Chapter 8 Discrete time system Description

8.1 systems

$$y(k) = \mathcal{O}_k\{x(k)\}$$

Example: unit delay

$$y(k) = \mathcal{O}_k\{x(k)\} = x(k-1)$$

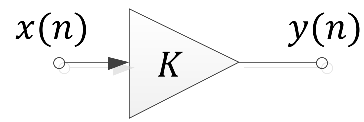
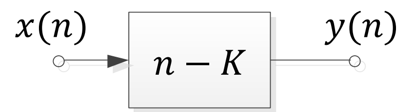
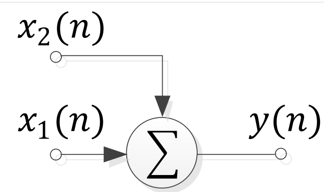
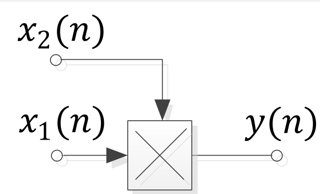
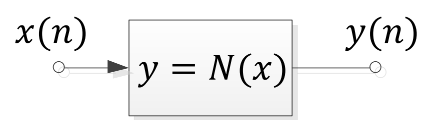
Name	*Transfer characteristic*	*Symbol*
Proportion	$y(n) = Kx(n)$	
Shift	$y(n) = x(n-k)$	
Adder	$y(n) = x_1(n) + x_2(n)$	
Multplier	$y(n) = x_1(n)x_2(n)$	
Nonlinear static element	$y(n) = N[x(n)]$	

Figure 8-1 Elementary digital systems

8.2 Classifications of discrete systems

1. Linear system

*observes the principle of superposition, the system parameters are independent of the magnitudes of the inputs described by linear difference equations.

Superposition principle: the operation to a weighted sum of inputs is the weighted sum of operations to each input.

If

Then

or equivalently

$$\mathcal{O}\left\{\sum_i a_i x_i(k)\right\} = \sum_i a_i \mathcal{O}\{x_i(k)\}$$

*Homogeneity

$$\mathcal{O}\{a x_i(k)\} = a\mathcal{O}\{x_i(k)\}$$

*additivity

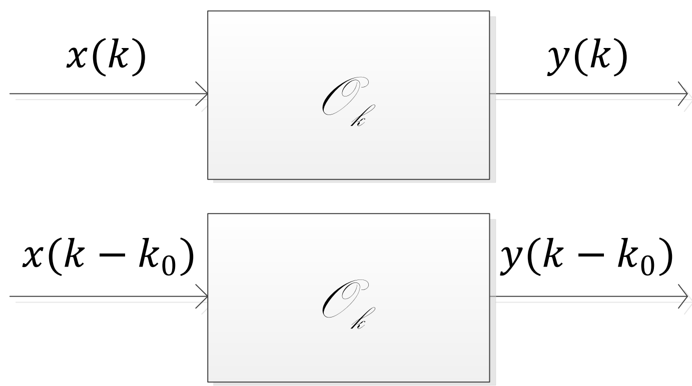
$$\mathscr{O}\left\{\sum_i x_i(k)\right\} = \sum_i \mathscr{O}\{x_i(k)\}$$

$$* \boldsymbol{linearity = homogeneity + additivity}$$

2. **Time-invariant systems**

* observes <u>principle of time-invariance</u>, the system parameter are independent of time variables, described by constant-coefficient difference equations.
  Principle of time-invariance: the time-shift in input causes the same time-shift in output.

  If



$$\boldsymbol{for\ any\ k_0}$$

Examples: check the linearity

(1).	$y(k) = x(k) - x(k-1)$	**yes**
(2).	$y(k) = \sum_n x(n)h(k-n)$	**yes**
(3).	$y(k) = \sin[x(k)]$	**no**
(4).	$y(k) = x^2(k)$	**no**
(5).	$y(k) = 3x(k) + 4$	**no**
(6).	$y(k) = \lvert x(k) \rvert \quad full - wave\ rectifier$	**no**

Examples: Check the time-invariance

(1).	$y(k) = 5x(k)$	**yes**
(2).	$y(k) = x(k) - x(k-1)$	**yes**
(3).	$y(k) = x(-k)$	**no**
(4).	$y(k) = kx(k)$	**no**

3. Instantaneous/non-instantaneous systems

* Instantaneous system – the output an any instant depends on the input at that instant only,
  - memoryless

  Example:

$$y(k) = ax(k)$$

* Non-instantaneous system – with memory

  Example:

$$y(k) = \sum_{n=-\infty}^{k} h(k-n)x(n)$$

  The output at k depends on the input for all it, in $\{-\infty, k\}$

* Can memoryless system operation be expressed as "convolution summation" ?

  Example:

$$y(k) = ax(k)$$

$$= \sum_{n=-\infty}^{k} a\delta(k-n)x(n)$$

$$= h(k) * x(k)$$

$$With\ h(k) = a\delta(k)$$

4. Causal/Non causal systems

* causal system – Non anticipative system.
  - The current output of the system depends on current and/or past input of the systems, **NOT** the future input.

$$\text{If} \quad x(t) = 0 \; for \; t \leq t_0$$
$$\text{then} \quad y(t) = 0 \; for \; t \leq t_0$$

* Non causal system/Anticipative system
  - The current output of the system may depends on future input as well as current and/or past input.

* ”Anti-causal” system – The current output depends on future and current input, **NOT** past input.

* ”Off-line” system may be modeled as non-causal since “future” input is available.

Examples:

(1). $y(k) = \sum_{n=-\infty}^{k} h(k-n)x(n)$

(2). $y(k) = \sum_{n=-\infty}^{k+1} h(k-n)x(n)$

(3). $y(k) = x(-k)$

(4). $y(k) = x(k-1) + x(k+1)$

Answers:

(1). Causal

(2). Non causal

(3). Non causal

(4). Non causal

Memoryless systems are causal, but not vice versa.
Causal system can be (and usually are) with memory.

5. Stable/Unstable systems

* stability – many definations
  BIBO – A system with a bounded input produces only bounded output

$$for\ |x(k)| < \infty, we\ have\ |y(k)| < \infty.$$

* Unstable system – Bounded input produces unbounded output.

* In a practical system, no signal can grow without limit, but variable can reach magnitudes that can overload the system and may cause physical damage.

* ***<u>Examples:</u>***

  Resistor **(Stable)**

$$v = i{\cdot}R\ if\ \ i\ is\ bounded\ then\ v\ is\ bounded$$

  Capacitor **(Unstable)**

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)u(t)dt = \left(\frac{i(t)}{C}\right)r(t)$$

  (In this sense)

6. Invertible system

∗ Input signal x(k) can be determined uniquely by observing output signal y(k), i.e. if and only if distinct input produce distinct output!

∗ ***Examples:***

(1). $y(k) = x^3(k)$

(2). $y(k) = 2x(k + 1) + 3$

(3). $y(k) = x^2(k)$

(4). $y(k) = \sin[x(k)]$

∗ ***Answers:***

(1)and(2) are invertible

(3)and(4) are non-vertible