}} = \frac{\alpha^2 A_c^2 m_a^2 \overline{s_n^2}}{2N_o B} = \eta CNR_{IN}$$

where

$$\eta = \frac{m_a^2 \overline{s_n^2}}{(1 + m_a^2 \overline{s_n^2})}$$

Since $\eta \le 0.5 \Rightarrow SNR_{AM}$ is at least 3 dB poorer than output SNR for DSB and SSB systems for the same CNR_{IN}

- The figure of merit of a conventional AM system is

$$\frac{SNR_{AM}}{CNR_{IN}} = \eta$$

Conventional AM Envelope Detection Performance

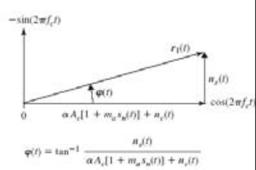
- The input to the envelope detector is

$$r_1(t) = \alpha A_c\left[1 + m_a s_n(t)\right]\cos\left(2\pi f_c t\right) + n_c(t)\cos\left(2\pi f_c t\right) - n_s(t)\sin\left(2\pi f_c t\right)$$

- The envelope and phase of $r_1(t)$ are given by

$$\left|r_1(t)\right| = \sqrt{\left(\alpha A_c\left[1 + m_a s_n(t)\right] + n_c(t)\right)^2 + n_s^2(t)}$$

$$\varphi(t) = \tan^{-1}\frac{n_s(t)}{\alpha A_c\left[1 + m_a s_n(t)\right] + n_c(t)}$$



Envelope Detection Performance (contd)

- Under high input CNR conditions, the signal component is much larger than the noise. That is,

$$P\left\{\left|\alpha A_c\left[1 + m_a s_n(t)\right] + n_c(t)\right| >> \left|n_s(t)\right|\right\} \approx 1$$

- Then with high probability

$$\left|r_1(t)\right| \cong \alpha A_c\left[1 + m_a s_n(t)\right] + n_c(t)$$

- The demodulated signal after envelope detection and DC removal is given by

$$y_D(t) = \alpha A_c m_a s_n(t) + n_c(t)$$

Identical to the output of the coherent demodulator

- Thus, the noise performance of coherent and envelope detectors is the same under high-CNR conditions

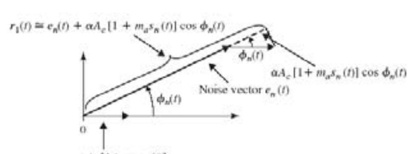
Envelope Detection Performance: Low CNRs

- Under low-input CNR conditions, the amplitude $\alpha A_c[1 + m_a s_n(t)]$ of the signal component is much smaller the noise amplitude

$$e_n(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

- From the phasor diagram in Figure, it is easy to write the following approximate expression for $r_1(t)$:

$$r_1(t) \cong e_n(t) + A_c[1 + m_a s_n(t)]\cos\phi_n(t)$$



Envelope Detection Performance: Low CNRs (contd)

- Under low-input CNR conditions, the signal term is multiplied by a random noise term $\cos\phi_n(t)$
- Since $\phi_n(t)$ is randomly varying in the range $-\pi$ to π, it is therefore not possible to recover the signal component
- As the CNR decreases from a high value, a **threshold** is reached
  - The term *threshold* signifies that for input CNR values above the threshold CNR, the output (demodulated) SNR is linearly related to the input (predetection) CNR
- However for CNR values below threshold, the output SNR decreases more rapidly than the input CNR
- The threshold in a conventional AM system occurs when CNR_{PD} is on the order of 10 dB or less, where CNR_{PD} is CNR at the envelope detector input

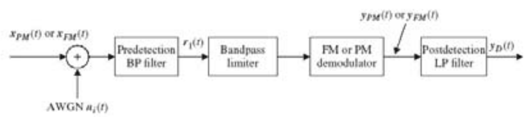
Performance of Angle-Modulation Systems

- The block diagram of a receiver for an arbitrary angle-modulated signal is displayed in Figure
- The predetection filter bandwidth is $BT \approx 2(D+1)B$ Hz, where B is the bandwidth of the message signal and D is the deviation ratio
- The input to the predetection filter is an angle-modulated random signal

$$\boldsymbol{x}(t) = \alpha A_c \cos\left[2\pi f_c t + \boldsymbol{\theta}(t)\right]$$

$$\text{PM:} \quad \boldsymbol{\theta}(t) = \Delta\phi_{\max} \boldsymbol{s}_n(t)$$

$$\text{FM:} \quad \boldsymbol{\theta}(t) = 2\pi \Delta f_{\max} \int_{-\infty}^{t} \boldsymbol{s}_n(\alpha) d\alpha$$



Performance of Angle-Modulation Systems (contd)

- The signal is embedded in the AWGN $\boldsymbol{n}_i(t)$ of zero mean and double-sided power spectral density $G_{n_i}(f) = N_o/2$
- The receiver input CNR is defined as

$$CNR_{IN} = \frac{\text{Power in the FM signal at the receiver input}}{\text{Input noise power in the message signal bandwidth}} = \frac{P_R}{N_o B} = \frac{\alpha^2 A_c^2}{2N_o B}$$

- The predetection filter output can be written using amplitude-phase representation of the narrowband noise as

$$\boldsymbol{r}_1(t) = \alpha A_c \cos\left[2\pi f_c t + \boldsymbol{\theta}(t)\right] + \boldsymbol{n}(t)$$
$$= \alpha A_c \cos\left[2\pi f_c t + \boldsymbol{\theta}(t)\right] + \boldsymbol{e}_n(t)\cos\left[2\pi f_c t + \boldsymbol{\phi}_n(t)\right]$$

- Under high-CNR conditions, $P\left\{\alpha A_c >> \boldsymbol{e}_n(t)\right\} \approx 1$

Angle-Modulation Systems: High SNRs

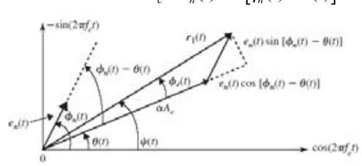
- The predetection filter output, as evident from the figure, can be approximated as

$$\boldsymbol{r}_1(t) \cong \left\{\alpha A_c + \boldsymbol{e}_n(t)\cos\left[\boldsymbol{\phi}_n(t) - \boldsymbol{\theta}(t)\right]\right\}\cos\left[2\pi f_c t + \boldsymbol{\psi}(t)\right]$$

where

$$\boldsymbol{\psi}(t) = \boldsymbol{\theta}(t) + \boldsymbol{\phi}_e(t)$$

$$\boldsymbol{\phi}_e(t) = \tan^{-1}\frac{\boldsymbol{e}_n(t)\sin\left[\boldsymbol{\phi}_n(t) - \boldsymbol{\theta}(t)\right]}{\alpha A_c + \boldsymbol{e}_n(t)\cos\left[\boldsymbol{\phi}_n(t) - \boldsymbol{\theta}(t)\right]} \cong \frac{\boldsymbol{e}_n(t)\sin\left[\boldsymbol{\phi}_n(t) - \boldsymbol{\theta}(t)\right]}{\alpha A_c}$$



Angle-Modulation Systems: High SNRs (contd)

- The amplitude of $\boldsymbol{r}_1(t)$ is irrelevant because the BP limiter removes such information prior to detection
- Observe that, under high-SNR conditions, the phase $\boldsymbol{\psi}(t)$ of the predetection filter output is the sum of two terms
  - Phase of the transmitted signal $\boldsymbol{\theta}(t)$
  - Undesirable random phase modulation $\boldsymbol{\phi}_e(t)$ due to noise
- As a result, the phasor randomly fluctuates near the signal phasor
  - These random phase fluctuations are translated by the discriminator into the noise in the demodulated signal
- The magnitude of random phase modulation term $\boldsymbol{\phi}_e(t)$ is inversely related to the signal amplitude αA_c. The higher the signal level, the lower the noise level $\Rightarrow$ **noise suppression**

Angle-Modulation Systems: High SNRs (contd.)

- The discriminator output can be expressed as

$$\text{PM:} \quad \boldsymbol{y}_{PM}(t) = K_{PD}\boldsymbol{\psi}(t) = K_{PD}[\boldsymbol{\theta}(t) + \boldsymbol{\phi}_e(t)] = K_{PD}\Delta\phi_{\max}\boldsymbol{s}_n(t) + \boldsymbol{n}_{PM}(t)$$

$$\text{FM:} \quad \boldsymbol{y}_{FM}(t) = \frac{K_{FD}}{2\pi}\frac{d\boldsymbol{\psi}(t)}{dt} = \frac{K_{FD}}{2\pi}\frac{d\boldsymbol{\theta}(t)}{dt} + \boldsymbol{n}_{FM}(t) = K_{FD}\Delta f_{\max}\boldsymbol{s}_n(t) + \boldsymbol{n}_{FM}(t)$$

where setting $\boldsymbol{\theta}(t) = 0$, we have

$$\boldsymbol{n}_{PM}(t) = K_{PD}\boldsymbol{\phi}_e(t) = K_{PD}\frac{\boldsymbol{e}_n(t)\sin\left[\boldsymbol{\phi}_n(t)\right]}{\alpha A_c} = \frac{\boldsymbol{n}_s(t)}{\alpha A_c}$$

$$\boldsymbol{n}_{FM}(t) = \frac{K_{FD}}{2\pi}\frac{d\boldsymbol{\phi}_e(t)}{dt} = \frac{K_{FD}}{2\pi\alpha A_c}\frac{d\boldsymbol{n}_s}{dt}$$

PM Noise Performance: High SNRs

- Signal power at the PM demodulator output:

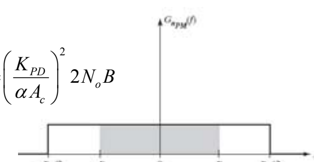
$$P_{out} = E\left\{K_{PD}^2\boldsymbol{\theta}^2(t)\right\} = E\left\{K_{PD}^2\left(\Delta\phi_{\max}\right)^2\boldsymbol{s}_n^2(t)\right\} = \left(K_{PD}\Delta\phi_{\max}\right)^2\overline{\boldsymbol{s}_n^2}$$

- Noise PSD at the at the PM demodulator output:

$$G_{n_{PM}}(f) = \begin{cases}\left(\dfrac{K_{PD}}{\alpha A_c}\right)^2 N_o, & |f| \le B_T/2 \\ 0, & \end{cases}$$

$$P_{n_{PM}} = \int_{-B}^{B} G_{n_{PM}}(f)df = \left(\frac{K_{PD}}{\alpha A_c}\right)^2 2N_o B$$

Spectrum of PM postdetection noise



PM Noise Performance: High SNRs

- Output (postdetection) SNR:

$$SNR_{PM} = \frac{\text{Signal power at the discriminator output}}{\text{Noise power at the discriminator output}} = \frac{P_{out}}{P_{n_{PM}}}$$
$$= (\Delta\phi_{\max})^2 \overline{s_n^2} CNR_{IN}$$

- The figure of merit of a PM system is given by

$$\frac{SNR_{PM}}{CNR_{IN}} = (\Delta\phi_{\max})^2 \overline{s_n^2}$$

- The output SNR is proportional to the square of the modulation index $\Delta\phi_{\max}$. Therefore, increasing $\Delta\phi_{\max}$ increases the PM output SNR

FM Noise Performance: High SNRs

- Signal power at the FM demodulator output:

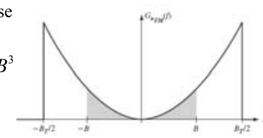
$$P_{out} = E\left\{\left(\frac{K_{FD}}{2\pi}\frac{d\theta(t)}{dt}\right)^2\right\} = (K_{FD}\Delta f_{\max})^2 E\{s_n^2(t)\} = (K_{FD}\Delta f_{\max})^2 \overline{s_n^2}$$

- Noise PSD at the at the FM demodulator output:

$$G_{n_{FM}}(f) = \begin{cases}\left(\frac{K_{FD}}{\alpha A_c}\right)^2 f^2 N_o, & |f| \le B_T/2 \\ 0, & \text{otherwise}\end{cases}$$

$$P_{n_{FM}} = \int_{-B}^{B} G_{n_{FM}}(f)df = \frac{2}{3}\left(\frac{K_{FD}}{\alpha A_c}\right)^2 N_o B^3$$

Spectrum of FM postdetection noise



FM Noise Performance: High SNRs

- The discriminator output noise PSD has a parabolic shape, that is, noise power increases as f^2
- $\Rightarrow$ the higher frequency components in the signal are subjected to higher noise levels than the lower frequency components
- FM output SNR (at the postdetection LP filter output) :

$$SNR_{FM} = \frac{\text{Signal power at the discriminator output}}{\text{Noise power at the discriminator output}} = \frac{P_{out}}{P_{n_{FM}}}$$
$$= 3\left(\frac{\Delta f_{\max}}{B}\right)^2 \overline{s_n^2}\frac{(\alpha A_c)^2}{2N_o B} = 3D^2\overline{s_n^2}\underbrace{\frac{P_R}{N_o B}}_{CNR_{IN}}$$
$$= 3D^2 \overline{s_n^2} CNR_{IN} \qquad (*)$$

FM Noise Performance: High SNRs

- (*) states that the output SNR in an FM system can be increased without bound by increasing the deviation ratio *D*.
  - Doubling of the deviation ratio improves the FM output SNR by 6 dB
- However, the transmission bandwidth requirement also increases, according to Carson's rule, as *D* is increased
- There is a limiting value of *D* after which the noise power becomes too large (due to increased bandwidth) and the threshold effect occurs
- Above threshold, the figure of merit of an FM system is given by

$$\frac{SNR_{FM}}{CNR_{IN}} = 3D^2 \overline{s_n^2}$$

SNR improvement in exchange for additional transmission bandwidth

FM Noise Performance: Low SNRs

- To analyze threshold effect, it is convenient to define CNR at the discriminator input (that is, predetection filter output)

$$CNR_{PD} = \frac{\text{Power in the FM signal at the predetection filter output}}{\text{Noise power in the predetection filter bandwidth } B_T}$$
$$= \frac{P_R}{N_o B_T} = \frac{(\alpha A_c)^2}{2N_o B_T}$$

- Since the transmission bandwidth $B_T = 2B(1+D)$, we have

$$CNR_{PD} = \frac{P_R}{N_o B_T} = \frac{P_R}{N_o 2B(1+D)} = \frac{CNR_{IN}}{2(1+D)}$$

- If the CNR at the discriminator input is low ($CNR_{PD} << 1$), it implies that the carrier amplitude is much smaller than the noise amplitude most of the time, that is, $P\{\alpha A_c << e_n(t)\} \approx 1$

FM Noise Performance: Low SNRs (contd)

- From the phasor diagram, we observe that

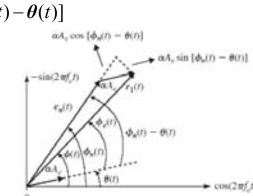
$$e_n(t)\sin[\phi_n(t) - \psi(t)] \cong \alpha A_c \sin[\phi_n(t) - \theta(t)]$$

- For $\phi_n(t) - \psi(t)$ small,

$$\sin[\phi_n(t) - \psi(t)] \approx \phi_n(t) - \psi(t)$$

or

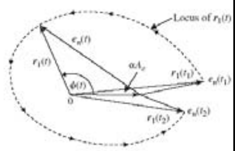
$$\psi(t) = \phi_n(t) - \frac{\alpha A_c}{e_n(t)}\sin[\phi_n(t) - \theta(t)]$$



- We observe that the phase $\psi(t)$ of the predetection filter output does not contain a distinct signal term that can be easily separated from the noise $\Rightarrow$ threshold condition

FM Threshold

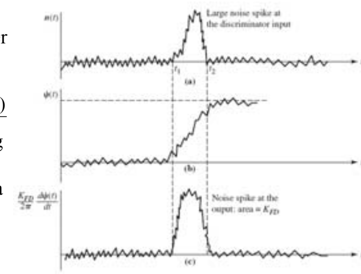
- Under low SNR conditions ($CNR_{PD} < 1$ and ↓), the magnitude of the noise phasor $\boldsymbol{e}_n(t)$ may exceed that of the carrier phasor αA_c
- The trajectory of the end point of the resultant vector $\boldsymbol{r}_1(t)$ follows phase variations of the noise and may occasionally encircle the origin resulting in changes of 2π in $\psi(t)$ over a short interval ([t_1, t_2] in the Figure)
- Figure illustrates the example when the trajectory of $\boldsymbol{r}_1(t)$ encircles the origin



FM Threshold (contd)

- The variation of the phase angle at or near the time of occurrence of this event is shown in Figure

- Noise spike changes $\psi(t)$ by 2π radians over the interval [t_1, t_2]
- The output of the discriminator $\frac{K_{FD}}{2\pi}\frac{d\psi(t)}{dt}$ changes by K_{FD} during the same interval, and the change appears as a **spike** in the output – can be heard as crackling or clicking sound



FM Threshold (contd)

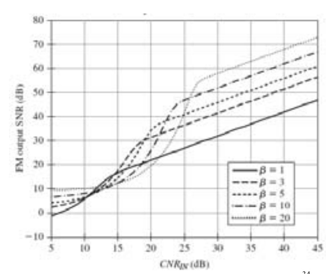
- For the case of sinusoidal modulation, a generalized expression which describes the behavior of output SNR near the threshold is given by

$$SNR_{FM} = \frac{\frac{3}{2}\beta^2 CNR_{IN}}{1+\frac{12\beta}{\pi} CNR_{IN} e^{-\frac{CNR_{IN}}{2(1+\beta)}}} \xrightarrow[\text{Threshold}]{\text{Above}} \frac{3}{2}\beta^2 CNR_{IN}$$

- The performance of FM systems deteriorates rapidly as CNR_{IN} falls below the threshold value (see Figure)
  - The threshold CNR_{IN} values depend on the choice of modulation index β
  - Larger values of β require larger values of CNR_{IN} to operate above the threshold

FM output SNR as a function of CNR_{IN}

- This sets a practical limit on maximum β that can be used to improve the FM SNR before the available CNR_{IN} falls below the threshold



FM Threshold Condition

- The onset of threshold occurs when $CNRPD \approx 10$ dB
- An equivalent condition for the onset of threshold in an FM system can be expressed as

$$(CNR_{IN})_{th} = (CNR_{PD})_{th}\, 2(D+1) \approx 20(D+1) \qquad (*)$$

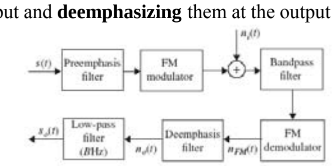
- (*) specifies the largest value of D that can be used for a given CNR_{IN} so that the system will operate above threshold
- Substituting (*) into SNR_{FM} expression (Slide 28) yields

$$(SNR_{FM})_{th} = 60D^2(D+1)\overline{s_n^2} \qquad (**)$$

- (**) provides the maximum value of deviation ratio D that can be used for a given SNR_{FM} specification so that the system will operate above threshold

Preemphasis and Deemphasis Filtering

- The noise PSD at the output of an FM discriminator has a parabolic shape, that is, noise power increases as f^2
- Therefore, an FM system exhibits better SNR performance for low-frequency components in the signal than for high-frequency components
- The noise performance of an FM system can be improved by **preemphasizing** the high frequencies in the message signal at the modulator input and **deemphasizing** them at the output of the demodulator



Preemphasis and Deemphasis (contd)

- The combination of preemphasis and deemphasis operations has no effect on the message signal; Deemphasis, however, attenuates high-frequency components in parabolic-shaped noise
- Noise power output at the discriminator output:
  - No deemphasis filtering: $P_{n_{FM}} = \int_{-B}^{B} G_{n_{FM}}(f)df$
  - With deemphasis filtering: $P_{n_o} = \int_{-B}^{B} |H_{DE}(f)|^2 G_{n_{FM}}(f)df$
- Assume single-pole LP deemphasis filter

$$H_{DE}(f) = \frac{1}{1 + j(f/f_1)}$$

Time constant

$f_1 = 1/2\pi\tau, \ \tau = RC$

3-dB frequency

Preemphasis and Deemphasis (contd)

- FM demodulator output noise with deemphasis filtering:

$$P_{n_o} = 2\left(\frac{K_{FD}}{A_c}\right)^2 N_o f_1^2 B$$

- FM demodulator output SNR with deemphasis filtering

$$(SNR_{FM})_{DE} = D^2 \left(\frac{B}{f_1}\right)^2 \overline{s_n^2} CNR_{IN}$$

- SNR improvement due to deemphasis

$$\text{SNR improvement} = \frac{(SNR_{FM})_{DE}}{SNR_{FM}} = \frac{D^2 \left(\frac{B}{f_1}\right)^2 \overline{s_n^2} CNR_{IN}}{3D^2 \overline{s_n^2} CNR_{IN}} = \frac{(B/f_1)^2}{3}$$

Preemphasis and Deemphasis (contd)

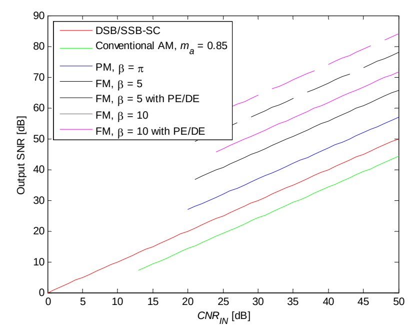
- In dB form, we have

$$\text{SNR improvement due to deemphasis} = 20\log_{10}(B/f_1) - 10\log_{10} 3 = 20\log_{10}(B/f_1) - 4.77 \text{ dB}$$

- **Commercial FM broadcasting**
  - $B = 15$ kHz, $\tau = 75\ \mu$sec
  - This yields $f_1 = 2.1$ kHz and $B/f_1 = 7.14$
  - Therefore, SNR improvement due to deemphasis

$$= 20\log_{10}(7.14) - 4.77 = 12.3 \text{ dB}$$

SNR Performance for Analog Modulation Schemes: Sinusoidal Modulating Signal



Performance Comparison of Analog Modulation Systems

Type of Modulation	Transmission Bandwidth	SNR Advantage	Equipment Complexity	Comment
Baseband	B	1	Minor	Short point-to-point link
DSB-SC	$2B$	1	Moderate; coherent demodulator required	
Conventional AM	$2B$	$\eta \le 1$	Minor; envelope detector is used	Low-cost receiver for broadcast application
SSB	B	1	Major; coherent demodulator required	
VSB	$B + f_v,\ f_v/B \approx 0.2 - 0.3$	1	Major; coherent demodulator required	Complex VSB filters
VSB + Carrier	$B + f_v,\ f_v/B \approx 0.2 - 0.3$	$\eta \le 1$	Moderate; envelope detector is used	Low-cost receiver for broadcast application
PM	$2(\Delta\phi_{max} + 1)B$	$(\Delta\phi_{max})^2 \overline{s_n^2}$	Moderate	$\Delta\phi_{max} \le \pi$ for certain modulation signal types
FM	$2(D + 1)B$	$3D^2 \overline{s_n^2}$	Moderate	CNR_{IN} above threshold value
FM with preemphasis	$2(D + 1)B$	$D^2 \overline{s_n^2} \left(\frac{B}{f_1}\right)^2$	Moderate; f_1 = 3 dB frequency of the deemphasis filter	CNR_{IN} above threshold value

Link Design

- Two factors set limit for the maximum link length
  - Signal attenuation
  - Additive noise
- The signal in a communications system suffers attenuation and distortion as it propagates along a communications link
  - The distortion of the signal results from the frequency-selective characteristics of the transmission medium
- Signal attenuation renders the communication signal more vulnerable to additive noise
- The minimum value of received power level (P_R) is a function of the SNR performance specification and varies with the modulation scheme used
- The loss budget, also called the **system gain**, of a point-to-point link is given by

Link Design (contd)

$$\text{System Gain (dB)} = P_T - P_R$$

P_T = transmitter output power level

- The system gain is allocated to transmission losses and link margin provided for temperature and aging effects.
- For wired media, the attenuation is a linear function of link length. Therefore, we can write

$$\text{System Gain (dB)} = P_T - P_R = \alpha . L + \text{link\_margin}$$

α = link attenuation in dB/km

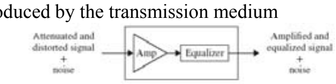
- The maximum link length can now be calculated using

$$\text{Link length } L = \frac{P_T - P_R - \text{link\_margin}}{\alpha} \text{ km}$$

Analog Repeater

- To transmit over longer distances, it is necessary to introduce **repeaters** periodically to compensate for the attenuation and distortion of the signal, as shown in Figure

- A repeater consists of an amplifier and an **equalizer** as shown.
  - The amplifier boosts the signal level to make up for the attenuation of the signal in the previous repeater section
  - The equalizer attempts to compensate for the distortion introduced by the transmission medium



Noise Contributed by Amplifiers

- All electronic amplifiers also add noise. The noise contributed by an amplifier is characterized by the noise figure F
- The available noise output power of the amplifier with gain $\mathscr{G}$ is given by

$$N_{out} = F\mathscr{G}N_oB_N \text{ Watts}$$

or in dB form

$$N_{out} = NF + G + 10\log_{10}(N_oB_N) \text{ dBW}$$

where

$G = 10\log_{10}\mathscr{G}$ = Amplifier gain in dB

$NF = 10\log_{10}F$ = Amplifier noise figure in dB

B_N = Noise-equivalent bandwidth of the amplifier

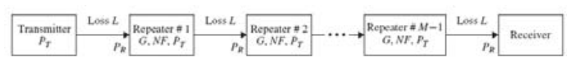
Performance of Link Using Cascade of Repeaters

- Consider an analog communication system consisting of M repeater sections in cascade as shown in Figure
- We assume that each repeater's gain makes up for the loss introduced by the associated cable section, i.e., $L = G$ (dB)
- The available noise output power of the first amplifier is

$$(N_{out})_1 = F\mathscr{G}N_oB_N \text{ Watts}$$

- The CNR at the output of the first repeater is given by

$$(CNR_{out})_1 = \frac{P_T}{(N_{out})_1} = \frac{P_T}{F\mathscr{G}N_oB_N}$$



Link Using Cascade of Repeaters (contd)

- The noise power output from the last repeater is simply M times the noise power output of the first. That is,

$$(N_{out})_M = M(N_{out})_1 = MF\mathscr{G}N_oB_N$$

- The output CNR of an analog communication system consisting of M repeater spans is, therefore, given by

$$(CNR_{out})_M = \frac{P_T}{MF\mathscr{G}N_oB_N} = \frac{(CNR_{out})_1}{M}$$

or in dB form

$$(CNR_{out})_M = (CNR_{out})_1 - 10\log_{10}M$$

- The output CNR in an analog communication system with M repeaters suffers a **penalty** of $10\log_{10}M$ dB